

Self-Fulfilling Debt Crises: A Quantitative Analysis*

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Abstract

We use a benchmark model of sovereign debt to measure the importance of belief-driven fluctuations in sovereign bonds markets. The model features debt maturity choices, risk averse lenders and rollover crises á la Cole and Kehoe (2000). In this environment, lenders' expectations of a default can be self-fulfilling, and their beliefs contribute to variation in interest rate spreads along with economic fundamentals. We identify these different sources of variation in interest rate spreads through the maturity choices of the government. When high interest rates are due to rollover risk, the government has incentives to lengthen its debt maturity in order to protect himself from these inefficient runs. A shortening of debt maturity, instead, is an optimal response when the government is facing a fundamental debt crisis. In an application, we fit the model to the Italian debt crises of 2008-2102. We estimate that rollover risk accounted for 23% of the movements in sovereign bond yields over this episode. Our results have implications for the effects of the liquidity provisions established by the European Central Bank during the summer of 2012.

Keywords: Sovereign Debt Crises, Rollover Risk, Maturity Choices, Risk Premia.

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1 Introduction

The idea that lenders' pessimistic beliefs about the solvency of a government can be self-fulfilling has been often used by economists to explain fluctuations in sovereign bonds yields. For instance, it has been a common explanation for the sharp increase in interest rate spreads of southern European economies in 2011, and for their subsequent decline upon the introduction of the OMT bond-purchasing program by the European Central Bank.¹ According to this view, the ECB interventions were desirable because they eliminated non fundamental fluctuations in bonds markets, protecting members of the euro-area from inefficient self-fulfilling crises.

Unfortunately, testing this hypothesis in the data is challenging, and this makes the evaluation of these "lender of last resort" type of policies problematic. The high interest rate spreads observed in southern Europe, for example, could have been due purely to bad economic fundamentals. In this second view, the fall in bond yields after the introduction of OMT would be evidence that the program implicitly offered bailouts guarantees for peripheral countries, guarantees that were priced by bondholders. This latter interpretation would lead to a less favorable assessment of the ECB intervention: implicit bailouts guarantees may induce governments to over borrow and they may introduce balance sheet risk for the central bank.

The main contribution of this paper is to bring a benchmark model of sovereign borrowing with self-fulfilling debt crises to the data, and to show how it can be used to disentangle fundamental and non-fundamental fluctuations in interest rate spreads. In our set-up, these two sources of default risk have opposing implications for the debt maturity structure chosen by the government. Our identification strategy consists in using these model's restrictions, along with observed maturity choices, to infer the likelihood of a self-fulfilling crisis.² We fit our model to the recent debt crisis in Italy (2008-2012), finding that non-fundamental risk accounted for 23% of the observed fluctuations in interest rate spreads. We then use our estimates to test whether the decline in interest rate spreads observed after the establishment of OMT was due to the elimination of their non-fundamental component, or whether it reflected implicit bailouts guarantees for southern

¹*Outright Monetary Transactions* (OMT), introduced in September 2012, allowed the ECB to purchases of sovereign bonds in secondary markets without explicit quantity limits. See Section 7 for details.

²While novel in the sovereign debt literature, the idea of using economic choices to learn about the types of risk agents are facing has been exploited in several other contexts. A classic example is the use of consumption data along with the logic of the permanent income hypothesis to separate between permanent and transitory income shocks. See [Cochrane \(1994\)](#) for an application on U.S. aggregate data, [Aguiar and Gopinath \(2007\)](#) for emerging markets, and [Guvenen and Smith \(2014\)](#) for a recent study using micro data.

European countries.

We consider the canonical model of sovereign borrowing in the tradition of [Eaton and Gersovitz \(1981\)](#), [Aguiar and Gopinath \(2006\)](#) and [Arellano \(2008\)](#). In our environment, a government issues debt of multiple maturities in order to smooth out endowment risk. The government lacks commitment over future policies and, as in [Cole and Kehoe \(2000\)](#), it cannot commit to repay the debt within the period. This last assumption leads to self-fulfilling debt crises: if lenders expect a default and do not buy new bonds, the government may find it too costly to service the stock of debt coming due, thus validating lenders' expectations. This can happen despite the fact that a default would not be triggered if lenders were holding more optimistic expectations about repayments. These rollover crises can arise in the model when the stock of debt coming due is sufficiently large and economic fundamentals are sufficiently weak.

As commonly done in the literature, we assume that this indeterminacy is resolved by the realization of a coordination device. In our set up, default risk varies over time because of "fundamental" and "non-fundamental" uncertainty. Specifically, default risk may be high because lenders expect the government to default in the near future irrespective of their behavior. Or, it may be high because of the expectation of a future rollover crisis. The goal of our analysis is to distinguish these different sources of default risk.

Our identification strategy consists in looking at the debt maturity choices of governments in periods of high interest rates. The reason why maturity choices provide information on the sources of default risk builds on basic properties of the canonical sovereign debt model. When choosing debt maturity, the government has to weight the contribution of three different forces: its lack of commitment, the incompleteness of the debt contracts, and the risk of roll-over crises.

Consider now a situation where high interest rates reflect mostly the prospect of a future rollover crisis. In this case, the government would optimally choose to *lengthen* its debt maturity. By back-loading payments, in fact, the government reduces the stock of debt that needs to be rolled over in the near future, minimizing in this fashion the possibility of a "run" on its debt. Hence, if high interest rates today are due to the expectation of future self-fulfilling crises, we should observe the government to actively increase the maturity of its debt.

On the contrary, the government would *shorten* the maturity of its debt when high interest rates are due to its fundamental inability to commit to future repayments. A shortening of the maturity structure of debt allows the government to improve the terms at which it borrows from lenders, and this is particularly valuable when the former is

facing high borrowing rates. As emphasized in [Arellano and Ramanarayanan \(2012\)](#) and [Aguilar and Amador \(2014b\)](#), this happens because short term debt is a better instrument for disciplining the borrowing behavior of future governments,³ and it allows the current government to reduce its time inconsistency problem over future policies. This makes the government better off from an *ex-ante* perspective. These gains are not counteracted by losses due to a decrease in the insurance that long term debt provides.⁴ As [Dovis \(2014\)](#) shows in a related environment, the higher ex-post volatility of bond prices attained in periods of high default risk allows the government to obtain the same amount of insurance with a shorter maturity structure of debt.

While this logic suggests that the behavior of debt maturity could be used to identify the sources of default risk, confounding factors may limit the feasibility of this approach. The relationship between interest rate spreads and debt maturity is, in fact, not only a product of government's incentives, but it also depends on lenders' attitude toward risk. [Broner, Lorenzoni, and Schmukler \(2013\)](#) document that risk premia over long term bonds typically increase during sovereign crises. Neglecting these shifts could undermine our identification strategy: rollover risk could be driving interest rate spreads and yet we could observe a shortening of debt maturity simply because lenders are not willing to hold long term risky bonds. To address this issue, we allow for time-varying term premia by introducing shocks to the lenders' stochastic discount factor, specifically the exponentially Gaussian approach of [Ang and Piazzesi \(2003\)](#).

We apply our framework to the recent sovereign debt crisis in Italy. We calibrate the lenders' stochastic discount factor by matching the behavior of risk premia on long term German's zero coupon bonds, measured using the [Cochrane and Piazzesi \(2005\)](#) predictive regressions. Implicit in our approach is the assumption that financial markets in the euro area are sufficiently integrated and that the lenders in our model are the marginal investors for other assets beside Italian government securities. The parameters of the government's decision problem are calibrated to match the cyclical behavior of Italian public debt, interest rate spreads and real economic activity over our sample. Our calibration strategy differ from previous work in the area and it is able to deliver high debt levels (100% of annual GDP), a countercyclical debt issuances, and key moments of the interest rate spreads distribution observed in our sample period (1999:Q1-2012:Q2).

³A government entering the period with mostly short term debt has less incentives to borrow because the associated increase in interest rates are applied on a larger fraction of the stock of debt.

⁴Long term debt provides insurance for the government because capital gains and losses imposed on holders of long term debt can approximate wealth transfers associated with state contingent securities, as the market value of debt falls when the marginal utility of the government is high. See [Angeletos \(2002\)](#), [Buera and Nicolini \(2004\)](#) and [Debortoli, Nunes, and Yared \(2015\)](#) for a similar mechanism in an environment without default risk.

Using the calibrated model, we next decompose the observed interest rate spreads into a component reflecting the expectation of a future rollover crisis and a component due to the fundamental shocks. We document that the combination of high risk premia and bad domestic fundamentals account for most of the run-up in interest rate spreads observed during the 2011-2012 period. Moreover, we show that neglecting the information content of maturity choices results in substantial uncertainty over the split between fundamental and non-fundamental sources of default risk, as the model lacks identifying restrictions to discipline the risk of a rollover crisis.

Finally, we show how our results can be used to evaluate the OMT program. We model OMT as a price floor schedule implemented by a deep pocketed central bank. We show that the central bank can design this schedule to eliminate the possibility of rollover crises without bond purchases being carried out on the equilibrium path. This design, which results in a Pareto improvement, is our normative benchmark. We use our model to test whether the OMT program is indeed implementing this benchmark. To test for this hypothesis, we use the model to construct the “fundamental” interest rate spread that would emerge in a world without rollover crises, and we compare it with the actual Italian spread observed after the policy announcements. We find that this counterfactual spread is 100 basis points above the observed one. We interpret this as evidence that the current spread partly reflect the expectation of future bailouts on the equilibrium path.

This paper contributes to the literature on multiplicity of equilibria in sovereign debt models. Previous works in this area like [Alesina, Prati, and Tabellini \(1989\)](#), [Cole and Kehoe \(2000\)](#), [Calvo \(1988\)](#), and [Lorenzoni and Werning \(2013\)](#) have been qualitative in nature. More recently, [Conesa and Kehoe \(2012\)](#), [Aguiar, Chatterjee, Cole, and Stangebye \(2015\)](#) and [Navarro, Nicolini, and Teles \(2015\)](#) considered more quantitative models featuring multiple equilibria. To best of our knowledge, this is the first paper that conducts a quantitative assessment of the importance of rollover risk in driving interest rate spreads.⁵ The main innovation relative to the existing literature is our identification strategy based on the behavior of debt maturity around default crises.

More generally, the paper is related to quantitative analysis of sovereign debt models. Papers that are related to our work include, among others, [Arellano and Ramanarayanan \(2012\)](#), [Chatterjee and Eyigungor \(2012\)](#), [Hatchondo, Martinez, and Sosa Padilla \(2015\)](#), [Bianchi, Hatchondo, and Martinez \(2014\)](#), [Borri and Verdhelan \(2013\)](#) and [Salomao \(2014\)](#). Relative to the existing literature, our model features rollover risk, endogenous maturity choices and risk aversion on the side of the lenders. Our analysis shows that the behavior of debt duration is necessary for the identification of rollover risk, while shocks to the

⁵There is also a reduced form literature that addresses this issue, see [De Grauwe and Ji \(2013\)](#).

stochastic discount factor of the lenders are introduced to control for confounding demand factors that may undermine our identification strategy. Our modeling of the maturity choices differ from previous research and builds on recent work by [Sanchez, Sapriza, and Yurdagul \(2015\)](#) and [Bai, Kim, and Mihalache \(2014\)](#). Specifically, the government in our model issues portfolios of zero coupon bonds with an exponentially decaying duration. The maturity choice is discrete, and it consists on the choice of the decaying factor. This modeling feature simplifies the numerical analysis of the model relative to the canonical formulation of [Arellano and Ramanarayanan \(2012\)](#).

Our analysis on the effects of liquidity provisions is related to [Roch and Uhlig \(2014\)](#) and [Corsetti and Dedola \(2014\)](#). These papers show that these policies can eliminate self-fulfilling debt crisis when appropriately designed. We contribute to this literature by using our calibrated model to test whether the drop in interest rates spreads observed after the announcement of OMT is consistent with the implementation of such policy or whether it signals a prospective subsidy paid by the ECB.

Finally, our paper is related to the literature on the quantitative analysis of indeterminacy in macroeconomic models, see the contributions of [Jovanovic \(1989\)](#), [Farmer and Guo \(1995\)](#) and [Lubik and Schorfheide \(2004\)](#). The closest in methodology is [Aruoba, Cuba-Borda, and Schorfheide \(2014\)](#) who use a calibrated New Keynesian model solved numerically with global methods to measure the importance of belief-driven fluctuations for the U.S. and Japanese economy.

Layout. The paper is organized as follows. Section 2 presents the model. Section 3 discusses our key identifying restriction, and Section 4 presents an historical example supporting our approach. Section 5 describes the calibration of the model and presents an analysis of its fit. Section 6 uses the calibrated model to measure the importance of rollover risk during the Italian sovereign debt crisis. Section 7 analyzes the OMT program. Section 8 concludes.

2 Model

2.1 Environment

Preferences and endowments: Time is discrete, $t \in \{0, 1, 2, \dots\}$. The exogenous state of the world is $s_t \in S$. We assume that s_t follows a Markov process with transition matrix $\mu(\cdot | s_{t-1})$. It is convenient to split the state into two components, $s_t = (s_{1t}, s_{2t})$ where s_{1t} is the *fundamental* component and s_{2t} is the *non-fundamental* component. The fundamental

component affects endowments and preferences while the non-fundamental components are coordination devices orthogonal to fundamentals.

The economy is populated by lenders and a domestic government. The lenders value flows using the stochastic discount factor $M(s_{1t}, s_{1t+1})$. Hence the value of a stochastic stream of payments $\{d\}_{t=0}^{\infty}$ from time zero perspective is given by

$$\mathbb{E}_0 \sum_{t=0}^{\infty} M_{0,t} d_t, \quad (1)$$

where $M_{0,t} = \prod_{j=0}^t M_{j-1,j}$.

The government receives an endowment (tax revenues) $Y_t = Y(s_{1t})$ every period and decides the path of spending G_t . The government values a stochastic stream of spending $\{G_t\}_{t=0}^{\infty}$ according to

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t U(G_t), \quad (2)$$

where the period utility function U is strictly increasing, concave, and it satisfies the usual assumptions.

Market structure: The government can issue a portfolio of non-contingent defaultable bonds to lenders in order to smooth its spending G_t in front of fluctuations in tax revenues. For computational convenience, we restrict the government to issue portfolios of zero-coupon bonds (ZCB) indexed by (B_t, λ_t) for $\lambda_t \in [0, 1]$. A portfolio (B_t, λ_t) at the end of period t corresponds to a stock of $(1 - \lambda_t)^{j-1} B_t$ ZCB of maturity $j \geq 1$. The variable λ_t captures the duration of the government stock of debt: higher λ_t implies that the repayment profile is concentrated at shorter maturities. For instance, if $\lambda_t = 1$, then all the debt is due next period. The variable B_t controls for the level of debt issuance. In particular, the total face value of debt is B_t / λ_t . We let the $q_{t,j}$ be the price of a ZCB of maturity j at time t .

The timing of events within the period follows [Cole and Kehoe \(2000\)](#): the government first issues new debt, lenders choose the price of newly issued debt, and finally the government decides to default or not, $\delta_t = 0$ or $\delta_t = 1$ respectively. We assume that if the government defaults, it is excluded from financial markets and it suffers losses in output. We denote by $\underline{V}(s_{1,t})$ the value for the government conditional on a default. Lenders that hold inherited debt and the new debt just issued do not receive any repayment.⁶ Differ-

⁶This is a small departure from [Cole and Kehoe \(2000\)](#), since they assume that the government can use the funds raised in the issuance stage even if it defaults. Our formulation simplifies the problem and it should not change its qualitative features. The same formulation has been adopted in other works, for

ently from the timing in [Eaton and Gersovitz \(1981\)](#), the government does not have the ability to commit not to default within the current period. As we will see, this allows for self-fulfilling debt crisis. The budget constraint for the government when it does not default is

$$G_t + B_t \leq Y_t + \Delta_t, \quad (3)$$

where Δ_t is the net amount of resources that the government raises in the period:

$$\Delta_t = \sum_{n=1}^{\infty} q_{t,n} \left[(1 - \lambda_{t+1})^{n-1} B_{t+1} - (1 - \lambda_t)^n B_t \right]. \quad (4)$$

In the expression above we are using the fact that if a government enters the period with a portfolio (B_t, λ_t) and wants to exit the period with a portfolio (B_{t+1}, λ_{t+1}) , then it must issue additional $(1 - \lambda_{t+1})^{n-1} B_{t+1} - (1 - \lambda_t)^n B_t$ ZCB of maturity n .⁷

2.2 Recursive Equilibrium

2.2.1 Definition

We now consider a recursive formulation of the equilibrium. Let $\mathbf{S} = (B, \lambda, s)$ be the state today and \mathbf{S}' the state tomorrow. The problem for a government that has not defaulted yet is

$$V(\mathbf{S}) = \max_{\delta \in \{0,1\}, B', \lambda', G} \delta \{U(G) + \beta \mathbb{E}[V(\mathbf{S}') | \mathbf{S}]\} + (1 - \delta) \underline{V}(s_1) \quad (5)$$

subject to

$$\begin{aligned} G + B &\leq Y(s_1) + \Delta(\mathbf{S}, B', \lambda'), \\ \Delta(\mathbf{S}, B', \lambda') &= \sum_{n=1}^{\infty} q_n(s, B', \lambda') \left[(1 - \lambda')^{n-1} B' - (1 - \lambda)^n B \right], \end{aligned}$$

where $q_n(s, B', \lambda')$ is the price of a defaultable ZCB of maturity n given the realization s for the exogenous state and the government's choices for the new portfolio (B', λ') .

The lender's no-arbitrage conditions require that

$$q_n(s, B', \lambda') = \delta(\mathbf{S}) \mathbb{E} \{ M(s_1, s'_1) \delta(\mathbf{S}') q_{n-1}(s', B'', \lambda'') | \mathbf{S} \} \text{ for } n \geq 1, \quad (6)$$

instance [Aguar and Amador \(2014b\)](#).

⁷When $(1 - \lambda_{t+1})^{n-1} B_{t+1} - (1 - \lambda_t)^n B_t$ is negative the government is buying back the ZCB of maturity n . Buy backs of government securities under our formulation are necessary whenever the government wants to shorten the duration of the debt. This is an unrealistic feature of the model as buy backs are hardly observed in the data, but it allows for a greater numerical tractability.

where $B'' = B'(s', B', \lambda')$, $\lambda'' = \lambda'(s', B', \lambda')$, and the initial condition is $q_0(s', B', \lambda') = 1$. The presence of $\delta(\mathbf{S})$ in equation (6) implies that new lenders receive a payout of zero in the event of a default today.

A *recursive equilibrium* is value function for the borrower V , associated decision rules $\{\delta, B', \lambda', G\}$ and a pricing function $q = \{q_n\}_{n \geq 1}$ such that $\{V, \delta, B', \lambda', G\}$ are a solution of the government problem (5) and q satisfies the no-arbitrage conditions (6).

2.2.2 Multiplicity of equilibria and Markov selection

This economy features multiple recursive equilibria. As in [Cole and Kehoe \(2000\)](#), for intermediate levels of inherited debt, lenders' expectations of a default may be self-fulfilling while if lenders believe the government will repay and lend at positive prices the government will indeed repay.

To understand how self-fulfilling crisis can arise, consider a situation in which it is optimal for the government to repay its debt if it can issue new debt at a positive price:

$$\max_{B', \lambda'} U(Y - B + \Delta(\mathbf{S}, B', \lambda')) + \beta \mathbb{E}[V(B', \lambda', s') | \mathbf{S}] \geq \underline{V}(s_1) \quad (7)$$

for $\Delta(\mathbf{S}, B', \lambda') > 0$. Suppose now that lenders expect the government to default today. By equation (6), for any portfolio (B', λ') that the government chooses, the price of newly issued debt is zero. The lenders' expectation is validated in equilibrium if default is optimal from the government's viewpoint. This second condition is met if⁸

$$U(Y - B) + \beta \mathbb{E}[V((1 - \lambda)B, \lambda, s') | \mathbf{S}] < \underline{V}(s_1), \quad (8)$$

that is if the government finds optimal to default when it cannot issue new debt. If both (7) and (8) hold, then the default decision of the government depends on the expectations of the lenders.

It is easy to see that for all λ and s there exist intermediate values of B such that both (7) and (8) hold.⁹ Debt crisis may thus be self-fulfilling, and outcomes indeterminate in this region of the state space: lenders may extend credit to the government and there will be no default, or the lenders may not roll-over because they expect no repayment, in which case the government would find it optimal to default validating lenders' expectations.

⁸If condition (8) is not satisfied, instead, lenders' expectations cannot trigger a default. This is because even if $q = 0$, it is still optimal for the government to repay its debt. Thus, lenders have no incentive to run: it is optimal for an individual lender to lend at a positive price even if other lenders do not, and so $q = 0$ cannot be an equilibrium price.

⁹See Proposition 1 in [Aguiar and Amador \(2014a\)](#) for a formal proof.

We follow most of the literature and use a parametric rule that selects among these possible outcomes. In order to explain our selection mechanism, it is useful to partition the state space in three regions (note that such regions are endogenous and depend on the selection mechanism). Following the terminology in [Cole and Kehoe \(2000\)](#), we say that the government is in the *safe zone*, $\mathcal{S}^{\text{safe}}$, if it does not find optimal to default even if lenders do not rollover its debt. That is,

$$\mathcal{S}^{\text{safe}} = \left\{ \mathbf{S} : (8) \text{ does not hold} \right\}.$$

We say that the government is in the *crisis zone*, $\mathcal{S}^{\text{crisis}}$, if (B, λ, s) are such that it is not optimal for the government to repay debt during a rollover crisis but it is optimal to repay if the lenders roll it over. That is,

$$\mathcal{S}^{\text{crisis}} = \left\{ \mathbf{S} : (7) \text{ and } (8) \text{ hold} \right\}.$$

Finally, the residual region of the state space, the *default zone*, $\mathcal{S}^{\text{default}}$, is the region of the state space in which the government defaults on its debt regardless of lenders' behavior,

$$\mathcal{S}^{\text{default}} = \left\{ \mathbf{S} : (7) \text{ does not hold} \right\}.$$

Indeterminacy in outcomes arises only when the economy is in the crisis zone.

We consider the following selection mechanism: let the non-fundamental state be $s_2 = (\pi', \xi)$. The variable π' is the probability that there will be a rollover crisis in the next period conditional on the economy being in the crisis zone. We assume that π' follows a first order Markov process, $\pi' \sim \mu_{\pi}(\cdot | \pi)$. The variable ξ indicates whether a rollover crisis takes place in the current period. Whenever the economy is in the crisis zone, if $\xi = 0$ then lenders roll-over the debt and there is no default. If $\xi = 1$, instead, the lenders do not roll-over the government debt and there is a default. Given our discussion above, $\xi = 1$ with probability π . Note that we do not need to keep track of π to characterize the decision problem of the government and the pricing functions. In fact, the realization of ξ is all the government cares about in the current period. We have to keep track of π' because it affects the probability of a crisis zone the next period and so it affects the government's decisions and the pricing functions today.

Conditional on this selection rule, the outcome of the debt auctions are unique in the

crisis zone.¹⁰ The equilibrium outcome is a stochastic process

$$\mathbf{y} = \{\lambda(s^t, B_0, \lambda_0), B(s^t, B_0), \delta(s^t, B_0, \lambda_0), G(s^t, B_0, \lambda_0), q(s^t, B_0, \lambda_0)\}_{t=0}^{\infty}$$

naturally induced by the recursive equilibrium objects. The outcome path depends on properties of the selection, i.e. the process for $\{\pi_t\}$, and on the realization of the non-fundamental state $\{s_{2t}\}$. Hence default risk is driven by both fundamental and non-fundamental sources. In our quantitative analysis we will use information from government's choices in order to distinguish these sources of default risk. As we will argue in the next section, government's choices regarding the maturity of debt are going to be informative for our exercise.

3 Maturity Choices and Sources of Default Risk

In this section, we explain why maturity choices provide information that is useful to distinguish between fundamental and non-fundamental sources of default risk. The key insight is that if rollover risk is large then the government has an incentive to reduce the probability of being in the crisis zone next period. As first shown in [Cole and Kehoe \(2000\)](#), to achieve this objective the government can lengthen the maturity of its debt since long term debt is less susceptible to rollover risk. Hence, we should expect the government to lengthen debt maturity if rollover risk is high. On the contrary, absent rollover risk, previous research - for instance [Arellano and Ramanarayanan \(2012\)](#), [Aguiar and Amador \(2014b\)](#) and [Dovis \(2014\)](#) - has shown that a shortening of maturity is typically an optimal response of the government when facing high interest rates driven by fundamental shocks: a shortening of debt maturity around a debt crisis would then indicate a more limited role for rollover risk.

3.1 Maturity choices: Trade offs in a three-period economy

3.1.1 Three-period economy

To illustrate in the most transparent way the key trade-offs that govern the optimal maturity composition of debt we consider a three-period version of the economy. At $t = 0$ the

¹⁰However, we cannot ensure that the equilibrium value function, decision rules and pricing functions are unique as the operator that implicitly defines a recursive equilibrium may have multiple fixed points. In order to overcome this issue, we restrict our attention to the limit of the finite horizon version of the model. Under our selection rule, the finite horizon model features a unique equilibrium and so does its limit.

government can issue two types of securities: a zero coupon bond maturing in period 1, b_{01} , and a zero coupon bond maturing in period 2, b_{02} . In period 1, the government issues only a zero coupon bond maturing in period 2, b_{12} . It is convenient to present the model starting from the last period. At $t = 2$, the government does not issue new debt and its only choice is whether to default on the previously issued debt ($\delta_2 = 0$),

$$V_2(b_{02} + b_{12}, Y_2) = \max_{\delta_2} \delta_2 U(Y_2 - b_{02} - b_{12}) + (1 - \delta_2) \underline{V}_2.$$

At $t = 1$, the government issues b_{12} and it decides whether to default ($\delta_1 = 0$). The decision problem at $t = 1$ is

$$V_1(b_{01}, b_{02}, s_1) = \max_{\delta_1, G_1, b_{12}} \delta_1 \{U(G_1) + \beta \mathbb{E}_1[V_2(b_{02} + b_{12}, Y_2)]\} + (1 - \delta_1) \underline{V}_1$$

subject to

$$G_1 + b_{01} \leq Y_1 + q_{12}(s_1, b_{02} + b_{12}) b_{12}$$

Finally at $t = 0$ the government issues both short and long term debt to solve

$$V_0(s_1) = \max_{G_0, b_{01}, b_{02}} U(G_0) + \beta \mathbb{E}_0[V_1(s_1, b_{01}, b_{02})]$$

subject to

$$G_0 + D_0 \leq Y_0 + q_{01}(s_0, b_{01}, b_{02}) b_{01} + q_{02}(s_0, b_{01}, b_{02}) b_{02},$$

with D_0 being the debt inherited from the past. To avoid issues associated with dilution of legacy debt, we assume that the government does not inherit long-term debt. We further assume that D_0 is sufficiently small that the government does not default at $t = 0$. Price schedules q_{01} , q_{02} , and q_{12} must be consistent with lenders no-arbitrage conditions. For simplicity we assume that lenders are risk neutral so $M(s_0, s_1) = M(s_1, s_2) = m$.

3.1.2 Maturity choices in absence of rollover risk

We start from the case in which rollover risk is absent and $\{\pi_t\}$ is identically equal to zero. Previous works on incomplete market models without commitment have emphasized two channels as the main determinants of the maturity composition of debt in the face of default risk: *incentives* and *insurance* channel.

The incentives channel makes short term debt relatively more desirable than long term debt because it is a better instrument to raise credit from the perspective of the $t = 0$ government. The government incentives to issue debt at any point in time depend, in fact,

on the maturity structure of inherited debt. When debt is long term, the government has more incentive to issue new debt- and therefore increase the probability of a future default- because the associated increase in interest rates are applicable only to new issuances, not on the stock of existing debt. When debt is short term, the incentives to issue new debt are lower because the higher interest rates are levied on the entire stock of debt. In equilibrium, the price of long term debt is more sensitive to new issuances relative to the price of short term debt because lenders anticipate higher default risk for the former.¹¹

We can isolate this channel by considering the problem of a government when there are no shocks at $t = 1$. In this case, the optimality conditions for b_{01} and b_{02} can be written as

$$\begin{aligned} mu'(c_0) &= \beta u'(c_1) \\ m \left(1 + \frac{\partial q_{12}}{\partial b_2} \frac{b_{02}}{q_{12}} \right) u'(c_0) &\leq \beta u'(c_1) \end{aligned}$$

where $b_{02} = 0$ whenever the last condition is a strict inequality. To write these conditions we used that there are no shocks in $t = 1$, and so the probability of a default at $t = 1$ is zero. This in turn implies that $q_{01} = m$ and $q_{02} = mq_{12}$. If there is default risk in period $t = 2$ then $\partial q_{12}/\partial b_2 < 0$ and so

$$\beta u'(c_1) = mu'(c_0) > m \left(1 + \frac{\partial q_{12}}{\partial b} \frac{b_{02}}{q_{12}} \right) u'(c_0)$$

This implies that the government does not issue long term debt in the first period, $b_{02} = 0$. The following proposition summarizes the argument above.

Proposition 1. *In the three period economy, if there is no rollover risk and there are no shocks in $t = 1$ then the optimal solution must have $b_{02} = 0$ if the probability of default in $t = 2$ is positive.¹²*

A formal proof can be found in Appendix A. The intuition is as follows. The government at $t = 1$ values issuance of new debt more than the government at $t = 0$ because it does not internalize the effect of its actions on the price of long-term debt at $t = 0$. Hence, it will tend to borrow more than what the time 0 government prefers, lowering its utility. This time inconsistency problem is not present if the government at $t = 0$ issues only short term debt. That is why the government at time zero prefers to issue short term debt only in our example.

¹¹The *debt-dilution* problem is not present if we consider the best SPE (which is history dependent) in which reputational costs prevent the government from deviating from its promised path of debt issuance.

¹²A sufficient condition for this is that β/m is sufficiently low or D_0 sufficiently large.

In general, it is not optimal to issue only short term debt in presence of fundamental risk only. Long term debt is desirable because it is a better asset than short term debt to provide insurance against shocks when there is no outright default. Capital gains and losses imposed on holders of long term debt can approximate wealth transfers associated with state contingent securities, as the market value of debt falls when the marginal utility of the government is high. We refer to this channel as *insurance channel*.¹³

In order to understand the insurance channel, consider the three-period economy where y_1 is stochastic. Consider a marginal variation that lengthens the maturity of government debt at $t = 0$ keeping constant its market value and the debt maturing at $t = 2$. That is, the variation decreases b_{01} by ε , increases b_{02} by $\varepsilon \frac{q_{01}}{q_{02}}$ to keep market value of issuance at $t = 0$ constant, and finally decreases b_{12} by $\varepsilon \frac{q_{01}}{q_{02}}$ so to keep the amount of debt to be repaid in the last period constant. Under this variation, government consumption is unaffected at $t = 0$ and $t = 2$ and the change in government consumption at $t = 1$ in state s_1 is

$$\Delta G_1(s_1) = \left[1 - \frac{q_{12}(s_1)}{\mathbb{E}[q_{12} | \delta_1 = 1]} \right] \varepsilon.$$

The equation shows that this variation leads to a decline in G_1 when the price of newly issued debt is above average at $t = 1$, while it leads to an increase in G_1 in states of the world in which the price is below its average. Since q_{12} is high in “good” states of the world, then lengthening the duration of debt at $t = 0$ provides more insurance for the government: lower consumption when marginal utility is relatively low and higher consumption when the marginal utility is relatively high. This makes issuance of long term debt desirable for a government seeking insurance.

The relative strength of these two forces shapes the optimal portfolio decision in the absence of rollover risk. We are interested in understanding how the maturity composition of debt changes with fundamental default risk. There is no full characterization for this comparative statics exercise. However, previous work and basic economic logic suggest that the government finds it optimal to reduce the maturity of its debt when facing higher fundamental default risk because of two reasons.

First, the time inconsistency problem associated with long term debt is more severe when default risk is high and so the government has a higher incentive to issue short term debt. Moreover, fundamental default risk is high if output is low and/or inherited debt is high, and these are states of the world where the government would like to issue more debt in order to smooth out consumption. As argued earlier, short term debt is a

¹³This channel was muted in the previous example because there were no shocks at $t = 1$ and so trivially consumption at $t = 1$ was not state dependent.

better instrument for this purpose because its price is less sensitive to new issuances. See [Aguiar and Amador \(2014b\)](#) for a similar argument.

Second, the need to hold long term debt for insurance reasons falls when default risk increases. As discussed in [Dovis \(2014\)](#), this happens because pricing functions are more sensitive to shocks when the economy approaches the default region. Hence the larger ex-post variance of the price of long-term debt allows for more insurance because the market value of long term debt falls more in future bad states. *Ceteris paribus*, this makes consumption in period 1 less sensitive to shocks. Thus both forces call for the government to tilt its maturity toward shorter maturity when fundamental default risk is high.

3.1.3 Maturity choices with rollover risk

With rollover risk, the government has an additional reason to actively manage the maturity of its debt. When $\pi > 0$, a rollover crisis can occur with positive probability if the economy happens to be in the crisis zone next period. Since these outcomes are inefficient, the government has an incentive to reduce the likelihood of falling into the crisis zone next period. As emphasized in [Cole and Kehoe \(2000\)](#), this can be achieved by reducing debt issuance and/or by lengthening the maturity of issued debt.

The logic of why lengthening the maturity of debt issued today helps avoiding the crisis zone in the next period can be best understood by looking at the condition defining the safe zone in the three-period economy:

$$U(Y_1 - b_{01}) + \beta \mathbb{E}_2[V_2(b_{02}, Y_2)] \geq \underline{V}_1. \quad (9)$$

Suppose the government today lengthens the maturity of its debt while keeping the amount of resources it raises constant. In our example this is achieved by increasing b_{02} and reducing b_{01} by the appropriate amount. By doing so, the government reduces the payments coming due in the next period at the cost of increasing future payments and reducing the continuation value $\beta \mathbb{E}_2[V_2(b_{02}, Y_2)]$. It is easy to show that this variation increases the left hand side of (9) because the marginal utility of consumption next period when there is a rollover crisis is higher than the marginal reduction in expected utility from period two onward (see [Appendix A](#) for a proof). Therefore, lengthening debt maturity reduces the likelihood of falling into the crisis zone next period.

This discussion suggests that the government has an incentive to lengthen the duration of its debt when rollover risk is sizable. In the extreme case where $\pi > 0$ and it is always optimal to repay the debt absent a rollover crisis (no fundamental default risk), the

government will issue only long term debt in this economy:

Proposition 2. *In the three period economy, if there is only rollover risk and fundamental default never happen at $t = 1, 2$ then $b_{01} = 0$ and all debt is long term.*

A formal proof for the proposition can be found in Appendix A. It is worth noticing that in absence of fundamental default default risk the government will first lengthen its maturity to the maximal extent before it starts to reduce the debt it issues. Lengthening the maturity of debt is, in fact, costless in absence of fundamental default risk while deleveraging has a cost. By continuity, we expect the government to use the maturity lever more than the issuance lever if his ultimate objective is to reduce the likelihood of being in the crisis zone next period. This provides a justification to our identification strategy because maturity choices will be more responsive to fluctuations in rollover risk relative to debt issuance.

3.2 Maturity choices and sources of default risk: IRFs

We now show that the logic of the three-period example carries over to the full model. Figure 1 plots the response of interest rate spreads and debt duration - measured as $1/\lambda'$ - to a negative income shock and to a positive shock to π' in a version of our full model.

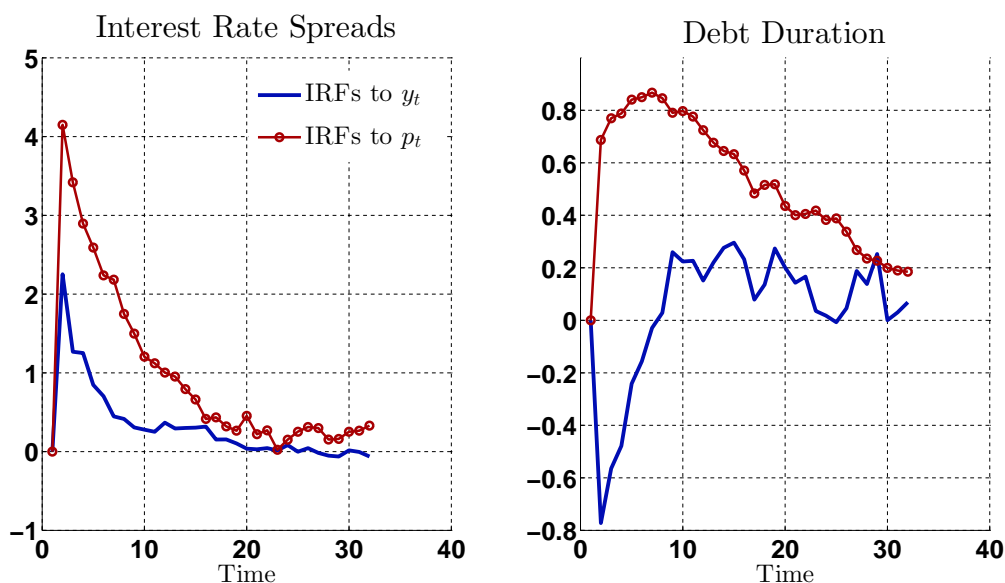
The solid blue line plots the response of interest rate spreads and debt duration to a negative shock to tax revenues, Y , conditional on the realization of $\pi_t = 0$ for all t . We can see that when interest rate spread goes up because of an increase in fundamental default risk the government shortens the maturity of its debt. This is consistent with our argument above. The circled line in Figure 1 plots the response of interest rate spreads and debt duration to an increase in π_t . As expected, an increase in the probability of future rollover crises leads to an increase in debt maturity.

In sum, this discussion suggests that when the sources of default risk are fundamental, interest rate spreads increase and the duration of debt declines. When default risk arises because of the prospect of a rollover crisis, instead, the government lengthens its debt maturity.

4 A case study: Italy in the early 1980s

Before turning to the quantitative analysis, it is useful to discuss in more details our main identifying restriction. Our approach builds on the hypothesis that governments would

Figure 1: The dynamics of interest rate spreads and debt duration



Notes: The blue solid line reports impulse response functions (IRFs) of interest rate spreads and debt duration to a 3 standard deviations income shock in the model without rollover risk. The red circled line reports IRFs to a 3 standard deviation increase in p_t . IRFs are calculated by simulation, and they are expressed as deviation from the ergodic mean. Interest rate spreads are expressed in annualized percentages while debt duration in years.

respond to heightened rollover risk by actively lengthening the maturity of their debt. However, previous cross-country studies have shown that the maturity of new issuances typically shortens around default crises (Broner, Lorenzoni, and Schmukler, 2013; Arellano and Ramanarayanan, 2012), and examples of governments extending the life of their debt in turbulent times are not well documented in the literature. In this section we discuss in details one of these examples. Using a narrative approach, we show how the Italian government in the early 1980s responded to heightened rollover risk (or refinancing risk in the Treasury parlance) by lengthening the duration of public debt.

Two main factors at the beginning of the 1980s contributed to place the Italian government at risk of a roll-over crisis. First, the average residual maturity of government debt was at its historical low, going from a peak value of 9.2 years in 1972 to 1.1 years in 1980.¹⁴ At that time, the Italian government needed to refinance the entire stock of debt, roughly 60% of gross domestic product, within the span of a year. Second, and in an effort to increase the independence of the central bank, a major institutional reform freed the Bank of Italy from the obligation of buying unsold public debt in auctions. This effectively meant that the government couldn't rely anymore on the central bank to finance its maturing

¹⁴This decline was due to the chronic inflation of the 1970s which discouraged investors from holding long duration bonds that were unprotected from inflation risk, see Pagano (1988).

debt and spending needs, and it had to use primarily private markets.¹⁵

Table 1: Auctions of Italian Treasury bills in the 1980s

	Private demand/Offered	Sold/Offered
1981	0.55	0.93
1982	0.71	0.93
1983	0.78	0.98
1984	0.90	0.98
1985	0.88	0.98
1986	0.84	0.98

Notes: Our calculations from Bank of Italy, Supplements to the Statistical Bulletin-Financial Markets.

The short duration of government debt coupled with the loss of central bank financing exposed the Italian government to rollover risk. Auction markets at the time were not well developed, and private demand of treasuries was weak and volatile (Campanaro and Vittas, 2004). In Table 1 we report two statistics: i) the average ratio between the demand of Italian treasury bills by private operators in auctions and the target set by the Treasury, and ii) the average ratio between the quantity of bond sold in the auctions and the target set by the Treasury between 1981 and 1986.¹⁶ We can see how in 1981 and in 1982 private demand of government bonds was substantially below the amount offered. This was exposing the Italian government to refinancing risk because it was not mandatory for the central bank to buy unsold public debt anymore. The potential of a default crisis became evident in the last quarter of 1982, when the weak demand in the auctions of government debt led the Treasury to reach the limit of the overdraft account it had with the Bank of Italy.¹⁷ The refusal of the newly independent Bank of Italy to buy unsold bonds in the auctions led to a budgetary crisis. While the Parliament later voted a law that allowed a temporal overshoot of the overdraft account (Scarpelli, 2001), this event revealed to policymakers the risks implicit in rolling over large amounts of debt in short periods of time.

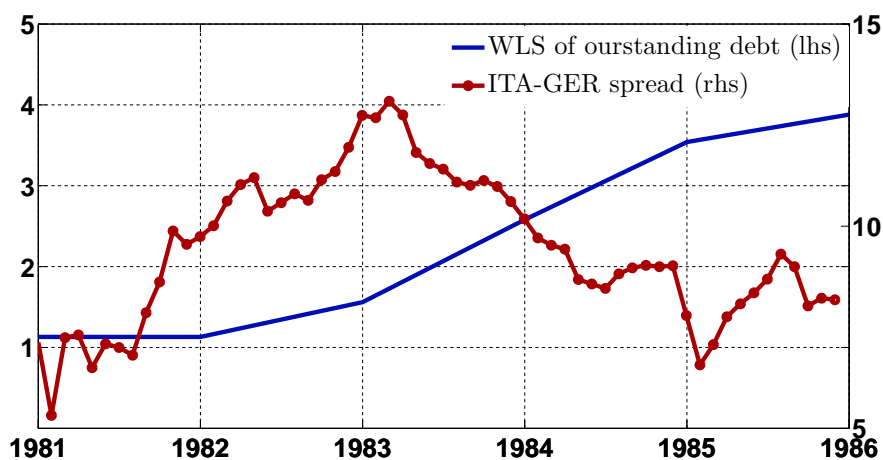
In such a context, the early 1980s saw a rapid increase in interest rate differentials between Italian and German government securities: as we can see from the circled line in

¹⁵Starting from 1975, the Bank of Italy was required to act as a residual buyer of all the public debt that was unsold in the auctions. This resulted in a massive increase in the share of public debt held by the Bank of Italy, reaching a maximum of 40% in 1976. See Tabellini (1988) for a discussion of the historical context underlying the “divorce” between the Bank of Italy and the Italian Treasury.

¹⁶The two differ because of the purchases in the auctions of Treasury bills by the Bank of Italy.

¹⁷This account allowed the Italian Treasury to directly borrow from the Bank of Italy up to a limit of 14% of the expenditures budgeted for the current year.

Figure 2: Debt duration and Interest rate spreads in the 1980s



Notes: The solid line stands for the weighted-average life of the outstanding central government debt. Data are reported in years (left scale), and they are obtained from the Italian Treasury. The circled line reports the yields differential between an Italian and a German zero coupon government bonds with a duration of twelve months. Data are reported in annualized percentages (right scale), and they are obtained from Bank of Italy and Bundesbank.

Figure 2, between January 1980 and March 1983, interest rate spreads rose from 500 to 1300 basis points.¹⁸ In light of the extremely short maturity of the stock of government debt, the institutional changes occurring at the time, and the low private demand for debt in auctions, it is plausible to believe that these tensions in the Italian bond markets were partly reflecting fears of rollover crises. In this respect, the response of the government is consistent with the predictions of our model. As documented in [Alesina, Prati, and Tabellini \(1989\)](#) and in [Scarpelli \(2001\)](#), the Italian government actively pursued throughout the 1980s a policy to extend the life of public debt. Specifically, the Treasury introduced new types of bonds whose interest payments were indexed to the prevailing nominal rate, thus offering to bondholders a protection from inflation risk. These *Certificati di Credito del Tesoro* (CCT) had longer maturity than the *Buoni Ordinari del Tesoro* (BOT), and they quickly replaced the latter as the main instrument used by the government to finance its spending needs.¹⁹ The solid line in Figure 2 shows that the weighted average life of Italian government debt more than tripled within the span of four years, going from 1.13 years in 1981 to 3.88 years in 1986.

¹⁸As Italy and Germany did not have a common currency at the time, these interest rate differentials reflect currency risk along with the risk of outright repudiation. To best of our knowledge, it is not possible to separate these two components of the spreads using existing methodologies ([Du and Schreger, 2015](#)) because of the unavailability of cross-currency swaps data for the early 1980s. It is worth noticing, however, that Italy and Germany were part of the European Monetary System at the time, an exchange rate regime which allowed for limited realignments between the currencies of their members.

¹⁹Indexed securities like CCT are not subject to refinancing and rollover problem but are essentially equal to short term debt for the incentive to generate ex-post inflation because any effort to generate ex-post inflation will not reduce the real value of debt. See [Missale and Blanchard \(1994\)](#).

Through the lens of the model, the actions of the Italian Treasury reduced its exposure to rollover risk without increasing the incentive to inflate away the debt. Consistent with this view, we can observe from Figure 2 that as the maturity of the stock of debt increased, the interest rate spreads between Italian and German government securities started to decline in 1983.

5 Calibration

We now apply our framework to Italian data during the recent debt crisis. This section proceeds in four steps. Section 5.1 describes the parametrization of the model and our calibration strategy. Section 5.2 describes the data. Section 5.3 reports the results of our calibration. Section 5.4 discusses the fit of the model.

5.1 Parametrization and Calibration Strategy

5.1.1 Lenders' stochastic discount factor

It is common practice in the sovereign debt literature to assume risk neutrality on the lenders' side. This specification, however, is not desirable given our objectives. First, several authors have shown that risk premia are quantitatively important to account for the level and volatility of sovereign spreads (Borri and Verdelan, 2013; Longstaff, Pan, Pedersen, and Singleton, 2011). Assuming risk neutrality implies that other unobserved factors in the model, for instance π_t , would need to absorb the variations in this component of the spread. Second, sovereign debt crisis are typically accompanied by a significant increase in term premia (Broner, Lorenzoni, and Schmukler, 2013). Neglecting these shifts could undermine our identification strategy: rollover risk could be driving interest rate spreads of peripheral countries in the euro-area and yet we could observe a shortening in debt maturity simply because high term premia made short term borrowing relatively cheaper.

Therefore, we introduce a stochastic discount factor that allows us to fit the behavior of risk premia on long term bonds observed in Europe during the period of analysis. We follow Ang and Piazzesi (2003) and assume that $m_{t,t+1} = \log M_{t,t+1}$ is given by the

conditionally Gaussian process

$$\begin{aligned}
m_{t,t+1} &= -(\tau_0 + \tau_1 \chi_t) - \frac{1}{2} \kappa_t^2 \sigma_\chi^2 - \kappa_t \varepsilon_{\chi,t}, \\
\chi_{t+1} &= \mu_\chi (1 - \rho_\chi) + \rho_\chi \chi_t + \varepsilon_{\chi,t} \quad \varepsilon_{\chi,t} \sim \mathcal{N}(0, \sigma_\chi^2), \\
\kappa_t &= \kappa_0 + \kappa_1 \chi_t.
\end{aligned} \tag{10}$$

When enriched with a process for payouts, one can use $m_{t,t+1}$ along with the pricing formula in equation (1) to express asset prices as a function of model parameters and of the state variable χ_t . As shown in [Ang and Piazzesi \(2003\)](#), the log price of non-defaultable ZCBs is linear in the state variable χ_t ,

$$q_t^{*,n} = a_n + b_n \chi_t, \tag{11}$$

where a_n e b_n are functions of the model's parameters and $q_t^{*,n}$ is the log price of a ZCBs maturing in n periods (See Appendix C).

Fluctuations in χ_t generate movements in bond prices which, depending on the model parametrization, can give rise to risk premia on long term bonds. To understand this point, we can write the excess log returns on a bond maturing in n periods as²⁰

$$\mathbb{E}_t[r x_{t+1}^n] + \frac{1}{2} \sigma_t [r x_{t+1}^n] = -\text{cov}_t[m_{t,t+1}, q_{t+1}^{*,n-1}]. \tag{12}$$

Long term bonds earn a risk premium when $\text{cov}_t[m_{t,t+1}, q_{t+1}^{*,n-1}] < 0$, that is when investors expect these assets to lose value in bad times. Different choices of model parameters imply different behavior for these risk premia. For example, when $\kappa_0 = \kappa_1 = 0$, the lenders are risk neutral and risk premia on long term bonds are identically zero.²¹ When $\kappa_1 \neq 0$, χ_t will affect the risk premium demanded by lenders to hold long term bonds.

For future reference, we let $\theta_1 = [\tau_0, \tau_1, \kappa_0, \kappa_1, \mu_\chi, \rho_\chi, \sigma_\chi]$ denote the parameters of the lenders' stochastic discount factor. It is important to stress that the stochastic discount factor is exogenous with respect to the risk of a government default, an assumption that is made mostly for tractability. In Appendix D we discuss potential limitations of this restriction, but we also argue that it does not invalidate our identification strategy.

²⁰In order to derive this equation, we make use of the lenders' no-arbitrage condition $\mathbb{E}_t[e^{m_{t,t+1} + r x_{t+1}^n}] = 0$, of the definition of excess log returns $r x_{t+1}^n = q_{t+1}^{*,n-1} - q_t^{*,n} + q_t^{*,1}$, and of the joint log-normality of the pricing kernel and excess returns.

²¹In order to see that, we can use equations (10) and (12) and write $-\text{cov}_t[m_{t,t+1}, q_{t+1}^{*,n-1}] = \kappa_t b_{n-1} \sigma_{\chi,t}$.

5.1.2 Government's decision problem

The government discounts future flow utility at the rate β . The utility function is parametrized as follows,

$$U(G_t) = \frac{(G_t - \underline{G})^{1-\sigma} - 1}{1-\sigma},$$

where \underline{G} is the non-discretionary level of public spending. We interpret \underline{G} as capturing the components of public spending that are hardly modifiable by the government in the short run, such as wages of public employees and pensions. As we will discuss in Section 5.4, this specification helps our model matching the level and cyclicity of public debt.

If the government enters a default state, it is excluded from international capital markets and it suffers an output loss d_t . These default costs are a function of the country's income, and they are parametrized following Chatterjee and Eyigungor (2012),

$$d_t = \max\{0, d_0 \exp\{y_t\} + d_1 \exp\{2y_t\}\}.$$

If $d_1 > 0$, then the output losses are larger when income realizations are above average. We also assume that, while in autarky, the government has a probability ψ of reentering capital markets. If the government reenters capital markets, it starts the decision problem with zero debt.

The country's endowment $Y_t = \exp\{y_t\}$ follows the stochastic process,

$$y_{t+1} = \rho_y y_t + \rho_{y\chi}(\chi_t - \mu_\chi) + \sigma_y \varepsilon_{y,t+1} + \sigma_{y\chi} \varepsilon_{\chi,t+1}, \quad \varepsilon_{y,t+1} \sim \mathcal{N}(0, 1). \quad (13)$$

In this formulation, output of the domestic economy depends on the factor χ_t and on its innovations, allowing us to match the observed correlation between risk premia and real economic activity over our sample.

The probability of lenders not rolling over the debt in the crisis zone next period follows the stochastic process $\pi_t = \frac{\exp\{\tilde{\pi}_t\}}{1 + \exp\{\tilde{\pi}_t\}}$, with $\tilde{\pi}_t$ given by

$$\tilde{\pi}_t = \pi^* + \sigma_\pi \varepsilon_{\pi,t}, \quad \varepsilon_{\pi,t} \sim \mathcal{N}(0, 1). \quad (14)$$

We let $\theta_2 = [\sigma, \psi, \rho_y, \rho_{y\chi}, \sigma_y, \sigma_{y\chi}, \underline{G}, \pi^*, \sigma_\pi, \beta, d_0, d_1]$ denote the parameters associated to the government decision problem.

5.1.3 Calibration strategy

Our strategy consists in calibrating $\theta = [\theta_1, \theta_2]$ in two steps. In the first step, we choose θ_1 to match the behavior of risk premia over non-defaultable long term bonds, measured using the term structure of German's ZCB. We focus on non-defaultable bonds rather than on the bonds issued by our government because this allows us to calibrate these parameters without solving the government decision problem, which is numerically challenging. In the second step, and conditional on θ_1 , we calibrate θ_2 by matching key facts about Italian public finances over our sample period.

5.2 Data

We use the Bundesbank online database to obtain information on the term structure of ZCBs for Germany. We collect monthly data on the parameters of the [Nelson and Siegel \(1987\)](#) and [Svensson \(1994\)](#) model, and we generate nominal bond yields for all maturities between $n = 1$ and $n = 20$ quarters. We convert these monthly series at a quarterly frequency using simple averages. We use the OECD *Main Economic Indicators* database to obtain a series for inflation, defined as the year-on-year percentage change in the German CPI index. These series, available for the period 1973:Q1-2013:Q4, are used in the first step of our procedure to calibrate θ_1 .

The endowment process y_t is mapped to detrended (cubic) log real Italian GDP. The quarterly GDP series is obtained from the OECD *Main Economic Indicators* starting from 1960. The interest rate spread series is the CDS spread on an Italian 5 years government bond, obtained from *Markit*, and it is available starting in 2001:Q1. We construct an indicator of debt duration for the outstanding bonds issued by the Italian central government, and we map it to $\frac{1}{\lambda}$.²² These series are used in the second step to calibrate θ_2 .

5.3 Results

The results are organized in two sections. First, we describe the parametrization of the pricing model. Then, we discuss the calibration of the remaining parameters.

²²Specifically, at any point in time t , we collect outstanding principal and coupon payments to bondholders for every maturity n ($C_{n,t}$) using data from the Italian treasury. Our indicator is then

$$\text{duration}_t = \sum_{n=1}^N n \frac{C_{n,t}}{V_t},$$

where V_t is the outstanding face value of bonds issued. This indicator, the weighted average of the times of principal and coupon repayments, maps exactly to $\frac{1}{\lambda}$ in our model.

5.3.1 Calibration of the pricing model

We choose the parameters of the lenders' stochastic discount factor to fit the behavior of risk premia on long term German ZCBs over our sample. Specifically, we calibrate θ_1 to match key features of the predictive regressions of [Cochrane and Piazzesi \(2005\)](#) (henceforth C-P). In order to explain the procedure, let $rx_{t+1}^n = q_{t+1}^{*,n-1} - q_t^{*,n} + q_t^{*,n}$ be the *realized* excess log returns on a ZCB maturing in n quarters, $f_t^n = q_t^{*,n-1} - q_t^{*,n}$ the time t log forward rate for loans between $t + n - 1$ and $t + n$, and $y_t^1 = -q_t^{*,1}$ the log yield on a ZCB maturing next quarter. We denote by \mathbf{r}_{t+1} and \mathbf{f}_t vectors collecting, respectively, excess log returns and log forward rates for different maturities. We proceed in two stages. In the first stage, we estimate by OLS a regression of the log excess returns averaged across maturities on all the forward rates in \mathbf{f}_t ,

$$\bar{rx}_{t+1} = \gamma_0 + \gamma' \mathbf{f}_t + \bar{\eta}_t. \quad (15)$$

In the second stage, we estimate the regressions

$$rx_{t+1}^n = a_n + b_n(\hat{\gamma}_0 + \hat{\gamma}' \mathbf{f}_t) + \eta_t^n, \quad (16)$$

where $[\hat{\gamma}_0, \hat{\gamma}]$ is the OLS estimator derived in the first stage. C-P document that that the linear combination of log forward rates obtained in the first stage has predictive power for the second stage regressions when applied to U.S. bond data. [Dahlquist and Hasseltoft \(2013\)](#) confirm this pattern for other countries, including Germany. Risk premia on a ZCB maturing in n period can then be measured using the fitted values of this second stage regression.²³

Our procedure consists in calibrating θ_1 so that the pricing model defined by the system in (10) satisfies three properties:

1. The factor χ_t equals $\hat{\gamma}_0 + \hat{\gamma}' \mathbf{f}_t^{\text{model}}$, where $\hat{\gamma}_0$ and $\hat{\gamma}$ are the OLS point estimates in equation (15) and $\mathbf{f}_t^{\text{model}}$ are the log forward rates generated by the model.
2. The model implied coefficients of equation (16) are equal to the OLS point estimates $(\hat{a}_n, \hat{b}_n, \hat{\sigma}_{\eta^n})$, for a five year bond ($n = 20$).
3. The mean and standard deviation of y_t^1 in the model matches that in the data.

The first requirement allows us to interpret χ_t as a shock directly moving risk premia

²³From equation (16) we can see that *expected* excess returns on a bond maturing in n period equal $\mathbb{E}_t[rx_{t+1}^n] = \alpha_n + \beta_n(\hat{\gamma}_0 + \hat{\gamma}' \mathbf{f}_t)$.

on long term bonds,²⁴ and it gives us a direct mapping between the state χ_t and the data, $\hat{\chi}_t = \hat{\gamma}_0 + \hat{\gamma}'\mathbf{f}_t$. The second requirement implies that our pricing model replicates the estimated time series of $\mathbb{E}_t[rx_{t+1}^{20}]$ once we feed it with $\hat{\chi}_t = \hat{\gamma}_0 + \hat{\gamma}'\mathbf{f}_t$ which we obtain from the data. The third requirement makes sure that the behavior of the risk free rate generated by the model is broadly consistent with the observed one.

Appendix C reports the details of the C-P regressions, and describes how we choose θ_1 to match the empirical targets. Panel A of Table 2 reports the numerical values of the parameters.

5.3.2 The government's decision problem

We next turn to the calibration of θ_2 . We fix σ to 2, a conventional value in the literature, and we set $\psi = 0.0492$, a value that implies an average exclusion from capital markets of 5.1 years following a sovereign default, in line with the evidence in Cruces and Trebesch (2013). The spending requirement, \underline{G} , is set to 0.70. This number is equal to the ratio of wages of public employees and transfers over government expenditures (net of interest payments) averaged over the 1999-2012 period.

We use our output series and the linear combination of log forward rates $\hat{\chi}_t = \hat{\gamma}_0 + \hat{\gamma}'\mathbf{f}_t$ to estimate the output process in equation (13) over the 1999:Q1-2012:Q2 period. We obtain the innovations to $\hat{\chi}_t$ by fitting an AR(1), and we subsequently estimate equation (13) by OLS. As $\rho_{y\chi}$ is not significantly different from zero in this specification, we impose the restriction $\rho_{y\chi} = 0$. The point estimates are $\rho_y = 0.939$, $\sigma_{y\chi} = -0.002$ and $\sigma_y = 0.008$.

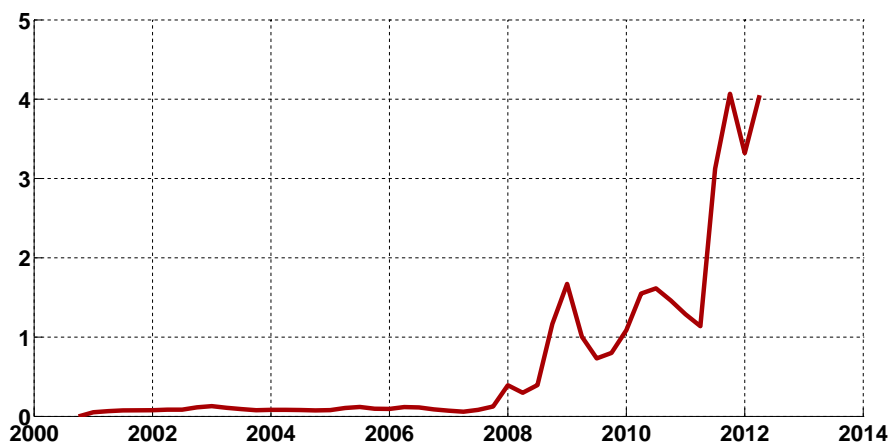
The calibration of $[\pi^*, \sigma_\pi]$ is challenging given the event under analysis. Figure 3 plots the interest rate spreads series over the 2001:Q1-2012:Q2 period. Interest rate spreads were essentially zero until 2008, implying that observables can hardly be used in the earlier part of the sample to obtain information over these two parameters.²⁵ We therefore follow a conservative approach and choose these two parameters so that roll-over risk could potentially explain the behavior of interest rate spreads during the crisis. Specifically, we set $\pi^* = -6.70$ and $\sigma_\pi = 1.15$. This calibration implies that, conditional on being in the crisis zone, the probability of a rollover crisis is roughly 0.50% in annualized terms. However, we allow for large deviations from this value. For example, $\pi = 0.0125$ is within two standard deviations. Hence, the model would not have difficulties in matching the peak annualized interest rate spreads of 400 basis points observed in Figure 3 through

²⁴As we show in Appendix C, equation (16) holds exactly in our pricing model for any n .

²⁵For example, maturity choices in our model are indeterminate in absence of default and interest rate risk. Hence, the behavior of debt duration in the early part of the sample is not likely to carry information over $\{\pi_t\}$.

variation in π_t .

Figure 3: Italian CDS spreads: 2001:Q1-2012:Q2



The remaining parameters are $[\beta, d_0, d_1]$. Typical calibrations of sovereign debt models select these parameters to target average debt to output ratio and first and second moments of the interest rate spreads distribution. However, a look at Figure 3 shows that this practice may lead to misleading results in our application. High and volatile interest rate spreads are not the norm for the Italian economy, and this information is hardly captured by first and second moments. For this reason, we enrich the set of targets considered in the literature with other statistics summarizing the tail behavior of interest rate spreads. Specifically, we choose $[\beta, d_0, d_1]$ so that our model matches the average debt to output ratio over the sample, and the mean, standard deviation, skewness and kurtosis of the observed interest rate spreads distribution. As the numerical solution of the model is challenging, we first experiment with these three parameters to obtain a range of values that is empirically relevant. We next solve our model on a grid of points for $[\beta, d_0, d_1]$, and select the specification that minimizes a weighted distance between sample moments and their model implied counterparts.²⁶

Panel B of Table 2 reports the calibrated values for the model's parameters. Our calibrated β is 0.9875, a value that is substantially higher than existing work that focus on emerging markets. In line with previous research, we find that convex output costs are necessary to fit the behavior of interest rate spreads. The numerical values of $[d_0, d_1]$ imply output losses upon defaults of 10.75% when output is at its average level, 6.25% when output is 9% below trend. These numbers are not inconceivable, given that a government

²⁶ Model implied moments are computed on a long simulation ($T = 5000$) of the model. When computing the statistics, we exclude the first 40 quarters after a default. We weight the distance between a sample moment and its model counterpart by the inverse of the sample moment standard deviation.

default in Italy would wipe out the balance sheet of domestic banks (Bocola, 2014), and most likely damage trade relations with other euro-area partners.²⁷

Table 2: **Model calibration**

Panel A: Pricing model		
Parameter	Value	Targets
τ_0	0.003	
τ_1	0.242	
κ_0	-0.228	C-P regressions
κ_1	-2322.111	and
μ_χ	0.008	Risk free rate
ρ_χ	0.906	
σ_χ	0.008	
Panel B: Government's decision problem		
ρ_y	0.939	
σ_y	0.008	Estimates from eq. (13)
$\sigma_{y\chi}$	-0.002	
σ	2.000	Conventional value
ψ	0.049	Cruces and Trebesch (2013)
\underline{G}	0.700	Non discretionary public spending
$\frac{\exp\{\pi^*\}}{1+\exp\{\pi^*\}}$	0.001	Numerical exploration
σ_π	1.150	Numerical exploration
β	0.988	Debt-to-output ratio
d_0	-0.415	and
d_1	0.522	Spreads distribution

5.4 Model Fit

We start by documenting the ability of our model to match the empirical targets. The first and second column of Table 3, panel A, report respectively the sample statistics of interest and their model counterpart at the baseline calibration. We can observe that our model is fairly successful in matching the targets. The face value of debt is 113% of annual GDP on average, close to the data figure. The model also captures key features of the interest rate spreads distribution we are fitting. On average, interest rate spreads in our model are 51 basis points in annualized terms, with a standard deviation of roughly 95 basis points. The model implied distribution is right skewed as in the data, with large excess kurtosis.

²⁷To best of our knowledge, Hebert and Schreger (2015) represents the only attempt in the literature to directly measure the output costs of a sovereign default. By using variation in legal rulings in the case of Republic of Argentina v. NML Capital, the authors estimate an output costs of sovereign default between 2.4% and 6% of GDP for the Argentinian economy.

Panel B of the Table reports a series of statistics that were not targeted in our calibration. Defaults are hardly observed in the model, as they are in the data, with a default rate of 0.06%. Also, debt issuances are countercyclical, with a correlation of -0.10 with output. Hence, our model is consistent with the observation that the Italian government was running deficits during the crisis, a phenomena that existing calibrations of models of sovereign debt would have hard time rationalizing. The standard deviation of the debt-to-output ratio is 3.21%, roughly 60% of what was observed over our episode. Another statistics of interest is the probability that the economy falls into the crisis zone next period. This is because movements in π_t can have substantial effects on interest rate spreads only if this probability is positive and large. The fact that our model predicts on average an 87% chance of being in the crisis zone is assuring that we are not a-priori restricting the role of rollover risk in driving interest rate spreads.

Table 3: **Calibration targets and additional statistics**

Panel A: Empirical Targets				
Statistic	Data	Baseline	High Impatience	CRRA Utility
Average debt-to-output ratio	102.20	113.20	30.00	47.96
Average spread	0.67	0.51	1.10	0.00
SD of spread	1.05	0.95	0.56	0.00
Skewness of spread	2.06	3.25	2.29	0.00
Kurtosis of spread	6.49	16.00	16.49	0.00
Panel B: Additional Statistics				
Default rate	0.00%	0.06%	0.10%	0.00%
Correlation $\Delta b'$ and y	-0.20	-0.10	0.20	0.29
SD of debt-output ratio	5.30	3.21	0.98	2.37
Average prob. of crisis zone		87.26%	0.00%	0.00%

Notes: The debt-to-output ratio is the quarterly outstanding debt of the central government scaled by annual GDP (Eurostat). Sample moments are computed over the 2001:Q1-2012:Q2 window. Moments in the model are computed as described in footnote 26.

In order to understand mechanically how our model generates these results, it is useful to consider how these statistics are affected by alternative parametrizations of $[\underline{G}, \beta, d_0, d_1]$. In Column 3 we set $\underline{G} = 0$, $\beta = 0.95$, $d_0 = -0.18$ and $d_1 = 0.24$. These last three values are borrowed from the calibration of a similar model to Argentina (Chatterjee and Eyigungor, 2012). In this latter specification, high impatience relative to the risk free rate and small consumption smoothing motives imply that the government behaves myopically, borrowing as much as possible in every period. This behavior generates two implications that would be counterfactual in our event. First, it generates procyclical debt issuance. When income increases, default risk becomes less of a concern, this leading to an outward shift

in the pricing schedule of debt. The government, thus, borrows more in good times because it *can*. Second, interest rates spreads are, on average, higher: precisely because the government borrows as much as it can, it is always at risk of a default.

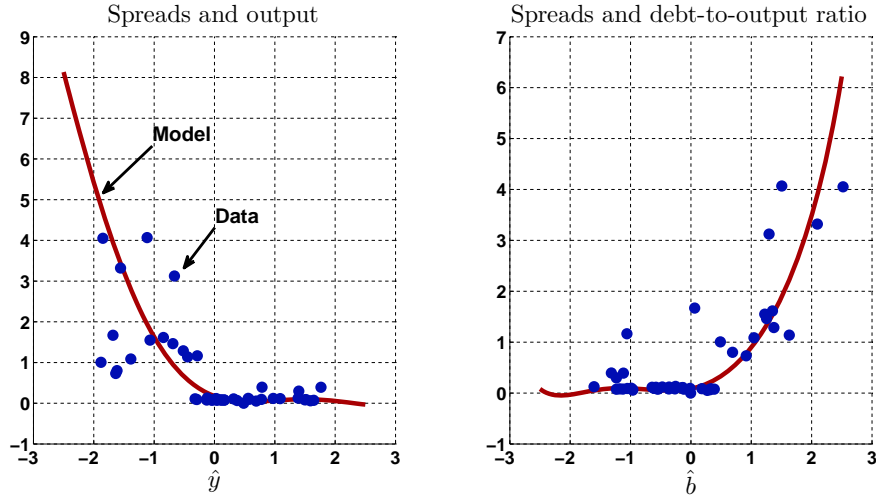
In our calibration, non-homotheticity in the utility function coupled with a lower impatience for the government are able to correct these two issues. Specifically, in good and normal times, the government does not borrow to its maximal capacity because of the high precautionary motives implied by our preference specification. As a result, default risk is very small in those states of the world, because the government is far from the default threshold. Hence, interest rate spreads in our calibration are, on average, very close to zero. A sufficiently bad income shock, however, pushes the government to issue more debt because of consumption smoothing purposes. As the government gets closer to its borrowing capacity, default risk becomes non-negligible, and interest rate spreads jumps. This feature generates countercyclical debt issuances and the right skewed distribution of spreads we observe in the data.

We can further evaluate the performance of our model by looking more closely at the behavior of interest rate spreads, output and debt. The solid line in the left panel of Figure 4 plots the average relation between interest rate spreads and output in our model. This relation is obtained by fitting, on simulated data, a polynomial regression of output on the interest rate spreads. The dots in the same figure represent combinations of output and interest rate spreads in the data. We can observe that the model implied elasticities of interest rate spreads to output are highly nonlinear: a decline in output when the economy is doing well has essentially no effects on interest rate spreads on average. However, this elasticity achieves a value of -3.75 when output is 1 standard deviation below its mean. This model implied elasticity appears empirically plausible, in terms of shape and magnitude. The right panel of the figure plots this same information for debt. Again, the implied elasticities of interest rate spreads to the debt-to-output ratio are highly nonlinear and they well capture the relation between these two variables in the data.

6 Decomposing Italian Spreads

[This section is based on a previous calibration of the model. we are in the process of updating it.] We now use the calibrated model to measure the importance of rollover risk in driving Italian spreads during the period of analysis. We proceed in two steps. In the first step, we use our calibrated model along with the data presented in Section 5 to estimate a time series for the exogenous shocks, $\{y_t, \chi_t, \pi_t\}$. In the second step, we use the

Figure 4: Interest rate spreads sensitivity to output and debt



Notes: The left panel is constructed as follows. We simulate a $T = 5000$ realization from the model. After standardizing the output series, we fit the following regression, $s_t = a_0 + \sum_{j=1}^5 a_j y_t^j + e_t$ on the model simulated data. The red solid line represents the fitting curve of this regression. The blue dots are combinations of interest rate spreads and linearly detrended GDP (standardized). The right panel reports the same experiment, this time for the debt-to-output ratio.

estimated path for the state variables and the model equilibrium conditions to calculate the component of interest rate spreads that is due to rollover risk. This exercise is reported in Section 6.1. In order to highlight the information content of maturity choices, Section 6.2 repeats the experiment, this time excluding the debt duration series from the set of observables.

6.1 Measuring rollover risk

Our model defines the nonlinear state space system

$$\begin{aligned} \mathbf{Y}_t &= \mathbf{g}(\mathbf{S}_t; \theta) + \eta_t \\ \mathbf{S}_t &= \mathbf{f}(\mathbf{S}_{t-1}, \varepsilon_t; \theta), \end{aligned}$$

with \mathbf{Y}_t being a vector of measurements, η_t classical measurement errors, the state vector is $\mathbf{S}_t = [B_t, \lambda_t, y_t, \chi_t, \pi_t]$ and ε_t are innovations to structural shocks. The first part of the system collects measurement equations, describing the behavior of observable variables while the second part collects transition equations, regulating the law of motion for the potentially unobserved states. We estimate the realization of the model state variables by applying the particle filter to the above system (Fernández-Villaverde and Rubio-Ramírez,

2007).²⁸ The set of measurements \mathbf{Y}_t includes the time series for interest rate spreads, linearly detrended real GDP, the previously estimated series for χ_t , and the weighted-average life of Italian debt. It is important to stress that the estimation of $[y_t, \chi_t]$ is disciplined by “actual” observations because the measurement equation incorporates empirical counterparts of these shocks. The truly unobservable process is the realization of π_t .

Equipped with the estimated path for the model state variables, we next use the structural model to measure the contribution of rollover risk to interest rate spreads. For this purpose, we use the lenders’ Euler equation and express interest rate spreads on a ZCB maturing next period as follows

$$\begin{aligned} \frac{r_{1,t} - r_t^*}{r_{1,t}} &= \Pr_t\{\mathbf{S}_{t+1} \in \mathcal{S}^{\text{default}}\} + \Pr_t\{\mathbf{S}_{t+1} \in \mathcal{S}^{\text{crisis}}\}\pi_{t+1} \\ &- \text{Cov}_t\left(\frac{M_{t,t+1}}{\mathbb{E}_t[M_{t,t+1}]}, \delta_{t+1}\right). \end{aligned} \tag{17}$$

The first two components in equation (17) represent the different sources of default risk in the model. As discussed in Section 2, the government can default because of two events. First, if $\mathbf{S}_{t+1} \in \mathcal{S}^{\text{default}}$, the government finds it optimal to default irrespective of the behavior of lenders. Second, the government may be in the crisis zone next period, in which case the conditional probability of observing a default is π_{t+1} . Finally, $\text{Cov}_t\left(\frac{M_{t,t+1}}{\mathbb{E}_t[M_{t,t+1}]}, \delta_{t+1}\right)$ reflects the premium that lenders demand to hold risky government securities. Our objective is to construct a time series for these three components of the interest rate spreads.

The left panels of Figure 5 reports the behavior of spreads and debt duration in the data and in the model. The model closely matches the behavior of interest rate spreads. This is not surprising because the variance of the measurement errors associated to this series in the state space model is small. The model tracks closely the behavior of the debt duration series, which decreases in the latter part of the sample. Between 2011:Q1 and 2012:Q2, the weighted average life of outstanding Italian debt dropped by half a year.²⁹ The model captures this pattern, but it cannot quantitatively fit this step because the grid for λ in the numerical solution is such that duration changes at least one year, see Appendix B.

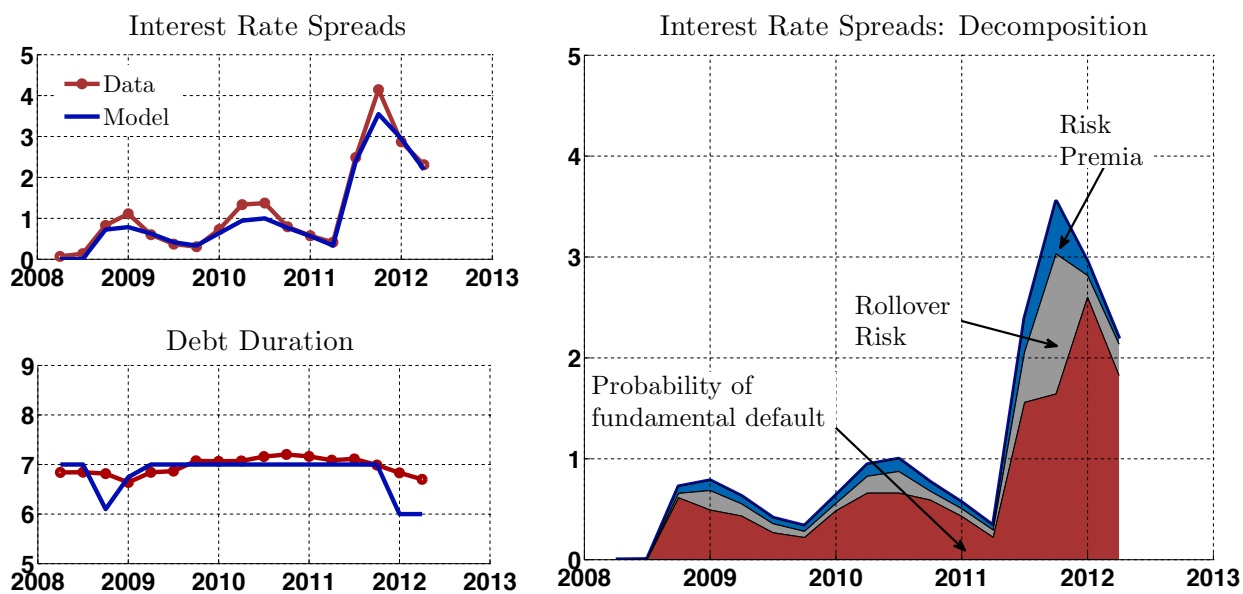
The right panel of Figure 5 reports the model implied decomposition of equation (17).

²⁸The measurement errors are Gaussian. The variance for the measurement error associated to interest rate spreads is set at 2.5% of the sample variance of the observables. The measurement errors for the other variables is set at 20% of their respective sample variance. The number of particles adopted is 50000.

²⁹While this may seem a small variation, it is important to stress that we are measuring the duration of the outstanding *stock* of debt. Hence, the change in duration for the flows (net issuances) are substantially larger.

The red shaded area represents the conditional probability of falling into the default region next period, the gray shaded area reports the conditional probability of a rollover crisis, and the blue shaded area denotes risk premia. From the figure, we can see that the risk premium component explains, on average, roughly 10% of the variation in interest rate spreads over our sample. The bulk of the variation in interest rate spreads arise because of fluctuations in the conditional probability of a fundamental default. Finally, rollover risk accounts for up to 38% of the observed movements in spreads, although its role is negligible at the end of the sample.

Figure 5: Contribution of rollover risk to interest rate spreads



Notes: The top left panel reports CDS spreads on 6 months Italian government bonds along with the point estimates for interest rate spreads on a one period ZCB implied by the model. The bottom left panel reports the same information for the weighted-average life of outstanding government debt. The right panel reports the decomposition of the filtered interest rate spreads given by equation (17). The red area represents $\{Pr_t\{\mathbf{S}_{t+1} \in \mathcal{S}^{default}\}\}$, the gray area $\{Pr_t\{\mathbf{S}_{t+1} \in \mathcal{S}^{crisis}\}\}$, and the blue area $\left\{Cov_t\left(\frac{M_{t,t+1}}{E_t[M_{t,t+1}]}\right)\right\}$.

6.2 The information content of maturity choices

We now repeat the filtering experiment, this time excluding the debt duration series from the set of observables. Table 4 reports several statistics for this specification. Specifically, the point estimates for the average of the three components of the interest rate spreads over the sample, along with the 5th and 95th percentile. We also report, as a comparison, the same statistics for the experiment of Section 6.1.

Absent data on debt duration, the model does not have clear identifying restrictions

that can be used to discipline π_t , and it attributes to this term variation in interest rate spreads that can not be accounted by $[y_t, \chi_t]$. Even in this specification, though, the model assigns *on average* a fairly limited role to rollover risk. This depends on the fact that detrended real GDP during the 2008-2012 period was well below average, and the χ_t factor signals increases in risk premia over the episode. Hence, the model requires little variation in π_t to fit the Italian spreads.

However, Table 4 documents also substantial uncertainty in this decomposition. This can be seen by looking at the standard errors of the three components. For example, the rollover risk component can account, on average, for almost all the variation in interest rate spreads (113 basis points vs an average spreads in the data of 120 basis points).

Table 4: Interest rate spreads decomposition

		Average	5 th Percentile	95 th Percentile
	$\Pr_t\{\mathbf{S}_{t+1} \in \mathcal{S}^{\text{default}}\}$	0.74	0.15	1.20
No duration	$\Pr_t\{\mathbf{S}_{t+1} \in \mathcal{S}^{\text{crisis}}\}$	0.32	0.00	1.13
	$\text{Cov}_t\left(\frac{M_{t,t+1}}{\mathbb{E}_t[M_{t,t+1}]}\right)$	0.10	0.03	0.25
	$\Pr_t\{\mathbf{S}_{t+1} \in \mathcal{S}^{\text{default}}\}$	0.75	0.55	1.13
Benchmark	$\Pr_t\{\mathbf{S}_{t+1} \in \mathcal{S}^{\text{crisis}}\}$	0.21	0.00	0.45
	$\text{Cov}_t\left(\frac{M_{t,t+1}}{\mathbb{E}_t[M_{t,t+1}]}\right)$	0.12	0.05	0.30

This second result is due to the combination of two factors. First, pricing schedules in models of sovereign debt are highly nonlinear, and small variations in $[y_t, \chi_t]$ can have sizable effects on interest rate spreads when default risk is non negligible. Second, the lack of discipline on π_t implies that the model can use this shock to fit variation in interest rate spreads that is not accounted by the fundamental shocks. Hence, small degrees of uncertainty over $[y_t, \chi_t]$, generated in our experiment by measurement errors, translates into sizable uncertainty over $\Pr_t\{\mathbf{S}_{t+1} \in \mathcal{S}^{\text{default}}\}$, and ultimately on the probability of a rollover crisis.

The introduction of the weighted-average life of outstanding debt in the set of observables helps resolving this identification problem. This can be seen by looking at the standard errors for the components of the spreads in our benchmark exercise, which are

substantially smaller than the specification without duration data. To understand why this is the case, we report in Table 5 the cross-sectional correlation between the filtered probability of a rollover crisis and the implied maturity choices that the government makes at these points in the state space. The Table shows that these two variables are negative correlated: realizations of the state vector in which rollover risk is high are associated to higher debt maturities (lower λ' chosen by the government). Hence, our benchmark estimation assigns low likelihood to these realizations because the implied debt maturity choices of the government are counterfactual.

Table 5: **Correlation between $\Pr_{i,t}\{\mathbf{S}_{t+1} \in \mathcal{S}^{\text{crisis}}\}$ and $\lambda_{i,t+1}$**

2011:Q2	2011:Q3	2011:Q4	2012:Q1	2012:Q2
-0.459	-0.453	-0.516	-0.523	-0.434

7 Evaluating OMT Announcements

As a response to soaring interest rate spreads in the euro-area periphery, the Governing Council of the European Central Bank (ECB) announced during the summer of 2012 that it would consider outright transactions in secondary, sovereign bond markets. The technical framework of these operations was formulated on September 6 of the same year. The Outright Monetary Transaction (OMT) program replaced the Security Market Program as a mean through which the ECB could intervene in sovereign bond markets.

OMTs consist in direct purchases of sovereign bonds of members of the euro-area in secondary markets.³⁰ These operations are considered by the ECB once a member state asks for financial assistance, and upon the fulfillment of a set of conditions.³¹ There are two important characteristics of these purchases. First, *no ex ante quantitative limits* are set on their size. Second, the ECB accepts the same (*pari passu*) treatment as private or other creditors with respect to bonds issued by euro area countries and purchased through OMTs.

Even though the ECB never purchased sovereign bonds within the OMT framework, the mere announcement of the program had significant effects on interest rate spreads

³⁰Transactions are focused on the shorter part of the yield curve, and in particular on sovereign bonds with a maturity of between one and three years. The liquidity created through OMTs is fully sterilized.

³¹A necessary condition for OMTs is a *conditionality* attached to a European Financial Stability Facility/European Stability Mechanism (EFSF/ESM) macroeconomic adjustment or precautionary programs. For a country to be eligible for OMTs, these programs should include the possibility of EFSF/ESM primary market purchases.

of peripheral countries. [Altavilla, Giannone, and Lenza \(2014\)](#) estimate that OMT announcements decreased the Italian and Spanish 2 years government bonds by 200 basis points. This decline in interest rate spreads was widely interpreted by economists and policy makers as a reflection of the success of this program in reducing non-fundamental inefficient fluctuations in sovereign bond markets of euro-area peripheral countries. Accordingly, OMT has been regarded thus far as a very successful program. In this Section we use our calibrated model to evaluate this interpretation.

We introduce OMTs in our model as a price floor schedule implemented by a Central Bank. Section 7.1 shows that an appropriate design of this schedule i) can eliminate the bad equilibria in our model, and ii) it does not require the Central Bank to ever intervene in bond markets. Therefore, along the equilibrium path the Central Bank can achieve a Pareto improvement without taking risk for its balance sheet. However, we also show that alternative formulations of the price floor may induce the sovereign to ask for assistance in the face of bad fundamental shocks. *Ex-ante*, this option leads the sovereign to overborrow. Under both of these scenarios, interest rate spreads decline once the Central Bank announces the price floor schedule: in the first scenario, the reduction in interest rate spreads is due the elimination of rollover risk. In the second scenario, this reduction reflects the option for bondholders to resell the security to the Central Bank whenever the sovereign is approaching a solvency crises. Section 7.2 proposes a simple procedure to test which of these two hypothesis better characterizes the observed behavior of Italian spreads after the announcements of the OMT program.

7.1 Modeling OMT

We model OMT as follows. At the beginning of each period, after all uncertainty is realized, the government can ask for assistance. In such case the Central Bank (CB) commits to buy government bonds in secondary markets at a price $q_{n,CB}(\mathbf{S}, B', \lambda')$ that may depend on the state of the economy, \mathbf{S} , on the quantity of debt issued, B' , and on the maturity of the portfolio, λ' . We assume that assistance is conditional on the fact that total debt issued is below a cap $\bar{B}_{n,CB}(\mathbf{S}, \lambda') < \infty$ also set by the CB. The limit can depend on the state of the economy and on the duration of the stock of the debt portfolio. This limit captures the conditionality of the assistance in the secondary markets. Moreover, it rules out Ponzi-scheme on the central bank. Hence OMT is fully characterized by a policy rule $(q_{n,CB}(\mathbf{S}, B', \lambda'), \bar{B}_{n,CB}(\mathbf{S}, \lambda'))$. We assume that the CB finances such transactions with a lump sum tax levied on the lenders. We further assume that such transfers are small enough that they do not affect the stochastic discount factor $M_{t,t+1}$.

The problem for the government described in (29) changes as follows. We let $a \in \{0, 1\}$ be the decision to request CB assistance, with $a = 1$ for the case in which assistance is requested. Then we have:

$$V(\mathbf{S}) = \max_{\delta, B', \lambda', G, a} \delta \{U(G) + \beta \mathbb{E}[V(\mathbf{S}') | \mathbf{S}]\} + (1 - \delta) \underline{V}(s_1) \quad (18)$$

subject to

$$\begin{aligned} G + B &\leq Y(s_1) + \Delta(\mathbf{S}, a, B', \lambda'), \\ \Delta(\mathbf{S}, a, B', \lambda') &= \sum_{n=1}^{\infty} q_n(s, a, B', \lambda') [(1 - \lambda')^{n-1} B' - (1 - \lambda)^n B] \\ B'_n &\leq \bar{B}_{n,CB}(\mathbf{S}, \lambda') \quad \text{if } a = 1. \end{aligned}$$

The lenders have the option to resell government bonds to the CB at the price $q_{n,CB}$ in case the government asks for assistance. The no-arbitrage conditions for the lenders (6) is modified as follows: The lender's no-arbitrage condition requires that

$$q_n(\mathbf{S}, a, B', \lambda' | \lambda) = \max\{a q_{n,CB}(\mathbf{S}, B', \lambda' | \lambda); \delta(\mathbf{S}) \mathbb{E}\{M(s_1, s'_1) \delta(\mathbf{S}') q'_{n-1} | \mathbf{S}\}\} \quad \text{for } n \geq 1 \quad (19)$$

where $B'' = B'(s', B', \lambda')$, $\lambda'' = \lambda'(s', B', \lambda')$, $a' = a(s', B', \lambda')$, $q'_{n-1} = q_{n-1}(s', B'', \lambda'')$, and the initial condition is $q'_0 = 1$. It is important to notice that the bonds prices now depend also on the current and future decision of the government to activate assistance.

Given a policy rule (q_{CB}, \bar{B}_{CB}) , a *recursive competitive equilibrium with OMT* is value function for the borrower V , associated decision rules δ, B', λ', G and a pricing function q such that V, δ, B', G are a solution of the government problem (18) and the pricing functions satisfy the no-arbitrage condition (19). For exposition, it is convenient to define also the *fundamental equilibrium outcome* $\mathbf{y}^* = \{\delta_t^*, B_{t+1}^*, \lambda_{t+1}^*, G_t^*, q_{n,t}^*\}$ as the equilibrium outcome that maximizes the utility for the government given an initial portfolio of debt. We denote the objects of a recursive competitive equilibrium associated with the fundamental outcome with a superscript “*”.

We now turn to show that an appropriately designed policy rule can uniquely implement the fundamental equilibrium outcome, our normative benchmark.³²

³²Clearly, the model has incomplete markets and all sorts of inefficiencies (especially when considering an environment with long-term debt). We are going to abstract from policy interventions that aims to ameliorate such inefficiencies. OMT is only targeted at eliminating “bad” equilibria. Such features will also survive in models with complete markets or in environment where some notion of constrained efficiency can be achieved as in [Dovis \(2014\)](#).

Proposition 3. *The OMT rule can be chosen such that the fundamental equilibrium outcome is uniquely implemented and assistance is never activated along the path. In such case, OMT is a weak Pareto improvement relative to the equilibrium without OMT (strict if the equilibrium outcome without OMT does not coincide with the fundamental equilibrium).*

Proof. An obvious way to uniquely implement the fundamental equilibrium outcome is to set $q_{n,CB}(\mathbf{S}, B', \lambda') = q_n^*(s, B', \lambda')$ and $\bar{B}_{n,CB}(\mathbf{S}, \lambda') \leq (1 - \lambda)^{n-1} B'(\mathbf{S})$ if $\lambda = \lambda^*(\mathbf{S})$ and zero otherwise. Such construction is not necessary. A less extreme alternative is to design policies such that for all \mathbf{S} for which there is no default in the fundamental equilibrium, $\delta^*(\mathbf{S}) = 1$, there exists at least one (B', λ') with $(1 - \lambda')^{n-1} B' \leq \bar{B}_{n,CB}(\mathbf{S}, \lambda')$ such that

$$U(Y - B + \Delta(\mathbf{S}, 1, B', \lambda')) + \beta \mathbb{E}V^*(B', \lambda', s') \geq \underline{V}(s_1), \quad (20)$$

and for all (B', λ') such that $(1 - \lambda')^{n-1} B' \leq \bar{B}_{n,CB}(\mathbf{S}, \lambda')$ the fundamental equilibrium is always preferable, in that

$$U(Y - B + \Delta(\mathbf{S}, 1, B', \lambda')) + \beta \mathbb{E}V^*(B', \lambda', s') \leq V^*(\mathbf{S}). \quad (21)$$

Under (20) and (21), it is clear that no self-fulfilling run is possible and there is no over-borrowing. Hence (20) and (21) are sufficient conditions to eliminate runs and to uniquely implement the fundamental equilibrium outcome. \square

Note that quantity limits (conditionality) are necessary to uniquely implement the fundamental equilibrium. In absence of \bar{B}_{CB} , the government would choose a B' that is larger than the one in the fundamental equilibrium because it acts as a price taker under OMT. So, a limit to B' is necessary to prevent overborrowing.

Proposition 3 gives us the most benevolent interpretation of the drop in Italian spreads after OMT was announced. If OMT follows the rule described in the proof of Proposition 3, it uniquely implements the fundamental equilibrium outcome. In this case the observed drop in spreads is due to the fact that lenders anticipate that no run can happen along the equilibrium path resulting in lower default probability and hence lower spreads.

However, the central bank does not want to support bond prices if they are low because of fundamental reasons. This entails a subsidy from the lenders to the borrower, reducing welfare for the lenders relative to the equilibrium without OMT (assuming lenders are the ones that have to pay for the losses of the central bank). Even in this scenario, bond prices may decline. To see this, suppose that in a given state the fundamental price for the portfolio of debt is q^{*} . Suppose now that the ECB sets an assistance price $q'_{CB} > q^{*}$. It is clear from (19) that the price today increases (the spread drops) relative to a counterfactual

world without OMT.

Thus, a decline in the spreads is not informative on whether ECB is following the benchmark rule, or whether it is providing some subsidy to peripheral countries. We now use the calibrated model to test between these two alternatives.

7.2 A Simple Test

[This section is based on a previous calibration of the model. we are in the process of updating it.] We now test for the hypothesis that the ECB did follow the policy described in Proposition 3. To explain our approach, suppose that the Central Bank credibly commits to our normative benchmark. The announcement of this intervention would eliminate all the rollover risk, and the spreads today would jump to their fundamental value, i.e. the value that would arise if rollover crisis were not conceivable from that point onward. This fundamental level of the interest rate spread represents a lower bound on the post-OMT spread under the null hypothesis that the program was directed exclusively to prevent runs on Italian debt. Our test consists in comparing the spread observed after the OMT announcements to their fundamental value: if the latter is higher than the observed one, it would be evidence against the null hypothesis that the ECB followed the policy described in Proposition 3.

We perform this test using our calibrated model. Our procedure consists in three steps:

1. Obtain decision rules from the fundamental equilibrium.
2. Feed these decision rules with our series for the fundamental shocks $\{\chi_t, y_t\}$. Obtain counterfactual post-OMT fundamental spreads.³³
3. Compare post-OMT spreads with the counterfactual ones.

Table 6 reports the results. In the first column we have the Italian spreads observed after the OMT announcements, while the second column presents the counterfactual spreads constructed with the help of our model. We can verify that the observed spreads lie below the one justified by economic fundamentals under the most optimistic interpretation of OMT. In 2012:Q4, the observed spread on our spread series was 222 basis points, while our model suggests that the spread should have been 354 basis points if the program was exclusively eliminating rollover risk. Therefore, our model suggests that the decline in the spreads observed after the OMT announcements partly reflects the anticipation of a future

³³ The estimates of the state vector ends in 2012:Q2. For the 2012:Q3-2012:Q4 period, we set y_t equal to linearly detrended Italian output and we filter out $\{\chi_t, v_t\}$ using our pricing model along with the data on the German yield curve and the euro-area price-consumption ratio.

intervention of the ECB in secondary sovereign debt markets. This is not surprising given our result in Section 6: since rollover risk was almost negligible in 2012:Q2, the observed drastic reduction in the spreads should partly reflect the value of an implicit put option for holders of Italian debt guaranteed by the ECB.

Table 6: **Actual and fundamental sovereign interest rate spreads in Italy**

	Actual spreads	Spreads justified by fundamentals
2012:Q3	348.24	379.80
2012:Q4	222.25	354.60

Clearly, it would be interesting to use our model to dig deeper into the implications of the OMT program. For example, we could try to measure the put option implicit in this intervention, to calculate the amount of resources that the ECB is implicitly committing under this policy or we could assess the moral hazard implications associated to this policy. This would not be an uncontroversial task, as it would require us to i) specify the policy rule followed by the ECB and to ii) specify how the selection mechanism responds to the policy intervention. The test we have described in this section is robust to these caveats, and we regard it as a first step for the evaluation of this type of interventions in sovereign debt models with multiple equilibria.

8 Conclusion

This paper has proposed a strategy to bring to the data the benchmark sovereign debt model of [Eaton and Gersovitz \(1981\)](#) modified to allow for self-fulfilling debt crises as in [Cole and Kehoe \(2000\)](#). In this class of models, the observed maturity choices of the government allow to distinguish between fundamental and non-fundamental sources of variation in interest rate spreads. We apply this identification strategy to Italian data during the debt crisis of 2008-2011. Our preliminary results indicate that fluctuations in non-fundamental risk accounted for a modest fraction of the increase in sovereign borrowing costs. This finding suggests that the sharp reduction in spreads observed upon the establishment of the OMT program most likely reflect the expectation of future bailouts of peripheral euro-area countries.

The analysis considered belief-driven fluctuations that arise from rollover risk as introduced in [Cole and Kehoe \(2000\)](#). We did not consider the type of multiplicity emphasized

in Calvo (1988) and recently studied by Lorenzoni and Werning (2013) and Navarro, Nicolini, and Teles (2015). Future research should investigate which feature of the data can be used to discipline empirically this and other sources of multiplicity.

Our approach is not limited to sovereign bond markets, and it can be used in other applications where self-fulfilling expectations may be important drivers of default risk. Of particular interest may be the analysis of whether bankruptcies of financial intermediaries in periods like the Great Depression were driven by solvency or whether they were due “bank runs” á la Diamond and Dybvig (1983).

References

- Aguiar, Mark and Manuel Amador. 2014a. *Sovereign Debt*. North-Holland, 647–87.
- . 2014b. “Take the Short Route: How to Repay and Restructure Sovereign Debt with Multiple Maturities.” Manuscript, Federal Reserve Bank of Minneapolis.
- Aguiar, Mark, Satyajit Chatterjee, Hal Cole, and Zachary Stangebye. 2015. “Sovereign Debt Crises.” Mimeo, University of Pennsylvania.
- Aguiar, Mark and Gita Gopinath. 2006. “Defaultable Debt, Interest Rates and the Current Account.” *Journal of International Economics* 69:64–83.
- . 2007. “Emerging Market Business Cycles: The Cycle is the Trend.” *Journal of Political Economy* 115:69–102.
- Alesina, Alberto, Alessandro Prati, and Guido Tabellini. 1989. “Public Confidence and Debt Management: A Model and a Case Study of Italy.” NBER Working Paper, No. 3135.
- Altavilla, Carlo, Domenico Giannone, and Michele Lenza. 2014. “The Financial and Macroeconomic Effects of OMT Announcements.” ECB Working Paper Series, No 1707.
- Ang, Adrew and Monika Piazzesi. 2003. “A No-Arbitrage Vector Autoregression of Term Structure Dynamics with Macroeconomic and Latent Variables.” *Journal of Monetary Economics* 50:745–787.
- Angeletos, George-Marios. 2002. “Fiscal Policy With Non-Contingent Debt And The Optimal Maturity Structure.” *Quarterly Journal of Economics* 117:1105–1131.
- Arellano, Cristina. 2008. “Default Risk and Income Fluctuations in Emerging Economies.” *The American Economic Review* 98 (3):690–712.
- Arellano, Cristina and A. Ramanarayanan. 2012. “Default and the Maturity Structure in Sovereign Debt.” *Journal of Political Economy* 120:187–232.
- Aruoba, Boragan, Pablo Cuba-Borda, and Frank Schorfheide. 2014. “Macroeconomic Dynamics near the ZLB: A Tale of Two Countries.” Manuscript, University of Pennsylvania.
- Bai, Yan, Seon Tae Kim, and Gabriel Mihalache. 2014. “The Maturity and Payment Schedule of Sovereign Debt.” Manuscript, University of Rochester.
- Bianchi, Javier, Juan Carlos Hatchondo, and Leonardo Martinez. 2014. “International Reserves and Rollover Risk.” Manuscript, Wisconsin University.

- Bocola, Luigi. 2014. "The Pass-Through of Sovereign Risk." Manuscript, Northwestern University.
- Borri, Nicola and Adrien Verdhelan. 2013. "Sovereign Risk Premia." Manuscript, MIT Sloan.
- Broner, Fernando A., Guido Lorenzoni, and Sergio L. Schmukler. 2013. "Why Do Emerging Economies Borrow Short Term?" *Journal of the European Economic Association* 11:67–100.
- Buera, Francisco and Juan Pablo Nicolini. 2004. "Optimal maturity of government debt without state contingent bonds." *Journal of Monetary Economics* 51 (3):531–554.
- Calvo, Guillermo. 1988. "Servicing the Public Debt: The Role of Expectations." *The American Economic Review* 78:647–661.
- Campanaro, Alessandra and Dimitri Vittas. 2004. "Greco-Roman Lessons for Public Debt Management and Debt Market Development." World Bank Policy Research Working Paper 3414.
- Chatterjee, Satyajit and Burcu Eyigungor. 2012. "Maturity, Indebtedness and Default Risk." *The American Economic Review* 6:2674–2699.
- Cochrane, John and Monika Piazzesi. 2005. "Bond Risk Premia." *American Economic Review* 95:138–160.
- Cochrane, John H. 1994. "Permanent and Transitory Components of GNP and Stock Prices." *Quarterly Journal of Economics* 109:241–265.
- Cole, Harald L. and Timothy J. Kehoe. 2000. "Self-Fulfilling Debt Crisis." *Review of Economic Studies* 67:91–116.
- Conesa, Juan Carlos and Timothy J. Kehoe. 2012. "Gambling for Redemption and Self-Fulfilling Crises." Research Department Staff Report 465, Federal Reserve Bank of Minneapolis.
- Corsetti, Giancarlo and Luca Dedola. 2014. "The "Mystery of the Printing Press". Monetary Policy and Self-fulfilling Debt Crises." Manuscript, Cambridge University.
- Cruces, Juan J. and Christoph Trebesch. 2013. "Sovereign defaults: the price of haircuts." *American Economic Journal: Macroeconomics* 5:85–117.
- Dahlquist, Magnus and Henrik Hasseltoft. 2013. "International Bond Risk Premia." *Journal of International Economics* 90:17–32.

- De Grauwe, Paul and Yuemei Ji. 2013. "Self-fulfilling Crises in the Eurozone: An Empirical Test." *Journal of International Money and Finance* 34:15–36.
- Debortoli, Davide, Ricardo Nunes, and Pierre Yared. 2015. "Optimal Time-Consistent Government Debt Maturity." Tech. rep., mimeo UPF.
- Diamond, Douglas W and Philip H Dybvig. 1983. "Bank runs, deposit insurance, and liquidity." *The Journal of Political Economy* :401–419.
- Dovis, Alessandro. 2014. "Efficient Sovereign Default." Manuscript, Penn State University.
- Du, Wenxin and Jesse Schreger. 2015. "Local Currency Sovereign Risk." Forthcoming, *Journal of Finance*.
- Eaton, Jonathan and Mark Gersovitz. 1981. "Debt with potential repudiation: Theoretical and empirical analysis." *Review of Economic Studies* 48:289–309.
- Farmer, Roger E. A. and Jang-Ting Guo. 1995. "The econometrics of indeterminacy: an applied study." *Carnegie-Rochester Conference Series on Public Policy* 43:225–271.
- Fernández-Villaverde, Jesús and Juan Rubio-Ramírez. 2007. "Estimating Macroeconomic Models: A Likelihood Approach." *Review of Economic Studies* 74:1059–1087.
- Guvenen, Fatih and Anthony A. Smith. 2014. "Inferring Labor Income Risk and Partial Insurance From Economic Choices." *Econometrica* 82:2085–2129.
- Hatchondo, Juan Carlos, Leonardo Martinez, and Cesar Sosa Padilla. 2015. "Debt Dilution and Sovereign Default Risk." Forthcoming, *Journal of Political Economy*.
- Hebert, Benjamin and Jesse Schreger. 2015. "The Costs of Sovereign Default: Evidence from Argentina." Manuscript, Harvard Business School.
- Jovanovic, Boyan. 1989. "Observable Implications of Models with Multiple Equilibria." *Econometrica* 57:1431–1437.
- Judd, Kenneth L., Lilia Maliar, Serguei Maliar, and Rafael Valero. 2014. "Smolyak method for solving dynamic economic models: Lagrange interpolation, anisotropic grid and adaptive domain." *Journal of Economic Dynamics and Control* 44:92–123.
- Longstaff, F. A., J. Pan, L. H. Pedersen, and K. J. Singleton. 2011. "How Sovereign is Sovereign Credit Risk?" *American Economic Journal: Macroeconomics* 3:75–103.
- Lorenzoni, Guido and Ivan Werning. 2013. "Slow Moving Debt Crisis." Manuscript, Northwestern University.

- Lubik, Thomas A. and Frank Schorfheide. 2004. "Testing for Indeterminacy: An Application to U.S. Monetary Policy." *American Economic Review* 94:190–217.
- Missale, Alessandro and Olivier Jean Blanchard. 1994. "The Debt Burden and Debt Maturity." *American Economic Review* 84:309–319.
- Navarro, Gaston, Junapa Nicolini, and Pedro Teles. 2015. "Sovereign Defaults: The Role of Expectations." Manuscript, Federal Reserve Bank of Minneapolis.
- Nelson, Charles R. and Andrew F. Siegel. 1987. "Parsimonious Modeling of Yield Curves." *Journal of Business* 60:473–489.
- Pagano, Marco. 1988. "The Management of Public Debt and Financial Markets." In *High public debt: the Italian experience*, edited by Francesco Giavazzi and Luigi Spaventa. Cambridge University Press.
- Roch, Francisco and Harald Uhlig. 2014. "The Dynamics of Sovereign Debt Crises and Bailouts." Manuscript, University of Chicago.
- Salomao, Juliana. 2014. "Sovereign Debt Renegotiation and Credit Default Swaps." Manuscript, Carlson School of Management.
- Sanchez, Juan M., Horazio Sapriza, and Emircan Yurdagul. 2015. "Sovereign Default and Maturity Choice." Manuscript, University of Rochester.
- Scarpelli, Giandomenico. 2001. *La Gestione del Debito Pubblico in Italia. Obiettivi e Tattiche di emissione dei titoli di Stato dagli anni Settanta ai giorni dell'euro*. Bancaria Editrice.
- Svensson, Lars E. O. 1994. "Estimating and Interpreting Forward Rates: Sweden 1992–1994." NBER Working Paper No. 4871.
- Tabellini, Guido. 1988. "Monetary and Fiscal Policy Coordination With a High Public Debt." In *High public debt: the Italian experience*, edited by Francesco Giavazzi and Luigi Spaventa. Cambridge University Press.

APPENDIX

A Proofs for the Three Period Economy

Proof of Proposition 1. It is helpful to use a primal approach to solve for the equilibrium outcome. Without rollover risk and uncertainty at $t = 0$, we can consider the following programming problem:

$$\max_{b_{01}, b_{02}, b_{12}, \delta_1, \delta_2} U(G_0) + \beta \mathbb{E}_0 \{ \delta_1 [U(G_1) + \beta (\delta_2 U(G_2) + (1 - \delta_2) \underline{V}_2)] + (1 - \delta_1) \underline{V}_1 \} \quad (22)$$

subject to budget constraints

$$\begin{aligned} G_0 + D_0 &\leq q_{01} b_{01} + q_{02} b_{02} + Y_0 \\ G_1 + b_{01} &\leq q_{12} b_{12} + Y_1 \\ G_2 + b_{02} + b_{12} &\leq Y_2 \end{aligned}$$

the pricing equations

$$\begin{aligned} q_{01} &= m, \quad q_{02} = mq_{12}, \\ q_{12} &= \mathbb{E}_1 [m_{12} \delta_2], \end{aligned}$$

the “default” constraints

$$\begin{aligned} U(G_1) + \beta \mathbb{E}_1 U(G_2) &\geq \underline{V}_1 \\ U(G_2) &\geq \underline{V}_2 \end{aligned}$$

and the “debt-dilution” constraint

$$U(G_1) + \beta \mathbb{E}_1 U(G_2) \geq V_1(b_{01}, b_{02}) \quad (23)$$

It is clear that a fundamental equilibrium outcome solves the above problem and the converse is also true. For notational simplicity we assume that $\underline{V}_2 = U((1 - \tau)Y_2)$ for some $\tau \in (0, 1)$.

We now show that short term debt is desirable because it relaxes the debt-dilution constraint (23). To this end, consider a relaxed version of (22) in which we drop the debt-

dilution constraint (23). Notice that such relaxed problem has a continuum of solutions since the split between long and short term debt issued in period zero is underdetermined. Let $\{b_{01}^*, b_{02}^*, b_{12}^*, \delta_1^*, \delta_2^*\}$ be a solution to this relaxed programming problem. The optimality condition for b_{12} for this relaxed problem is

$$0 = m \frac{\partial q_{12}}{\partial b_2} b_{02}^* U'(c_0^*) + \left(q_{12}^* + \frac{\partial q_{12}}{\partial b_2} b_{12}^* \right) U'(Y_1 - b_{01}^* + q_{12}^* b_{12}^*) - \int_{\{Y_2 \geq \frac{b_{02}^* + b_{12}^*}{\tau}\}} U'(Y_2 - b_{02}^* - b_{12}^*) d\mu_{Y_2} \quad (24)$$

This implies that if $b_{02}^* = 0$ then the government at $t = 0$ can achieve the value of this relaxed problem in the more constrained problem (22). To see this it is sufficient to check that the dilution constraint is met at $\{b_{01}^*, b_{02}^*, b_{12}^*, \delta_1^*, \delta_2^*\}$. To this end notice that starting at (b_{01}^*, b_{02}^*) in period $t = 1$ the optimal b_{12} chosen by period 1 government is such that

$$0 = \left(q_{12}^* + \frac{\partial q_{12}}{\partial b_2} b_{12} \right) U'(Y_1 - b_{01}^* + q_{12} b_{12}) - \int_{\{Y_2 \geq \frac{b_{02}^* + b_{12}}{\tau}\}} U'(Y_2 - b_{02}^* - b_{12}) d\mu_{Y_2} \quad (25)$$

where $q_{12} = m \Pr\left(Y_2 \geq \frac{b_{02}^* + b_{12}}{\tau}\right)$. Hence the allocation $\{b_{01}^*, b_{02}^*, b_{12}^*, \delta_1^*, \delta_2^*\}$ satisfies the debt-dilution constraint if and only if it satisfies (25) with $b_{12} = b_{12}^*$ and $q_{12} = q_{12}^*$. Now, from (24) and $b_{02}^* = 0$ it follows that (25) is satisfied. Hence the solution to the relaxed problem can be implemented when $b_{02}^* = 0$.

The final step in the proof is to show that $b_{02}^* = 0$ is necessary when the solution to (22) is such that there are defaults in $t = 2$ in some states. Note that (24) and (25) can be jointly satisfied if and only if $\frac{\partial q_{12}}{\partial b_2} b_{02}^* = 0$ so at least one of the following conditions must be satisfied: i) $b_{02}^* = 0$, ii) no default at $t = 2$ so that $\partial q_{12} / \partial b_2 = 0$. Hence if there are defaults in $t = 2$ then it must be that $b_{02}^* = 0$. *Q.E.D.*

Proof of Proposition 2. The proof is by contradiction. Suppose $b_{01} > 0$. Consider then the following variation: increase b_{02} by $\varepsilon/q_{02} > 0$, and decrease b_{01} by $\varepsilon/q_{01} > 0$ so that G_0 is unchanged. Moreover, notice that - under the assumption that there is no fundamental default risk - the optimal allocation that can be achieved at $t = 1$ starting from (b_{01}, b_{02}, s) can be achieved with $(b_{01} - \varepsilon/q_{01}, b_{02} + \varepsilon/q_{02})$. In fact, since there are no shocks at $t = 1, 2$ we have that $q_{12} = m$ and at the original allocation the following Euler equation is satisfied:

$$mU'(G_1) = \beta U'(G_2) \quad (26)$$

and so it is clear that achieving same G_1, G_2 is budget feasible and optimal.

We next turn to show that the proposed variation reduces the crisis zone. In fact, at $t = 1$ there can be a rollover crisis only if

$$U(Y - b_{01}) + \beta EV_2(b_{02}, Y_2) \leq \underline{V}_1 \quad (27)$$

$$U(Y - b_{01} + \varepsilon/q_{01}) + \beta EV_2(b_{02} + \varepsilon/q_{02}, Y_2) \leq \underline{V}_1 \quad (28)$$

The fact that (26) holds at the original allocation implies that if $b_{12} > 0$ then

$$q_{12}U'(Y - b_{01}) > \beta EV_2'(b_{02} + b_{12}) \iff \frac{1}{q_{01}}U'(Y - b_{01}) > \frac{1}{q_{02}}\beta EV_2'(b_{02} + b_{12})$$

So we have that

$$\begin{aligned} U(Y - b_{01} + \varepsilon/q_{01}) + \beta EV_2(b_{02} + \varepsilon/q_{02}, Y_2) &\approx [U(Y - b_{01}) + \beta EV_2(b_{02}, Y_2)] \\ &\quad + \left[\frac{1}{q_{01}}U'(Y - b_{01}) + \frac{1}{q_{02}}\beta EV_2'(b_{02}, Y_2) \right] \varepsilon \\ &> U(Y - b_{01}) + \beta EV_2(b_{02}, Y_2) \end{aligned}$$

Hence if the second inequality is satisfied so it is the first but not vicesa. Hence the variation reduces the probability of default (rollover crisis) at $t = 1$ because (28) is less likely to hold than (27). Then we have that consumption in the first period is larger and so the variation increases utility, a contradiction. *Q.E.D.*

B Numerical Solution

Let $\mathbf{S} = [B, \lambda, y, \chi, p]$ be the vector collecting the model's state variables. Before explaining the numerical solution, it is convenient to rewrite the decision problem for the government as follows

$$V(\mathbf{S}) = \max_{\delta \in \{0,1\}, B', \lambda', G} \delta \{U(G) + \beta \mathbb{E}[V(\mathbf{S}') | \mathbf{S}]\} + (1 - \delta)\underline{V}(s_1) \quad (29)$$

subject to

$$\begin{aligned} G + B &\leq Y(s_1) + \Delta(\mathbf{S}, B', \lambda'), \\ \Delta(\mathbf{S}, B', \lambda') &= q(s, B', \lambda' | \lambda') B' - q(s, B', \lambda' | \lambda) (1 - \lambda) B, \end{aligned}$$

where $q(s, B', \lambda' | \lambda)$ is the per unit value of a portfolio of ZCBs with decay parameter λ given the realization s for the exogenous state, and given the government's choices for the new portfolio is (B', λ') . The price of this portfolio of ZCBs can be written as

$$q(s, B', \lambda' | \lambda) = \delta(\mathbf{S}) \mathbb{E} \{ M(s_1, s'_1) \delta(\mathbf{S}') [1 + (1 - \lambda)q(s, B'', \lambda'' | \lambda)] | \mathbf{S} \}, \quad (30)$$

where $B'' = B'(s', B', \lambda')$ and $\lambda'' = \lambda'(s', B', \lambda')$.

We define the value of repaying the debt conditional on lenders rolling over the debt, $V_{\text{roll}}^R(\mathbf{S})$, as follows

$$V_{\text{roll}}^R(\mathbf{S}) = \max_{B', \lambda'} \{ U(Y - B + \Delta(\mathbf{S}, B', \lambda')) + \beta \mathbb{E}[V(B', \lambda', s') | \mathbf{S}] \}. \quad (31)$$

The value of repaying conditional on lenders not rolling over the debt, $V_{\text{no roll}}^R(\mathbf{S})$, is

$$V_{\text{no roll}}^R(\mathbf{S}) = \{ U(Y - B) + \beta \mathbb{E}[V(B(1 - \lambda), \lambda, s') | \mathbf{S}] \}, \quad (32)$$

while the value of defaulting, $V^D(y, \chi)$, is

$$V^D(y, \chi) = \{ U(Y[1 - \tau(Y)]) + \beta \{ \psi \mathbb{E}[V(0, \bar{\lambda}, y', \chi') | \mathbf{S}] + (1 - \psi) \mathbb{E}[V^D(y', \chi') | \mathbf{S}] \} \}. \quad (33)$$

The value function for the government decision problem can then be written as

$$V(\mathbf{S}) = \begin{cases} V_{\text{roll}}^R(\mathbf{S}) & \text{if } V_{\text{no roll}}^R(\mathbf{S}) \geq V^D(y, \chi) \\ V_{\text{roll}}^R(\mathbf{S}) & \text{if } V_{\text{no roll}}^R(\mathbf{S}) < V^D(y, \chi) \text{ and } \xi < p \\ V^D(y, \chi) & \text{otherwise} \end{cases}$$

When coupled with the pricing function q , the knowledge of $\{V_{\text{roll}}^R(\mathbf{S}), V_{\text{no roll}}^R(\mathbf{S}), V^D(y, \chi)\}$ is sufficient to solve for the policy functions of the model. The numerical solution consists in approximating these three value functions and the pricing schedule q .

The inverse duration for the debt portfolio, λ , is assumed to be a discrete variable from the set $\Lambda = \{\lambda_1, \lambda_2, \dots, \lambda_N\}$. The value functions are approximated using piece-wise smooth functions. Specifically, $V_{\text{roll}}^R(\cdot)$, is approximated as follows,

$$V_{\text{roll}}^R(\lambda_j, \tilde{\mathbf{S}}) = \gamma_{\text{roll}, \lambda_j}^R \mathbf{T}(\tilde{\mathbf{S}}),$$

where $\tilde{\mathbf{S}} = [B, y, \chi, p]$ is a realization of state variables that excludes λ , $\gamma_{\text{roll}, \lambda_j}^R$ is a vector of coefficients and $\mathbf{T}(\cdot)$ is a vector collecting Chebyshev's polynomials. The value of repaying

conditional on the lenders not rolling over the debt, and the value of defaulting are defined in a similar fashion, and we denote by $\gamma_{\text{no roll}, \lambda_j}^R$ and ψ^D the coefficients parametrizing those values. The pricing schedule q is approximated over a grid of possible debt choices, $\mathcal{B} = [b_1, \dots, b_K]$. Letting $s = [y, \chi, p]$ be a realization of the exogenous states, we have that the price of a λ portfolio given the government's choices (B', λ') is $q(s, B', \lambda' | \lambda)$, as defined in equation (30).³⁴

Letting $\Gamma = [\{\gamma_{\text{roll}, \lambda_j}^R, \gamma_{\text{no roll}, \lambda_j}^R\}_j, \gamma^D]$ collect the coefficients that parametrize the value functions, we can index the model's numerical solution by (Γ, q) . Our procedure consists in using the government's decision problem and the lenders' no arbitrage condition to iterate over (Γ, q) until a convergence criterion is achieved. Specifically, the algorithm for the numerical solution of the model is as follows:

- **Step 0: Defining the grid and the polynomials.** Specify the set of values in Λ . Set upper and lower bounds for the state variables $\tilde{\mathbf{S}} = [B, y, \chi, p]$. Given these bounds, construct a μ -level Smolyak grid and the associated Chebyshev's polynomials $\mathbf{T}(\cdot)$ following [Judd, Maliar, Maliar, and Valero \(2014\)](#). Let $\tilde{\mathcal{S}}$ denote the set of points for the state variables $\tilde{\mathbf{S}}$.
- **Step 1: Update value functions.** Start with a guess for the value and pricing functions, (Γ^c, q^c) . For each $\mathbf{S}^i \in \Lambda \times \tilde{\mathcal{S}}$, update the value functions using the definitions in equation (31)-(33). Denote by Γ^u the updated guess, and by $[r_{\text{roll}}^R, r_{\text{noroll}}^R, r^D]$ the distance between the initial guess and its update using the sup-norm.
- **Step 2: Update pricing function.** For each exogenous state s^i in the relevant subset of $\tilde{\mathcal{S}}$ and for each $(B^{i'}, \lambda^{i'}) \in \mathcal{B} \times \Lambda \times \Lambda$, evaluate the right hand side of equation (30) using (Γ^u, q^c) . Denote by $\hat{q}^u(s^i, B^{i'}, \lambda^{i'})$ this value, and by r^Q the distance between q^c and \hat{q}^u under the sup norm. Update the pricing schedule as

$$q^u(\cdot) = \theta \hat{q}^u(\cdot) + (1 - \theta) q^c(\cdot) \quad \theta \in (0, 1).$$

- **Step 3: Iteration.** If $\max\{r_{\text{roll}}^R, r_{\text{noroll}}^R, r^D\} \leq 10^{-6}$ and $r^Q \leq 10^{-3}$, stop the algorithm. If not, set (Γ^u, q^u) as the new guess, and repeat Step 1-2. \square

³⁴A complication of our approach to maturity choices relative to the set up in [Arellano and Ramanarayanan \(2012\)](#) is that we need to price an arbitrary λ portfolio, given government choices (B', λ') , in order to know the market value of the portfolio repurchased by the government. See [Sanchez, Sapriza, and Yurdagul \(2015\)](#) for a discussion.

Regarding the specifics of the algorithm, we generate \mathcal{S} using an anisotropic Smolyak grid of $\mu = 6$ in the B dimension and $\mu = 3$ on the other dimensions. The upper and lower bound for B are $[0, 2\mu_y]$, while the upper and lower bounds for $s = (y, \chi, p)$ are equal to ± 3 times the standard deviation of these stochastic processes. The grid for λ contains 5 values: ± 2 years round an average duration of seven years (the Italian pre-crisis level). The grid for debt choices over which the pricing function is defined, \mathcal{B} , consists of 100 equally spaced values between $[0, 2\mu_y]$. Expectations over future outcomes are computed using Gauss-Hermite quadrature, with $n = 15$ sample points on each random variable. The smoothing parameter for the updating of the pricing schedule is set at $\theta = 0.05$.

In the numerical solution, we introduce a small cost for adjusting debt maturity,

$$\alpha \left(\frac{4}{\lambda'} - \bar{d} \right)^2.$$

We set $d = 7$, and $\alpha = 0.001$. We introduce this adjustment cost for two purposes. First, it ameliorates the convergence properties of the algorithm as it breaks down indifference in region of the state space where default risk and risk premia on long term bonds are small.³⁵ Second, we make sure that in this region of the state space the maturity choice is consistent with the pre-crisis level of the weighted average life of Italian outstanding debt.

C The Lenders' Stochastic Discount Factor

We now derive some results concerning the lenders' stochastic discount factor, and describe in more details our calibration. Let $q_t^{*,n}$ be the log price of a non-defaultable ZCB maturing in n periods. These bond prices satisfy the recursion

$$\exp\{q_t^{*,n}\} = \mathbb{E}_t[M_{t,t+1} \exp\{q_{t+1}^{*,n-1}\}],$$

where $M_{t,t+1}$ is defined in the system (10), and the initial condition is $q_t^0 = 0$. Ang and Piazzesi (2003) show that $\{q_t^{*,n}\}$ are linear functions of the state variable χ_t ,

$$q_t^{*,n} = A_n + B_n \chi_t,$$

³⁵Maturity choices in the model are not determined absent default risk and with risk neutral lenders.

where A_n and B_n satisfy the recursion

$$\begin{aligned} B_{n+1} &= -\tau_1 + B_n \phi^*, \\ A_{n+1} &= -\tau_0 + A_n + B_n \mu^* + \frac{1}{2} B_n^2 \sigma_\chi^2, \end{aligned} \tag{34}$$

with $A_0 = B_0 = 0$, $\phi^* = [\phi - \sigma_\chi^2 \kappa_1]$ and $\mu^* = [\mu(1 - \phi) - \sigma_\chi^2 \kappa_0]$. It is important to highlight that A_n and B_n are implicitly functions of $\theta_1 = [\tau_0, \tau_1, \kappa_0, \kappa_1, \mu_\chi, \rho_\chi, \sigma_\chi]$.

We now discuss in details the restrictions described in Section 5.3.1. We then present the results of the C-P regressions, and the calibration of θ_1 .

C.1 Restrictions on the Stochastic Discount Factor

C.1.1 Log forward rates and χ_t

By definition, the log forward rate at time t for loans between $t + n - 1$ and $t + n$ equals

$$f_t^n = q_t^{*,n-1} - q_t^{*,n} = \underbrace{(A_{n-1} - A_n)}_{\tilde{A}_n} + \underbrace{(B_{n-1} - B_n)}_{\tilde{B}_n} \chi_t. \tag{35}$$

Given equation (35), we can express $\hat{\gamma}_0 + \hat{\gamma}' \mathbf{f}_t^{\text{model}}$ as

$$\hat{\gamma}_0 + \hat{\gamma}' \mathbf{f}_t^{\text{model}} = \hat{\gamma}_0 + \sum_{j=1}^6 \hat{\gamma}_j \tilde{A}_{4(j-1)} + \left(\sum_{j=1}^6 \hat{\gamma}_j [\tilde{B}_{4(j-1)}] \right) \chi_t.$$

Therefore, one has that $\chi_t = \hat{\gamma}_0 + \hat{\gamma}' \mathbf{f}_t^{\text{model}}$ if the following conditions hold

$$\begin{aligned} \hat{\gamma}_0 + \sum_{j=1}^6 \hat{\gamma}_j \tilde{A}_{4(j-1)} &= 0, \\ \left(\sum_{j=1}^6 \hat{\gamma}_j [\tilde{B}_{4(j-1)}] \right) &= 1. \end{aligned} \tag{36}$$

C.1.2 Cochrane and Piazzesi (2005) regressions

By definition, holding period excess log returns on a ZCB maturing in $n = 20$ quarters equal $rx_{t+1}^{20} = q_{t+1}^{*,19} - q_t^{*,20} + q_t^{*,1}$. Substituting the expression for log prices, we can rewrite

it as

$$rx_{t+1}^{20} = \underbrace{[A_{19} + B_{19}\mu(1 - \phi) - A_{20} + A_1]}_{a_{20}^{model}} + \underbrace{[B_{19}\phi - B_{20} + B_1]}_{b_{20}^{model}} \chi_t + B_{19}\varepsilon_{\chi,t+1}. \quad (37)$$

If the restrictions described in the previous subsection hold, Equation (37) has the same form of the C-P predictive regressions we have estimated in Section 5.3.1, and it will exactly reproduce the results reported in Table A-2 if the following conditions hold

$$\begin{aligned} a_{20}^{model} &= \hat{a}_{20}, \\ b_{20}^{model} &= \hat{b}_{20}, \\ B_{19}\sigma_{\chi}^2 &= \hat{\sigma}_{\eta^{20}}^2. \end{aligned} \quad (38)$$

C.1.3 The risk free rate

By definition, log-yields on a bond maturing next quarter equal $y_t^1 = -q_t^{*,1}$. We can express it as

$$y_t^1 = \tau_0 + \tau_1 \chi_t. \quad (39)$$

The mean and variance of y_t^1 can then be easily derived as a function of deep model parameters

$$\mathbb{E}[y_t^1] = \tau_0 + \tau_1 \mu \quad \text{var}[y_t^1] = \tau_1^2 \frac{\sigma_{\chi}^2}{(1 - \phi^2)}. \quad (40)$$

C.2 Calibration of θ_1

We use our data on the term structure of German's ZCBs to construct time series for the realized excess log returns and the log forward rates for $n = 4, 8, 12, 16, 20$, following the above definitions. Table A-1 reports summary statistics on yields and realized excess returns at different horizons. We can verify that the yield curve slopes up on average: yields on 5 years bonds are, on average, 80 basis points higher than yields on bonds maturing next quarter. We can also see that long term bonds earn a positive excess return over our sample. For example, holding a 5 year bond and selling it off next quarter earns, on average, an annualized premium of 2.40% relative to investing the same amount of money in a bond that matures next quarter. Excess returns on long term bonds increase monotonically with n , and so does their Sharpe ratio.

Table A-2 reports the results of the C-P regressions. The top panel reports OLS estimates of equation (15), where \bar{rx}_{t+1} are realized excess log returns averaged across

Table A-1: **Summary statistics: yields and holding period returns**

	Mean	Standard deviation	Sharpe Ratio
$y_t^1 - \text{infl}_t$	2.16	1.93	
$y_t^{20} - \text{infl}_t$	2.94	1.72	
rx_{t+1}^4	0.21	2.05	0.11
rx_{t+1}^8	0.94	4.22	0.22
rx_{t+1}^{12}	1.54	6.08	0.25
rx_{t+1}^{16}	2.02	7.70	0.26
rx_{t+1}^{20}	2.40	9.14	0.26

Notes: The sample period is 1973:Q1-2013:Q4. Variables are reported as annualized percentages (multiplied by a factor 400).

$n = 4, 8, 12, 16, 20$ and the vector \mathbf{f}_t includes the risk free rate and the log forward rates for our five maturities. The bottom panel reports the individual bond regressions of equation (16). Differently from the analysis of [Cochrane and Piazzesi \(2005\)](#) on U.S. data, the estimated vector $\hat{\gamma}$ is not “tent” shaped. However, we confirm using German data the finding that a single linear combination of log forward rates has predictive power for excess log returns, and that the sensitivity of the latter to this factor (the estimated b_n 's) increases with maturity.

Table A-2: **C-P regressions**

	γ_0	γ_1	γ_2	γ_3	γ_4	γ_5	γ_6	R^2
Estimates of equation (15)	-0.002 (-0.27)	-1.65 (-2.89)	5.00 (2.92)	-21.70 (-2.10)	47.20 (1.58)	-45.18 (-1.19)	16.53 (0.95)	0.12
			a_n		b_n			R^2
	4		-0.001 (-2.06)		0.46 (5.48)			0.20
	8		-0.000 (-0.37)		0.77 (4.92)			0.13
Estimates of equation (16)	12		0.000 (0.14)		1.02 (4.60)			0.11
	16		0.001 (0.30)		1.27 (4.55)			0.11
	20		0.001 (0.34)		1.48 (4.56)			0.11

Notes: The sample period is 1973:Q1-2013:Q4. Robust t -statistics in parenthesis.

The parameters in θ_1 are chosen so that i) the conditions in (36) are satisfied, ii) the

model reproduces the predictive regressions in Table A-2 (the equations in (38) hold), and iii) the mean and volatility of the risk free rate in the model, defined in equation (40), equal the associated sample moments reported in Table A-1.

D Endogeneity of SDF and output.

For tractability, we have assumed that lenders' stochastic discount factor is independent on the probability of a government default. While this assumption may be uncontroversial if one considers a small open economy, this might be problematic for a country like Italy. In fact, it is natural to think that a default of a large economy would have adverse consequences on bondholders, and that the prospect of this event may alter their attitude toward risk.³⁶ Because of that, one may think that our procedure underestimates the importance of rollover risk: by making a sovereign default more likely, an increase in the probability of a rollover crisis could lead to an increase in the risk aversion of lenders, and impact interest rate spreads through risk premia. Our approach could erroneously misinterpret this indirect effect of rollover risk as a shock to the lenders' stochastic discount factor. We next show that this is not to be expected.

To understand why, it is important to stress that the quantitative importance of rollover risk is identified in the model from the joint behavior of debt duration and interest rate spreads. Consider a version of the model in which future default probabilities affect the lenders' stochastic discount factor. In particular, assume that the prospect of a government default makes the stochastic discount factor more volatile,

$$M(s, s') = \begin{cases} \frac{\Pr(s'|s)}{1+r^*} \frac{1}{\mathbb{E}[(1+m)(1-\delta(s'))]} & \text{if } \delta(s') = 1 \\ \frac{\Pr(s'|s)}{1+r^*} \frac{(1+m)}{\mathbb{E}[(1+m)(1-\delta(s'))]} & \text{if } \delta(s') = 0 \end{cases}, \quad m > 0. \quad (41)$$

From equation (41) we have that the risk free rate is constant and equal to $1 + r^*$ and the price of risk is increasing in the probability of default. If $\mathbb{E}_t[1 - \delta]$ increases then $M_{t,t+1}$ increases in a second order stochastic dominance sense. When facing the pricing kernel in equation (41) the government has an *extra motive to lengthen* debt maturity because, by doing so, it reduces not only rollover risk, but also its price. This discussion indicates that our calibration would assign a more limited role to rollover risk if we were to incorporate this feedback in the model, because the model would imply an even more counterfactual behavior for debt maturity over the sample.

³⁶For example, this prediction would arise in a set up where lenders are exposed to government debt and they face occasionally binding constraints on their funding ability, see [Bocola \(2014\)](#).

The same exact argument can be made for output. Previous literature has suggested that the prospects of a future sovereign default are recessionary, see [Bocola \(2014\)](#). One may then argue that expectations of future rollover crises can reduce output, and we could misinterpret this indirect effect as an endowment shock. However, this is ruled out by our identification strategy. If output depends negatively on the probability of future default, then a government facing a higher prospects of rollover risk would have an extra incentive to lengthen the duration of its debt, because that would reduce rollover risk and mitigate the fall in output.

In sum, by looking at the behavior of debt duration, our identification strategy is not likely to underestimate rollover risk because of concerns on the endogeneity of output and of the lenders' stochastic discount factor.