Sovereign Debt Restructurings and the Short-term Debt Curse\textsuperscript{1}

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Abstract:
We present novel evidence that, in present value terms, creditors with short maturity securities suffer significantly more than creditors with long maturity securities during recent sovereign debt restructuring episodes. We also document a new stylized fact, consistent with this novel evidence, that the sovereign yield curve becomes systematically inverted as countries are approaching sovereign default or restructuring. We then show, using a simple model of sovereign debt, how model-implied differential NPV haircuts between short- and long-term creditors, can explain the observed dynamics of the sovereign yield curve.

Key words: Sovereign Debt; Default; Debt Restructuring; Maturity Structure; Yield Curve; Creditor Losses

JEL classification: F34, F41, H63

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1. Introduction

A long standing issue is whether creditors with different types of securities are experiencing different losses in sovereign debt restructuring. Intuitively if during a restructuring all outstanding bonds are exchanged, possibly after the same face value haircut, against a single bond with long maturity, then creditors with short maturity securities will experience bigger losses in net present value (NPV) terms, than the creditors with long maturity securities. In this paper, we document empirically that short term debt creditor, where short term debt is understood in terms of remaining maturity, are systematically suffering higher NPV losses than long term debt creditors. This establishes, as a novel fact, that the effective haircut curve is typically downward sloping, and therefore that there is curse in facing restructuring with short-term debt: the short-term debt curse. To our knowledge, our paper is the first to document this new stylized evidence and as such contributes to the empirical literature on creditor losses during sovereign debt restructurings and their distribution among different debt holders.4

The existence of a short-term debt curse has important implications for understanding the dynamic of the yield curve. The standard factor explaining an upward sloping sovereign yield curve is that creditors holding long-term bonds are facing higher uncertainty as they are exposed to default risk over a much longer period than short-term bond.5 This is consistent with the empirical findings that these countries pay a higher risk premium on long-term than on short-term bonds. On the contrary to this, our new findings bring about a new factor affecting the slope of the yield curve that goes in the opposite direction: shorter creditors are losing disproportionately more than long term creditors in the event of restructuring. As a consequence, the actual slope of the yield curve would depend on the relative importance of these two factors. In particular, we should expect that the short-term debt curse plays a much more important role as the country is facing a near-default situation. Indeed, we document, a second novel stylized fact, that the sovereign yield curve becomes systematically inverted in the run-up to sovereign default and restructuring.6

The two new stylized facts documented in this paper – the downward sloping haircut curve and the inversion of the yield curve in near default situations – are fully consistent from a debt pricing perspective. As default risk increases, sovereign spreads increase across the


5 See Broner et al. (2013), Arellano and Ramanarayanan (2012), and Chatterjee and Eyigungor (2012).

6 The yield curve could become inverted in non-default or non-restructuring cases. Commonly, the inversion has lasted only short horizon and the yield curve has returns to upward sloping with lower level of yields than 1 month prior to the inversion.
spectrum of maturity of instruments but even more so for short maturity securities which will be disproportionately more affected by the default. Equivalently, the premium to get insured against default risk, using credit default swap (CDS), will increase by more for short maturities. The objective of the theoretical model presented in the second part of the paper is therefore not only to explain how these two facts are internally consistent but also to show how they emerge in equilibrium in a sovereign debt model in which restructuring decisions and the implied NPV haircuts are endogenous.

Our theoretical model of sovereign consists in a 3-periods model in the style of Broner et al. (2013). In the first period, a sovereign issues short- and long-term debt to risk-averse international creditors. In the second and third periods, the government experiences fiscal shocks that affect its ability to repay debt. At the end of the second period, the government optimally decides whether to repay to short-term creditors or to declare default and restructuring. At restructuring, initial one-period bonds will be exchanged for two-period bonds and the same face value haircut is applied to all bonds. This symmetric treatment across creditors in nominal term reflects legal constraints for sovereign debtors stated in the *pari passu* clause “ranking all creditors equally.” As a result, the “effective” NPV haircut on the initial short-term bond will exceed that of the long-term bond, creating a short term debt curse as observed in the data. The advantage of the model is to allow then to explore the implication of the differential NPV haircut between short- and long-term creditors, on the initial pricing of debt. Solving the model backward, we show that the slope of the initial yield curve depends the probability of future negative fiscal shocks, and thus of the probability to face a default situation. If the probability of negative fiscal shocks is below some threshold, then long term debt spreads are higher than short-term debt spreads and a “normal” positively sloped yield curve arises. If, on the contrary, the same probability is high, then the short term debt curse becomes more likely and the slope of the yield curve get reversed because the sovereign is more likely to restructure its debt in period 1, and by doing so generate the short-term debt curse. Finally, we can explore, among other sensitivity analysis, how the persistence and mean reversion of fiscal shocks across the second and third periods affects this result. Our analysis incorporates the role of differential creditor losses in the analysis of risk factors driving the dynamic of the sovereign yield curve.

Our empirical findings on the yield curve dynamics prior to inversion are in line with those in Broner et al. (2013) and Arellano and Ramanarayanan (2012); initially, the yield curve stays at low level of yields and is upward sloping – yields on long-term bonds are higher than

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7 See Bucheheit and Pam (2004) for the *pari passu* clause in sovereign debt and Asonuma et al. (2014) for the *pari passu* clause in Belize restructurings in 2012-13.

8 This assumption is consistent with an observed pattern of nominal haircut treatment across creditors in recent sovereign debt restructurings.

those on long-term bonds. As the sovereign approaches close to debt distress, both yields on short-term and long-term bonds increase, raising the yield curve at the higher level of yields. In contrast, our new finding on the yield curve evolution after inversion associated with debt restructuring contributes to the literature. Once the creditors anticipate risks of restructuring with larger losses on short-term bonds, an increase in short-term bond yields outweighs one on long-term bond yields, inverting the yield curve. At the time of defaults or announcement of the restructuring, the yield curve becomes more elevated and a steeply downward sloping, which coincides with larger creditor losses on short-term bonds in ex-post at debt restructuring.

The rest of the paper is structured as follows. Section 2 reviews the related literature. We present the new empirical findings in Section 3. Section 4 introduces the theoretical model of sovereign debt. Section 5 shows theoretical model results, and Section 6 concludes our discussion.

2. Related literature

Creditor losses at sovereign debt restructurings have been discussed intensively in the academic literature. Previously, some studies measure the interest rates of return on sovereign bonds over longer periods such as Eichengreen and Portes (1986, 1989), Lindert and Morton (1989). Recently, Sturzenegger and Zettelmeyer (2006, 2008, see SZ hereafter) first propose a more refined measure of welfare losses for creditors in present value term and estimate for 22 recent restructurings. Following a similar approach, Cruces and Trebesch (2013) expand coverage of haircut measures to 180 restructurings over 1980-2010.11 Benjamin and Wright (2009) provide haircut estimates for 90 cases since 1990, which are not computed in present value terms but rather based on aggregate face value reduction and interest forgiven. Further haircut estimates for several specific cases are provided by Cline (1995) and Rieffel (2003).12 This paper complements the literature in that we focus instrument-specific credit losses.


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10 See also Klingen, Weder, Zettelmeyer (2004).

11 Asonuma (2012) find that creditors demand a larger increase in interest spreads to compensate larger losses (haircuts) at debt restructurings. Moreover, Cruces and Trebesch (2013) show that restructurings involving higher haircuts are associated with significantly higher subsequent bond yield spreads and longer periods of capital market exclusion.

12 See also Finger and Mecagni (2007), Diaz-Cassou et al. (2008), and Asonuma et al. (2014).
(2008a). Broner et al. (2013) empirically find that short-term bonds are cheaper than long-term bonds in normal times, while during the crisis, the relative cost of long-term bonds increases substantially and sovereigns reduce the maturity of bond issuance making curve steeper and shorter. Ramanarayanan (2012), Broner et al (2013), Chatterjee and Eyigungor (2012, 2013) and Hatchondo and Martinez (2009) theoretically show that the maturity structure reflects a trade-off between the relative incentive benefit of short-term debt and the hedging benefit of long-term debt reflected in these price functions. On the contrary, Niepelt (2014) characterizes equilibrium in closed form and proves that short-term debt issuance decreases in the quantity of outstanding debt, which is crucial for the equilibrium maturity structure. Current paper fills the gap of literature by inversion of yield curve reflecting creditor losses at debt restructurings.


3. Empirical Section: Four Stylized Facts on Creditor Losses and Yield Curves

Our empirical analysis focuses on creditor losses across instruments with different remaining maturity at sovereign debt restructurings and evolution of yield curves around restructurings. We newly find that (i) haircuts on short-term bonds are larger than those on long-term bonds; (ii) yield curve of sovereign bonds becomes inverted before the announcement of

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13 Chatterjee and Eyigungor (2012) also show that if the model allows a small probability of a self-filling rollover crisis, then the result is reversed: a sovereign that issues long-term debt is less vulnerable to a rollover crisis than one that issues short-term debt.

14 Bi (2008a) examines maturity choice in a model with one and two-period debt with renegotiation on defaulted debt and shows the maturity structure of debt shortens when approaching a crisis.

15 See more on Estrella (2005), Estrella and Adrian (2008), Rosenberg and Maurer (2008), Chinn and Kucko (2010).

16 Borri and Verdelhan (2011) empirically find that the higher the correlation between past bond returns and US corporate default risk, the higher the average bond returns.

17 Gilchrist, Yue, and Zakrajsek (2012) examine the relationship between sovereign bond spreads, local economic activity, and global financial risk.
restructurings; and (iii) “inverted” yield curve coincides with a downward sloping curve of haircuts.

3.1. Definition of Creditor Losses, Remaining Maturity and Data

We start from defining the welfare loss of each creditor, i.e., net present value haircuts by exchanges. Throughout this paper, we follow Sturzenegger and Zettelmeyer (2006, 2008) approach to measure welfare losses for creditors in present value term. Their haircuts compare the net present value of the new and the old debt in a hypothetical scenario in which the sovereign kept servicing old bonds that are not tendered in the exchange on a pari passu basis with the new bonds (SZ 2008, 783) and are defined as follows:

\[ H_t^i = 1 - \frac{\text{NPV}(\text{New Debt}, r_t^{\text{new}})}{\text{NPV}(\text{Old Debt}, r_t^{\text{new}})} \]  

(1)

where \( r_t^{\text{new}} \) is the exit yield of new debt i. Net present value of both the new and the old debt is evaluated at the same “exit yield” of the new debt in order to reflect the increased debt servicing capacity resulting from the exchange.

Over 1999-2010, Cruces and Trebesch (2013) document 17 external private debt restructurings. In addition, IMF (2013) report 3 external private debt restructurings and 3 domestic private debt restructurings from 2011 to 2013. To compute instrument-by-instrument haircuts, we collect information on old and new bonds from several sources, e.g., offering memoranda, the press releases from the governments, financial sector database and reports, the IMF staff reports, Sturzenegger and Zettelmeyer (2006, 2008) and Cruces and Trebesch (2013). We compute instrument-by-instrument haircuts for almost all restructuring episodes in the recent decade.

Table 1 reports a difference in haircuts on short-term and long-term debt and maturity of these two instruments. Over our sample of restructurings, there exists a substantial difference in haircuts on short-term and long-term debt with an average of 17.2 percent. \(^{18}\) Haircuts on short-term debt are larger than those on long-term debt. This difference in haircuts is associated with a difference in maturity of instruments.

Obviously, this trend is the case for restructurings which the same menu of new instruments was provided across creditors, for instance Belize 2006-2007 and Greece 2011-2012. \(^{19}\) In such cases, present value of new instruments is identical across creditors, while the percent value of old instrument varies substantially depending on remaining life of old instruments.

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\(^{18}\) In Jamaica 2013 debt restructuring, haircuts vary substantially across instruments mostly due to asymmetric treatment of maturity extension across old instruments.

\(^{19}\) See Asonuma et al. (2014) for Belize 2006-2007 restructuring and Zettelmeyer et al. (2013) for Greece 2012-2013 restructuring.
Moreover, even in restructurings which different new instruments were offered across creditors for example Cyprus 2013, Ecuador 1998-2000 and Pakistan 1999, we clearly witness substantially larger haircuts on short-term debt than those on long-term debt.\textsuperscript{20}

Table 1: Differences in Haircuts on Short- and Long-term Bonds and Maturity for Selected Restructurings

<table>
<thead>
<tr>
<th></th>
<th>Diff. in Haircuts on Short-and Long-term Debt (percent)</th>
<th>Maturity of Short-term and Long-term Debt (years)</th>
<th>Maturity of All Instruments Exchanged (years)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Argentina 2001-2005</td>
<td>13.4</td>
<td>0.1 and 22.6</td>
<td>[0-18.1, 22.6-26.3]</td>
</tr>
<tr>
<td>Belize 2006-2007</td>
<td>22.4</td>
<td>0 and 8.0</td>
<td>[0, 1, 3, 4, 5, 7, 8]</td>
</tr>
<tr>
<td>Cyprus 2013</td>
<td>14.7</td>
<td>0 and 2.7</td>
<td>[0, 0.5, 1.5, 1.6, 1.8, 2.2, 2.4, 2.7]</td>
</tr>
<tr>
<td>Ecuador 1999-2000</td>
<td>28.2</td>
<td>1.7 and 24.5</td>
<td>[1.7, 2.7, 3.7, 10.6, 24.5]</td>
</tr>
<tr>
<td>Ecuador 2008-2009</td>
<td>7.0</td>
<td>3.5 and 21.5</td>
<td>[3.5, 21.5]</td>
</tr>
<tr>
<td>Greece 2011-2012</td>
<td>45.4</td>
<td>0 and 28.5</td>
<td>[0-10.6, 12.13.4, 14.0, 18.4, 25.5, 28.5]</td>
</tr>
<tr>
<td>Grenada 2004-2005</td>
<td>21.4</td>
<td>0 and 13.0</td>
<td>[0-9, 10, 13]</td>
</tr>
<tr>
<td>Jamaica 2013</td>
<td>0.9</td>
<td>0 and 33.2</td>
<td>[0-9.5, 11, 14, 17, 19, 27, 33.2]</td>
</tr>
<tr>
<td>Pakistan 1999</td>
<td>25.7</td>
<td>0 and 2.5</td>
<td>[0, 0.5, 2.5]</td>
</tr>
<tr>
<td>Russia 1998-2000</td>
<td>19.2</td>
<td>-0.1 and 10.9</td>
<td>[-0.1, 0, 9.1, 10.9]</td>
</tr>
<tr>
<td>St. Kitts and Nevis 2011-2012</td>
<td>10.3</td>
<td>0 and 8</td>
<td>[0, 1, 4, 8]</td>
</tr>
<tr>
<td>Ukraine 1998</td>
<td>6.2</td>
<td>0.1 and 3.0</td>
<td>[0.1, 0.4, 0.7, 1.1, 3.0]</td>
</tr>
<tr>
<td>Uruguay 2003</td>
<td>8.4</td>
<td>0.5 and 24.2</td>
<td>[0-8.7, 17.8, 24.2]</td>
</tr>
<tr>
<td><strong>Average</strong></td>
<td><strong>17.2</strong></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Source: Asonuma et al. (2014), Struzenegger and Zettelmeyer (2006, 2008) and authors’ computations

Figure 1 plots haircuts and maturity of all old instruments exchanged in 4 restructuring cases. Haircuts for instruments with short maturity are remarkable higher than those with long maturity. In our sample of restructurings, an 1-year increase in maturity leads to a reduction of haircuts by 2.7 percent. This is quite the same if we omit representatives of restructurings with large face-value reductions (Argentina and Greece).\textsuperscript{21} Roughly speaking, in average, creditors holding instruments with remaining life of 10 years will receive 27-percent higher haircuts than those holding instruments which are close to due at the time of restructurings. Clearly, creditors with short-term bonds suffer substantially larger haircuts.


\textsuperscript{21} Zettelmeyer et al. (2013) first document that in the Greek debt restructuring, holders of short-term bonds suffered much higher haircuts (up to 80 percent) than longer-term bonds because all creditors were offered exactly the same (and only one) package of new securities.
Figure 1: Haircuts and Maturity Structure of Old Debt

(i) Pakistan 1999

(ii) Ecuador 1999-2000

(iii) Russia 1998-2000

(iv) Cyprus

Source: Struzenegger and Zettelmeyer (2006, 2008) and authors’ computations

- **Stylized fact 1**: Creditors holding short-term bonds suffer larger haircuts than those holding long-term bonds. In average, creditors holding instruments with remaining life of 10 years will receive 27-percent higher haircuts than those holding instruments close to due at restructurings.

### 3.2. Yield Curve around Restructuring Events

Next, this subsection overviews evolution of yield curve around restructuring events. We assemble collected information on daily yields of old bonds exchanged and daily credit default swap (CDS) spreads for recent restructuring episodes from different financial market database, e.g., Bloomberg, Datastream and Markit. There are two difficult issues on constructing the data. First, when sovereigns are to announce restructurings, bonds are sometimes no longer traded at the market, i.e., no bond yields available. Second, CDS...
spreads are only available for advanced and emerging countries with well-developed bond markets. Even for some emerging countries where CDS spreads are available, time series coverage of CDS spreads is sometimes limited, e.g., Dominican Republic from February 2005, Serbia from August 2009 and Uruguay from 2008. Therefore, we end up with a smaller restructuring sample on yield curves relative to that of haircuts.

On restructuring events, we use Asonuma and Trebesch (2014), IMF (2013), press release from the authorities to identify dates of (i) start of restructuring (default/announcement of restructurings), (ii) exchange offer proposed to creditors, (iii) end of restructurings (completion of exchanges). Asonuma and Trebesch (2014) define the start of a restructuring whenever the government misses first payments to private creditors beyond the grace period (default date) or whenever a key member of government publicly announces a restructuring of government debt to private creditors. Similarly, the end month of a restructuring is defined as the date in which either an official signing ceremony took place (in the case of bank debt restructurings), or as the date in which the debt was ultimately exchanged on the market (in the case of bond restructurings). IMF (2013) and press release from the authorities provide information on dates of exchange offer made.

In addition, we define dates of changes in yield curves: (i) initial normal yield curve (1 month and 3 month before inversion), (ii) first date of inversed yield curve, and (iii) the first date of normal yield curve after restructurings. The first one is defined as 1 month and 3 months before inversion of the yield curve when the yield curve was “normal”, i.e., yields on long-term bonds (10-year) are higher than those on short-term bonds (1-year). The second is specified as the first date when the yield curve became “inversed”, i.e., yields on short-term bonds are higher than those on long-term bonds. Intuitively, the inverted yield curve reflects creditors’ expectation on restructurings, while, the initial normal yield curve does not reflect risks of potential restructurings. The last one is set as the day when the yield curve first returns to upward sloping after restructurings.

Figure 2 displays how yields curve evolved as sovereigns are proceeding toward restructurings. For 1 or 2 years before start of restructurings, yield curve is normal as shown in Panel (A). Though yields at 1 month before inversion of yield curve are slightly higher than at 3 months before inversion, both yields are in general at low level. When creditors anticipate possible restructurings, we obviously witness sharp hikes in both short-term bond and long-term bonds, but higher spike in short-term bonds reported in Panel B. This shifts yield curve at the higher level and make it inverted with higher yields on short-term bonds. Clearly both yields on short-term and long-term bonds are higher than those at normal upward sloping yield curve. The yield curve continues to elevate and remain inverted until the announcement of restructurings. Yields at the time of announcement of restructurings are remarkably higher than those at the first inversion, in particular those on short-term bonds.

During the restructuring, the yield curve remains elevated and inverted as reported in Figure A1 in the Appendix 1. At the completion of restructurings, the yield curve continues to be inverted and coincides with the downward sloping haircuts reflecting ex-post creditor losses in Figure 1. Then, it takes several months after completion of restructuring to bring the yield
curve back to the normal upward sloping curve with lower levels of spreads (Figure A2 in Appendix 1).

- **Stylized fact 2: Yield curve of sovereign bonds becomes inverted before the announcement of restructurings anticipating possible restructurings.** When creditors anticipate possible restructurings in near future, yields on both long-term and short-term bonds spike with higher spike on short-term bonds.

**Figure 2: Yield Curve before Restructurings**

(I) Russia 1998-2000

(A) Initial Normal Curve

(B) Inverted Curve

(II) Greece 2011-2012

(A) Initial Normal Curve

(B) Inverted Curve

Source: Asonuma and Trebesch (2014), Bloomberg, Datastream, IMF (2013), Markit, press release from the authorities

Inversion of the yield curve is not specific to sovereign defaults or restructurings. We have witnessed several cases of inversion of the yield curve in advanced and emerging market countries, which are associated with neither sovereign defaults nor restructurings as reported
in Figure A4 in Appendix 5. In these cases, inversion has lasted only a short horizon, i.e., 3-6 months. Moreover, at the time when the yield curve has returned to upward sloping, spreads along the yield curve are lower than those 1 month before the inversion.

3.3. Stylized Evidence on Yield Curve and Haircuts

Finally, we combine two data sets of haircuts on restructurings and of yield curve evolution. Our sample of restructuring episodes, however, are constrained due to availability of (1) detailed instrument-by-instrument information on old and new debt, (2) multiple instruments with different maturity (at least 2 instruments) exchanged, (3) daily credit default swap (CDS) spreads at least before 1.5 year before the announcement of restructurings, and (4) CDS spreads for different maturity (at least 2 spreads). As a result, we have 7 debt restructuring episodes (6 external and 1 domestic) over 1999-2013; Pakistan 1999, Ecuador 1999-2000, Russia 1998-2000, Argentina 2001-2005, Ecuador 2008-2009, Greece 2011-2012, and Cyprus 2013.

In Figure 3, we contrast changes in yield curve with haircuts on instruments with different maturity. An interesting observation emerge from Figure 3: the “inverted” yield curve clearly coincides with a downward sloping curve of haircuts, i.e., realized costs of restructurings. This feature is common for all restructuring episodes in our sample and with more prominent for restructuring with symmetric new instruments (Argentina and Greece). Figure A1 in the Appendix confirms that the yield curve remains inserted during the restructuring, i.e., from the announcement of restructuring to completion of exchanges. The inverted yield curve at completion of exchanges clearly matches with haircuts which are ex-cost (actual) costs of restructurings. In ex-ante before announcement of restructurings, the inverted yield curve has already incorporated risks of potential costs of restructurings.

During tranquil period, the yield curve is “normal” implying that long-term bonds are more expensive than short-term bonds because of higher risk of repayment on long-term bond. On the contrary, when a sovereign is more likely to default and restructure debt in the near future, creditors anticipate higher losses on short-term bonds than long-term bonds at restructurings. This makes both short-term bonds and long-term bonds more expensive but even more for short-term bonds. Therefore, the yield curve becomes “inverted” reflecting higher potential loss on shorter bonds than long-term bonds.

- **Stylized fact 3:** The “inverted” yield curve clearly coincides with a downward sloping curve of haircuts. The inverted yield curve reflects creditors’ expectation on higher potential losses on short-term bonds at restructurings.

22 Given a lack of exit yields on new bonds prior to exchange, ex-ante NPV haircuts are not available. Though different from haircuts of bonds at exchange, Hatchondo et al. (2014) compute the evolution of market value of debt stock.
4. Theoretical Model

In this section, we show a simple theoretical model of sovereign debt and restructuring. Basic structure of our model follows closely 3-period model in Broner et al. (2013). However, there is one crucial element newly introduced in our model: we consider the government’s optimal restructuring choice in intermediate period (period 1), whereas no restructuring is assumed in Broner et al. (2013). At the restructuring, the government applies the same face-value haircuts on both short- and long-term bonds. The government’s choice of restructuring with
symmetric nominal haircuts rather than only defaults is consistent with empirical findings in
the literature of sovereign debt restructurings (Cruces and Trebesch, 2013, Asonuma and
Trebesch 2014) that sovereigns opt to take restructurings and there is a large variation in
NPV haircuts. This is in line with recent theoretical studies of sovereign debt which model a
bargaining game between a sovereign debtor and its creditors.23

4.1. Timing of the Model

There are three periods, dated 0, 1, and 2. Timing of events and decisions is summarized in
Figure 4.

1. In period 0, the government adjusts its debt maturity structure (d_S, d_L). We are in node (A).

2. In period 1, fiscal revenue y_1 realizes.

(a) If fiscal revenue is high (y_1 = \bar{y}), we move to the upper branch of the tree. We are
in node (B). The government repays its old debt (short-term bonds - d_S) and
issues new short-term debt (d'_{S,H}).

(b) If fiscal revenue is low (y_1 = 0), we move to the lower branch of the tree. We are in
node (C). The government decides whether to pay its debt and to restructure debt
based on expected restructuring costs E_0[\zeta_1].

(i) If the government decides to pay its debt, it repays short-term bonds (d_S) and
issues new short-term debt (d'_{S,L}).

(ii) If the government chooses to restructure debt, restructuring cost (\zeta_1)
materializes.

3. In period 2, fiscal revenue y_2 realizes.

(a) We are in note (B).

(i) If fiscal revenue is high (y_2 = \bar{y}), we move to the upper branch of a sub-tree.
The government repays both short-term and long-term bonds (d'_{S,H}, d_L).

(ii) On the contrary, if fiscal revenue is low (y_2 = 0), we move to the lower
branch of a sub-tree. The government defaults its debt.

(b) If the government repays its debt after realization of low fiscal revenue, we are in
node (D).

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23 Bulow and Rogoff (1989), Benjamin and Wright (2009), Kovrijnykh and Szentes (2007), Bi (2008b), Bai and
Zhang (2010), D’Erasmo (2010), Yue (2010), Pitchford and Wright (2012), Asonuma (2012), Asonuma and
Trebesch (2014), and Hatchondo et al. (2014)
(i) If fiscal revenue is high \((y_2 = \bar{y})\), we move to the upper branch of a sub-tree. The government repays both short-term and long-term bonds \((d_{s,t}, d_{l,t})\).

(ii) On the contrary, if fiscal revenue is low \((y_2 = 0)\), we move to the lower branch of sub-tree. The government defaults its debt.

(c) If the government restructures its debt after realization of low fiscal revenue, we are in node (E).

(i) If fiscal revenue is high \((y_2 = \bar{y})\), we move to the upper branch of a sub-tree. The government pays recovered short-term \((1 - \alpha)d_s\) and long-term bonds \((1 - \alpha)d_L\).

(ii) On the contrary, if fiscal revenue is low \((y_2 = 0)\), we move to the lower branch of sub-tree. The government defaults its debt.

Figure 4: Timing of the Model
4.2. Debtor Country

The government starts with period 0 with an initial debt which is due in periods 1 and 2. The stock of one-period (short-term) bonds due in period 1 is $d_S$ and the stock of two-period (long-term) bonds due in period 2 is $d_L$. In period 0, the government can adjust its debt maturity structure. The new stocks of short-term and long-term bonds, $d_S'$ and $d_L'$, must satisfy the budget constraint

$$p_S d_S + p_L d_L = p_S d_S' + p_L d_L'$$

where $p_S$ and $p_L$ are bond prices in period 0.

In period 1, the government has access to an exogenous flow of fiscal revenue $y_1$. Without loss of generality, we assume that the government never issues so much debt that it is unable to repay in full when $y_1 = \bar{y}$, namely,

$$d_S \leq \bar{y}$$

(3)

The fiscal revenue is stochastic and can take two values, $\bar{y}$ and 0. If the fiscal revenue is $\bar{y}$, the government pays off the maturing short-term bonds and issues new short-term bonds which are due in period 2. The budget constraint is

$$\bar{y} + p_{S,H}' d_{S,H}' = d_S$$

(4)

where $d_{S,H}'$ is the number of short-term bonds issued in period 1 and $p_{S,H}'$ is their price. Short-term bonds issued in period 1 are junior to existing long-term bonds.24 If the government receives 0, it decides whether to pay its debt or to restructure debt anticipating restructuring cost $E_0(\zeta_1)$. In the case the government chooses to repay its debt, it pays off the maturing short-term bonds and issues new short-term bonds which are due in period 2. The budget constraint is

$$p_{S,L}' d_{S,L}' = d_S$$

(5)

On the contrary, in the case the government opts to restructure debt, it postpones payments of recovered debt to period 2. There is no budget constraint in period 1 in this case. In period 2, the government again has access to an exogenous low of fiscal revenue $y_2$, which takes two values $\bar{y}$ and 0. The extreme case of zero realization in a bad income state simplifies the

---

24 This assumption is standard and ensures that the government can not dilute the claims of the holders of long-term bonds. It is also consistent with our assumption that defaults are due to government’s inability to repay as opposed to strategic considerations.
analysis since it eliminates the possibility of partial default in period 2. The government assigns all necessary fiscal revenue to debt payments and the reminder to public spending. We thus abstract from issues of strategic default and assume instead that the government defaults only when it is unable to repay. Without loss of generality, we assume that the government never issues so much debt that it is unable to repay in full when $y_2 = \bar{y}$, namely,

$$d_L + d_{S,H}' \leq \bar{y} \quad \& \quad d_L + d_{S,L}' \leq \bar{y}$$

(6)

$$\left(1 - \alpha \right)d_L + \left(1 - \alpha \right)d_S = \bar{y}$$

(7)

When $y_2 = \bar{y}$, thus all debt is repaid and public spending equals $\bar{y} - d_L - d_{S,H}'$, or $\bar{y} - d_L - d_{S,L}'$. When $y_2 = 0$, on the other hand, the government defaults on all its debt and public spending equals 0.

Uncertainty about the fiscal revenue $y_2$ is resolved as follows. In period 2, $y_2$ is realized and is equal to $\bar{y}$ with probability $\pi_2$ and 0 with $1 - \pi_2$. The probability $\pi_2$ is publicly revealed in period 0.

In the case of repayment under high fiscal revenue ($y_1 = \bar{y}$) in period 1 (node B), the government chooses its debt policy to maximize the expected fiscal surplus in period 2,

$$W_B = E_1[\pi_2 \cdot (\bar{y} - d_L - d_{S,H}')]$$

(8)

Similarly, in the case of repayment under low fiscal revenue ($y_1 = 0$) in period 1 (node D), the government chooses its debt policy to maximize similar expected fiscal surplus in period 2.

$$W_D = E_1[\pi_2 \cdot (\bar{y} - d_L - d_{S,L}')]$$

(9)

On contrary, in the case of restructuring under low fiscal revenue ($y_1 = 0$) in period 1 (node E), its objective function is

$$W_E = E_1[\pi_2 \cdot (\bar{y} - (1 - \alpha)d_L - (1 - \alpha)d_{S,L}')]$$

(10)

where $\alpha$ denoted nominal haircut on both long-term and short-term bonds.

4.3. International Investors and Bond Prices

International investors are risk-averse and they price assets using the stochastic discount factors $m_1$ and $m_2$, respectively, in period 0 and 1. We assume that investors’ stochastic discount factors do not vary depending on periods $m_1 = m_2 = m$. We also assume that
these stochastic discount factors are unaffected by the maturity structure chosen by the
government, yet they are negatively correlated with bond returns. We simplify notations by
assuming that the risk-free short-term rate is 0 in both periods, that is $E_0[m] = E_1[m] = 1$. In
period 1, the government repays its debt when fiscal revenue is high $y_1 = \bar{y}$. On the contrary,
when fiscal revenue is low $y_1 = 0$, the government decides whether it repays debt or
restructure debt.

The price of short-term bonds in period 0 is

$$p_s = E_0[m \cdot R_s]$$

where $R_s$ is an indicator variable which denotes repayment in period 1 expressed as

$$R_s = \begin{cases} 
1 & \text{if } y_1 = \bar{y} \\
1 & \text{if } y_1 = 0 \text{ and } I_{\text{REST}} = 0 \\
\pi_2 \cdot m \cdot (1 - \alpha) & \text{if } y_1 = 0 \text{ and } I_{\text{REST}} = 1 
\end{cases}$$

where $\pi_2 \cdot m \cdot (1 - \alpha)$ is effective recovery rates on short-term bonds in period 1. When the
government restructures bonds, creditors receive recovered payments only if fiscal revenue at
period 2 is high. Otherwise, they will not receive any payments at all. As mentioned above,
we assume that $R_s$ and $m$ are negatively correlated, so that

$$p_s < E_0[R_s] = (\pi_1 + (1 - \pi_1)\{(1 - I_{\text{REST}}) + I_{\text{REST}} \cdot \pi_2 \cdot m \cdot (1 - \alpha)\})$$. $I_{\text{REST}}$ is index
function of sovereign’s decision of restructuring or not defined in Section 4.4. To simplify
the analysis, we further assume that correlation between the risk premium and expected
return is constant, i.e., $\gamma = \text{corr}(p_s, R_s)$ so that,

$$p_s = \gamma(\pi_1 + (1 - \pi_1)\{(1 - I_{\text{REST}}) + I_{\text{REST}} \cdot \pi_2 \cdot m \cdot (1 - \alpha)\})$$

for some scalar $\gamma < 1$. In both node B and C in period 1 i.e., conditional on realization of
high and low fiscal revenue ($\pi_1 = 1$ or $\pi_1 = 0$), the price of this short-term bonds is

$$p_{s,B} = p_{s|\pi_1=1} = \gamma$$

$$p_{s,C} = p_{s|\pi_1=0} = \gamma((1 - I_{\text{REST}}) + I_{\text{REST}} \cdot \pi_2 \cdot m \cdot (1 - \alpha))$$

where expected (and actual) return on bonds is 1 in node B and $(1 - I_{\text{REST}}) + I_{\text{REST}} \cdot \pi_2 \cdot m \cdot
(1 - \alpha)$ in node C respectively.

In period 2, while the government repays its debt when fiscal revenue is high $y_2 = \bar{y}$, it
defaults both bonds when fiscal revenue is low $y_2 = 0$. The price of short-term bonds in both
node B and D in period 1 is
\[ p_{S,i}^t = E_0[m \cdot R'_{S,i}] \quad \text{for } i = B,D \] (14)

where \( R'_S \) is an indicator variable which denotes repayment in period 2 expressed as

\[ R'_{S,i} = \begin{cases} 
1 & \text{if } y_2 = \bar{y} \\
0 & \text{if } y_2 = 0 
\end{cases} \quad \text{for } i = B,D \] (15)

We again assume that \( R'_{S,i} \) and \( m \) are negatively correlated, so that \( p_{S,i}^t < E_0[R'_{S,i}] = \pi_2 \) for \( i = B,D \). To simplify the analysis, we further assume that correlation between the risk premium and expected return is constant, i.e., \( \rho = \text{corr}(p'_{S,i}, R'_{S,i}) \), so that

\[ p_{S,i}^t = \rho \cdot \pi_2 \quad \text{for } i = B,D \] (16)

for some scalar \( \rho < 1 \). After realization of low fiscal revenue in period 1 (\( y_1 = 0 \)), the government can only issue short-term bonds when it opts to repay its current short-term debt \( d_S \). Price of short-term bonds in node C prior to sovereign’s restructuring choice can be expressed by

\[ p_{S,C}^t = (1 - I_{REST}) \rho \cdot \pi_2 \] (16a)

Next, we consider long-term bonds. Price of long-term bonds issued in period 0 is

\[ p_L = E_0[m \cdot m \cdot R_L] \] (17)

where \( R_L \) is an indicator variable which denotes returns in period 2 expressed as

\[ R_L = \begin{cases} 
1 & \text{if } y_1 = \bar{y} \text{ and } y_2 = \bar{y} \\
0 & \text{if } y_1 = \bar{y} \text{ and } y_2 = 0 \\
0 & \text{if } y_1 = 0 \text{ and } I_{REST} = 0 \text{ and } y_2 = \bar{y} \\
(1 - \alpha) & \text{if } y_1 = 0 \text{ and } I_{REST} = 1 \text{ and } y_2 = \bar{y} \\
0 & \text{if } y_1 = 0 \text{ and } I_{REST} = 1 \text{ and } y_2 = 0 
\end{cases} \] (18)

where \((1 - \alpha)\) is returns on long-term bonds in period 2 if the sovereign opts to restructure debt in period 1. In line with previous literature, for instance Arellano and Ramanarayanan (2012), Chatterjee and Eyigungor (2012) and Bi (2008a), we assume that creditors can not exchange long-term bonds before maturity. Once again, \( R_L \) and \( m \) are assumed to be negatively correlated, so that \( p_L < E_0[R_L] = \pi_1 \cdot \pi_2 + (1 - \pi_1)(1 - I_{REST}) \cdot \pi_2 + \)
$I_{\text{REST}} \cdot \pi_2 \cdot (1 - \alpha)$. Correlation between the risk premium and expected return is constant over 2 periods, i.e., $\chi = \text{corr}(p_L, R_L)$, so that As a result,

$$p_L = \chi[\pi_1 \cdot \pi_2 + (1 - \pi_1)\{(1 - I_{\text{REST}}) \cdot \pi_2 + I_{\text{REST}} \cdot \pi_2 \cdot (1 - \alpha)\}]$$

(19)

for some scalar $\chi < 1$. Given uncertainty in both period 1 and period 2 fiscal revenue, correlation between the risk premium and expected return over two periods is smaller than the multiple of correlation between the risk premium and expected return on period 1 or 2, i.e., $\chi < \gamma \cdot \rho$. Similarly, price of long-term bonds in node B in node C (in period 1 and with realization of high and low fiscal revenue) is shown by

$$p_{LB} = p_L|\pi_1 = 1 = \chi \cdot \pi_2$$

(19a)

$$p_{LC} = p_L|\pi_1 = 0 = \chi\{(1 - I_{\text{REST}}) \cdot \pi_2 + I_{\text{REST}} \cdot \pi_2 \cdot (1 - \alpha)\}$$

(19b)

Following Cochrane and Piazzesi (2005, 2008), ex-ante term premium on short-term and long-term bonds at node A is expressed as:

$$E_0\left[\frac{p_S \cdot p_S'}{p_L} - 1\right] = \frac{\gamma(\pi_1 + (1 - \pi_1)\{(1 - I_{\text{REST}}) + I_{\text{REST}} \cdot \pi_2 \cdot m \cdot (1 - \alpha)\}) \cdot \rho \cdot \pi_2}{\chi[\pi_1 \cdot \pi_2 + (1 - \pi_1)\{(1 - I_{\text{REST}}) \cdot \pi_2 + I_{\text{REST}} \cdot \pi_2 \cdot (1 - \alpha)\}]} - 1$$

(20)

From our assumption that $m$ is exogenous and negatively correlated with bond returns, this implicitly assumes that the bonds issued by the country are a small fraction of the portfolio of international investors, yet their returns are positively correlated with the returns of this portfolio.

Similarly, conditional term premia on the same bonds at node B and C evaluated after realization of fiscal revenue in period 1 are shown

$$E_1\left[\frac{p_{S,B} \cdot p_{S,B}'}{p_{LB}} - 1|y_1 = \bar{y}\right] = \frac{\gamma \cdot \rho \cdot \pi_2}{\chi \cdot \pi_2} - 1$$

(21)

$$E_1\left[\frac{p_{S,C} \cdot p_{S,C}'}{p_{LC}} - 1|y_1 = 0\right] = \frac{\gamma(1 - I_{\text{REST}}) + I_{\text{REST}} \cdot \pi_2 \cdot m \cdot (1 - \alpha)) \cdot \rho \cdot \pi_2}{\chi(1 - I_{\text{REST}}) \cdot \pi_2 + I_{\text{REST}} \cdot \pi_2 \cdot (1 - \alpha)} - 1$$

(22)

"Effective haircuts" for bonds, i.e., NPV haircuts for short-term and long-term bonds evaluated based on cash flows of new and old instruments at maturity of bonds are defined as follows:
where $\alpha$ is a nominal haircut on both short-term and long-term in period 2. NPV of new short-term bond is $\pi_2 \cdot m \cdot (1 - \alpha)$ which is expected return on bonds by restructuring in period 1. NPV of old short-term bond is 1 as it is maturing in current period (period 1). Similarly, NPV of new long-term bond is $\pi_2 \cdot (1 - \alpha)$ which is expected return on new long-term bonds in period 2, and NPV of old long-term bond is $\pi_2 \cdot 1$, expected return on old long-term bonds in period 2. During the restructuring in period 1, short-term bonds are exchanged with long-term bonds maturing in period 2, i.e., extending maturity of existing bonds, while long-term bonds with long-term bonds maturing at the same period (period 2). In addition, both of them are symmetrically subject to the nominal haircut. Though these bonds are treated equally in nominal term, i.e., subject to the same nominal haircut, in present value term, they are treated asymmetrically creating a difference in NPV haircuts.

Lemma 1. (Haircuts): NPV haircut on short-term bonds is higher than that on long-term bonds.

$$\text{NPV}_{\alpha_S} - \text{NPV}_{\alpha_L} = 1 - \pi_2 \cdot m \cdot (1 - \alpha) - (1 - (1 - \alpha)) = 1 - \pi_2 \cdot m + \pi_2 \cdot m \cdot \alpha - \alpha$$

$$= (1 - \pi_2 \cdot m)(1 - \alpha) \geq 0$$

(25)

where equality holds when either $\pi_2 = 0$ or $\alpha = 0$. Thus, as long as nominal haircut is higher than 0 or probability of high fiscal revenue is higher than 0, NPV haircut on short-term bonds is larger than that on long-term bonds.

Proposition 1 (Dynamics of Yield Curve).

The term premia satisfy the following properties:

1. The conditional term premia at node B is positive.

2. The conditional term premia at node C is negative under moderate or low probability of high fiscal revenue realization ($\pi < \bar{\pi}_C$).

3. The ex-ante term premia at node A is positive when the probability of high fiscal revenue realization is high enough ($\pi > \bar{\pi}_A$) and negative otherwise. ($\pi < \bar{\pi}_A$).

4. The threshold for ex-ante term premia to be negative is lower than the threshold for conditional term premia at node C ($\bar{\pi}_A < \bar{\pi}_C$).
Proof: See Appendix.

4.4. Sovereign’s Restructuring Choice

In this subsection, we solve the government’s optimal debt maturity policy and restructuring decision. Its optimal debt maturity problem is to choose $d_L$, $d_S$, $d_S$ to maximize its objective function (8)-(10) subject to (2)-(7). Though optimal maturity problem, the government decides whether it restructures its debt or to repay its debt at node C. We solve this problem backward, deriving first the optimal borrowing in period 1. Let us start with a case where the government receives high income in period 1 ($y_1 = \bar{y}$). Equation (5) indicates that the realized fiscal revenue $\bar{y}$ is high enough to cover the payment for maturing short-term debt $d_L$.

$$d_{S,H} = 0$$

(26)

The government’s payoff is shown as follows;

$$\pi_2 \cdot (\bar{y} - d_L)$$

Next, we consider the case the government receives low fiscal revenue in period 1 ($y_1 = 0$) and decides to repay its maturing short-term debt. The maximum amount of short-term debt that the government can issue in period 1 is $d_S/p_S$ which is valued at $p_{S,D} = \rho \cdot \pi_2$. The government’s payoff is shown as follows;

$$\pi_2 \cdot \left( \bar{y} - d_L - \frac{d_S}{\rho \cdot \pi_2} \right)$$

Lastly, we consider the case the government receives low fiscal revenue in period 1 ($y_1 = 0$) and decides to restructure its debt. In this case, the government can not issue new short-term debt. Its payoff is shown respectively:

$$\pi_2 \cdot (\bar{y} - (1 - \alpha)d_L - (1 - \alpha)d_S)$$

Next, we consider the government choice at note (C). The decision of restructuring is a rational choice by comparing values of repayment and values of restructuring. The government opts to restructure when $W_D < W_E$ is higher than a restructuring cost $\xi_1$, as in Kumhof et al., (2014) and Pouzo and Presno (2012). The government’s choice of restructuring can be expressed as

$$I_{REST} = \arg \max_{\{0,1\}} \{W_E - \xi_1, W_D\}$$

(27)

Restructuring decision depends upon the distribution of $\xi_1$. We have the simple equation:
where $\Xi(.)$ is the cumulative distribution function of $\zeta_1$. We assume that $\Xi(.)$ takes the modified logistic form:

$$
\Xi(x) = \begin{cases} 
\frac{\psi}{1 + e(-\theta x)} & \text{if } x < \bar{x} \\
1 & \text{if } x \geq \bar{x}
\end{cases}
$$

where $\bar{x}$ is assumed to be arbitrarily large positive number, and that $\psi < 1$. Together this implies that, over the economically relevant range, restructuring occurs with positive probability, but never with certainty. The parameter $\psi$ helps to determine the mean level of restructuring probability over the sample, while $\theta$ determines the curvature of restructuring probability with respect to the difference $W_E - W_D$.\(^{25}\)

Then, the government’s indirect utility function in period 1 can be written as

$$
V_1(d_S, d_L) = \begin{cases} 
\pi_2 \cdot (\bar{y} - d_L) & \text{if } y_1 = \bar{y} \\
\pi_2 \cdot (\bar{y} - d_L - \frac{d_s}{\rho \cdot \pi_2}) & \text{if } y_1 = 0 \& \ I_{REST} = 0 \\
\pi_2 \cdot (\bar{y} - (1 - \alpha)d_L - (1 - \alpha)d_s) - \zeta_1 & \text{if } y_1 = 0 \& \ I_{REST} = 1
\end{cases}
$$

Similarly, its indirect utility function in period 0 can be written as

$$
V_0(d_S, d_L) = \pi_1 \cdot \pi_2 \cdot (\bar{y} - d_L) + (1 - \pi_1) \left[ \left(1 - I_{REST}\right) \cdot \pi_2 \cdot \left(\bar{y} - d_L - \frac{d_s}{\rho \cdot \pi_2}\right) + I_{REST} \cdot \pi_2 \cdot (\bar{y} - (1 - \alpha_S)d_L - (1 - \alpha_S)d_s) - E[\zeta_1] \right]
$$

Note that the utility function $V(\ldots)$ is increasing and concave. Using the period-0 budget constraint and short-term debt function using long-term debt $d_S(d_L)$, we can write its problem in period 0 as

$$
\max_{d_L} V_0(d_S(d_L), d_L)
$$

\(^{25}\) When $\theta$ approaches $+\infty$, restructuring probability converges to $\psi$. Despite the fact that expected cost of restructuring responds infinitely to any marginal difference between two values, premia does not deviate substantially from that under the baseline case.
where \( d_S(d_L) = \bar{d}_S + \frac{p_L}{p_S}(d_L - \bar{d}_L) \).

### 4.5. Equilibrium

We define a stationary recursive equilibrium of the model as a set of functions for, (a) the sovereign's asset position (short-term and long-term) and restructuring choice, (b) the price of both short-term and long-term bonds such that

1. sovereign's asset position (short-term and long-term) and restructuring choice satisfy its optimization problem (2)-(10), (26)-(30)

2. bond prices (both short-term and long-term) satisfy the international creditors’ conditions (11)-(19)

### 4.6. Discussion – Symmetric NPV haircuts

In the conventional restructurings, the government applies the same face-value haircuts to all instruments. This symmetric treatment across creditors in face value term reflects legal constraints for the government stated in the *pari passu* clause that the debtor should rank all creditors equally. Lemma 1 explains that in the benchmark case, NPV haircut on short-term bonds is higher than that on long-term bonds. This is because of the difference in discounted present value of new instruments associated with restructuring: while creditors with long-term bonds receive payments on new instruments at maturity of “old” long-term bonds, creditors with short-term bonds receive only one period after maturity of “old” short-term bonds.

Alternative restructuring rule would be to impose symmetric NPV haircuts across instruments by offering asymmetric menu of nominal (face-value) haircuts \((\alpha_S, \alpha_L)\). As a consequence, all creditors suffer the same costs in present value term. Our theoretical model implies that the rule compensates creditors holding short-term bonds more by a multiple of a discount rate and probability of high fiscal revenue.

\[
\text{NPV}_{\alpha_S} = \text{NPV}_{\alpha_L} \Leftrightarrow (1 - \alpha_S) = \frac{(1 - \alpha_L)}{\pi_2 \cdot m}
\]

\[(25')\]
To explore welfare implications of symmetric NPV rule, we differentiate the indirect utility function at period 0 (equation 30) with respect to nominal haircut on the short-term bonds:

\[
\frac{\partial V_0(d_s, d_l)}{\partial \alpha_s} |_{\alpha_s, \alpha_l} = (1 - \pi_1) \left[ I_{\text{REST}} \cdot \pi_2 \cdot \tilde{d}_s + \{W_E - E[\xi_1] - W_D\} \frac{\partial I_{\text{REST}}}{\partial \alpha_s} \right]
\]

(30'')

An increase in haircut on the short-term bonds influences the ex-ante welfare in two contradicting directions: On the one hand, if the government is more likely to default \((I_{\text{REST}} = 1)\), the ex-ante welfare increases by reducing restructuring payments on short-term bonds in period 2, \((1 - \pi_1) \cdot \pi_2 \cdot \tilde{d}_s\) (corresponding to the first term inside the bracket). On the other hand, when an increase in haircut affects the government’s decision of restructuring, the ex-ante welfare decreases because the ex-ante value of repayment is higher than the ex-ante value of restructuring, i.e., \(W_E - E[\xi_1] < W_D\).

5. Theoretical Explanations for Stylized Facts

In this section, we explain theoretically the observed changes in yield curve based on model of sovereign debt and restructuring provided in Section 4. Rather than applying rigorous quantitative analysis and obtaining the statistic moments from calibration exercise, we take an approach similar to Broner et al. (2013) to understand intuitively the mechanism of flip in yield curve using 3-period model with restructuring provided in Section 4. In particular, we focus on conditions on “normal” and “inverted” yield curve in ex-ante (at node A in period 0) and explore how these numerically generated yield curves match with what we observe in the data in Section 3.

We find that (i) when probability of high fiscal revenue is high, risk of repayment on long-term bonds dominates risk of restructuring and this results in positive ex-ante term premia, i.e., “normal” yield curve; (ii) when probability of high fiscal revenue is low, risk of restructuring outweighs risk of repayments on long-term bonds generating negative ex-ante term premia, i.e., “inverted” yield curve; (iii) our estimated positive and negative ex-ante term premia coincide with “initial normal” yield curve and “inverted” yield curve in the data.

5.1. Baseline case

Under our benchmark case, we assume that conditional probability of high fiscal revenue in period 2 is symmetric between high and low fiscal revenue realizations in period 1 shown as:

---

26 By the envelop theorem, both \(d_s, \tilde{d}_l\) take values, which maximize the indirect utility function: \(\frac{\partial V_0(d_s, d_l)}{\partial d_s} = 0\) and \(\frac{\partial V_0(d_s, d_l)}{\partial \tilde{d}_l} = 0\).
\[
\begin{bmatrix}
P_{H,H} & P_{L,H} \\
P_{H,L} & P_{L,L}
\end{bmatrix} = \begin{bmatrix}
p & p \\
1-p & 1-p
\end{bmatrix}
\]

(31)

where \( p_{ij} \) is probability of fiscal revenue realization of \( i \) in period 2 conditional on fiscal revenue realization of \( j \) in period 1. Using this transitional matrix, we compute “unconditional provability of fiscal revenue realization in period 1 as

\[
\begin{bmatrix}
P_H \\
P_L
\end{bmatrix} = \begin{bmatrix}
1-p \\
p
\end{bmatrix}
\]

(32)

where \( p_i \) is unconditional probability of fiscal revenue realization of \( i \) in period 1.

We explore how the yield curve becomes inverted in our benchmark case.\(^{27}\) To understand determinants of ex-ante term premia in period 0 (evaluated with “unconditional” probability at node A), we first see how sub-components of ex-ante term premia are determined in period 1, particularly at node B and node C respectively. Conditional term premia at node B and node C evaluated after realization of fiscal revenue in period 1 are defined in equation (21) and (22).

Panel (i) in Figure 5 reports that conditional term premia at node B (red dot line overlapped with green dot line) is positive and stays constant at any level of probability of high fiscal revenue in period 2. As explained in Section 4.3, positive term premia corresponds to long-term bond yields higher than short-term bond yields, equivalent to “normal” yield curve. Given realization of high fiscal revenue in period 1 observed at node B, price of short-term bond issued in period 0 is deterministic and constant at 1. On the contrary, prices of both long-term bonds issued in period 0 and of short-term bonds issued in period 1 are subject to the same uncertainty in return in period 2, i.e., influenced by the same probability of high fiscal revenue and payments in period 2. Clearly, while premium on short-term bonds is partially influenced (only on bonds issued in period 1) by uncertainty, premium on long-term bonds are fully influenced. This leads to higher long-term bond yields than short-term bond yields, i.e., normal yield curve. Whether fiscal revenue is highly persistent or mean reverting does not affect the term premium because these only change the probability of high fiscal revenue in period 2 symmetrically as shown in blue solid and green dot lines.

Panel (ii) in Figure 5 shows that conditional term premia at node C (red dot line) is negative, except at high level of probability of high fiscal revenue in period 2 \( (\pi < \bar{\pi}_C \) where \( \bar{\pi}_C \) is the level of probability of fiscal revenue which term premia is zero shown by a black vertical line). After realization of low fiscal revenue in node C, a sovereign optimally decide whether to repay short-term bonds issued in period 0 (proceeding to node D) or restructure both short-term and long-term bonds (proceeding to node E). At normal level of probability of fiscal revenue in period 2, the sovereign is more prone to restructure its bonds. In this case, expected return on short-term bonds conditional on restructuring expressed as \( \pi_2 * m * (1 - \)

\(^{27}\) Model parameters are reported in Appendix 3.
The value of $\alpha$ is lower than expected return on long-term bonds denoted by $\pi \times (1 - \alpha)$. Premium on short-term bonds is higher than that of short-term bonds anticipating the larger costs on short-term bonds at restructuring. This corresponds to “inverted” yield curve and this is consistent with “yield curve at announcement of defaults/restructurings as in Figure 2 and 3. In contrast, at high level of fiscal revenue in period 2 (above 70 percent), the sovereign is willing to repay short-term bonds issued in period 0 and issue new short-term bonds. This is exactly the same with positive term premia at node B, but with smaller magnitude.

Finally, we combine these two conditional term premia with using unconditional probability at node A. Panel (iii) in Figure 5 shows ex-ante term premia at node A. Ex-ante term premia (expressed in red dot line) is positive above a threshold of probability of high fiscal revenue ($\tilde{\pi}_A$ equivalent to 0.35), while negative below the threshold ($\tilde{\pi}_A$). If probability of high fiscal revenue is above the threshold ($\pi > \tilde{\pi}_A$), positive conditional term premia at node B outweighs negative conditional term premia at node C. When probability of high fiscal revenue is high, risk of repayments on long-term bonds is higher than risk of restructuring in ex-ante. This part is similar to findings in Borner et al. (2013), Arellano and Ramanarayanan (2012) and Chatterjee and Eyigungor (2012). Current yield curve coincides with “Initial normal” yield curve in Figure 2 and 3 when creditors anticipate low probability of default/restructuring in ex-ante.

On the contrary, if probability of high fiscal revenue is low, i.e., below the threshold ($\pi < \tilde{\pi}_A$), negative conditional term premia at node C exceeds positive conditional term premia at node B. As probability of high fiscal revenue becomes low, expected losses at restructuring become higher for short-term bonds than for long-term bonds. Obviously, risk of restructuring with higher losses on short-term bonds outweighs risk of repayments on long-term bonds generating inverting yield curve. The yield curve in this case is consistent with “Inverted yield curve” in Figure 2 and 3. This theoretical explanation is a new observation in the literature of sovereign debt default and restructuring.

Figure 5: Ex-ante Term Premia and Conditional Term Premia

(i) Conditional Term Premia at Node B

(ii) Conditional Term Premia at Node C
5.2. Sensitivity Analysis

In the sensitivity analysis, we consider two cases: (1) high persistency and (2) high mean reversion. In contrast to transitional matrix under the baseline case, we make adjustments only on probability of fiscal realization in period 2 conditional on low fiscal revenue realization in period 1 (keeping probability of fiscal realization in period 2 conditional on high fiscal revenue in period 1 unchanged) shown as:

\[
p_{t, t+1}^{L, L} = \frac{p_{t-1}^{L, L}}{1 - p_{t-1}^{L, L}}
\]

where \(\delta\) captures both persistency and mean reversion of bad fiscal revenue. If \(\delta > 0\), conditional on low fiscal revenue in period 1, it is less likely that fiscal revenue is high in period 2, i.e., fiscal revenue is highly persistent. On the contrary, \(\delta < 0\), conditional on low fiscal revenue in period 1, it is more likely that fiscal revenue is high in period 2, i.e., fiscal revenue is highly mean reverting. Unconditional probability of fiscal revenue realization in period 1 is shown as:

\[
\begin{bmatrix}
    p_{H,H} & p_{L,H} \\
    p_{H,L} & p_{L,L}
\end{bmatrix} = \begin{bmatrix}
p & p - \delta \\
1 - p & 1 - p + \delta
\end{bmatrix}
\]
Let us start from a case of high persistency. Panel (i) and (ii) in Figure 6 report conditional term premia at node B and C. As shown in Panel (i), the conditional term premium at node B does not depend on whether fiscal revenue is highly persistent or not as shown in blue solid line. On the contrary, when realization of fiscal revenue becomes highly persistent, i.e., more likely that fiscal revenue in period 2 is low as in period 1, expected return in period 2 is less given the sovereign’s repayment or restructuring decision in node C (period 1). This is because the sovereign does not repay anything at upon realization of low fiscal revenue in period 2. As a result, conditional term premia at node C (blue solid line) becomes higher in Panel (ii), i.e., long-term bond yields higher than short-term bond yields (normal yield curve) given probability of fiscal revenue.

As a result, ex-ante term premia at node A highlighted in blue solid line in Panel (iii) is higher than the baseline and positive implying higher long-term yields than short-term bond yields. This is driven solely by higher conditional term premia at node C due to lower expected return aforementioned. Figure A3 in Appendix shows ex-ante term premia at different degree of high persistency in probability of fiscal revenue.

Next, we consider a case of high mean reversion. Similar to the case of high persistency, mean reversion of fiscal revenue does not affect the conditional term premium at node B because these only change the probability of high fiscal revenue in period 2 symmetrically as shown in green dot line. In contrast, if outcome of fiscal revenue is highly mean-reverting, i.e., that fiscal revenue in period 2 is more likely to be high, expected return in period 2 becomes higher. However, when the sovereign opts to restructure its debt, this makes ex-ante return on both long-term and short-term bonds higher, but much more for long-term bonds due to discount rate. It leads to lower conditional term premia at node C (green dot line), i.e., short-term bond yields higher than long-term bond yields (highly inverted yield curve).

Therefore, at node A, we have lower and more negative ex-ante term premia than the benchmark case corresponding to higher short-term bond yields than long-term bond yields. Clearly, lower and more negative conditional term premia influences this result because of higher expected return on long-term bonds conditional on restructuring as explained above.

\[
\begin{bmatrix}
    p_H \\
    p_L
\end{bmatrix} = \begin{bmatrix}
    1 - p \\
    1 - \delta \\
    p - \delta \\
    1 - \delta
\end{bmatrix}
\]
Figure 6: Ex-ante Term Premia and Conditional Term Premia

(i) Conditional Term Premia at Node B

(ii) Conditional Term Premia at Node C

(iii) Ex-ante Term Premia at Node A
5.3. Welfare analysis

For welfare analysis, we explore how different menu of nominal haircuts on the short-term debt influence the ex-ante welfare of the government. We specify the following spectrum of face-value haircuts on short-term debt ranging from -5% to 50%. Nominal haircut of 50% corresponds to the case of symmetric face value haircuts applied to both short- and long-term bonds, whereas nominal haircut of -5% corresponds to the case of symmetric NPV haircuts. In the case of symmetric NPV haircuts, the nominal haircut on short-term bonds of -5% is obtained from equation (25’). Panel (i) in Figure 7 displays that a NPV differential increases as the short-term haircut increases from -5%. As explained in Lemma 1, with the short-term haircut of 50%, a substantial NPV differential emerges due to the difference in discounted present value of new instruments associated with restructuring.

Panel (ii) in Figure 7 shows that when the short-term haircut deviates from -5%, the ex-ante probability of restructuring increases. Recovered debt payments on short-term bonds in period 2 conditional on restructuring choice are reduced. Thus, the government is more willing to opt restructuring in period 1.

Lastly, panel (iii) in Figure 7 reports that the ex-ante welfare is higher under the symmetric NPV haircuts (short-term haircut of -5%) than under the symmetric face value haircuts (short-term haircut of 50%). As the short-term haircut increases from -5%, the ex-ante probability of restructuring gets higher. As explained in Section 4.6, an increase in short-term haircut influences the ex-ante welfare in two channels: on the latter channel, when an increase in short-term haircuts affects the government’s decision of restructuring, the ex-ante welfare in period 0 decreases because in period 1, the ex-ante value of full repayments is higher than that of restructuring due to expected costs of restructuring, i.e., \( W_E - E[\zeta_1] < W_D \). Due to high likelihood of restructuring choice, expected costs of restructuring become higher.
Figure 7: NPV Differential, Restructuring Probability, and Ex-ante Welfare

(i) NPV Differential (Short – Long)  
(ii) Probability of Restructuring  
(iii) Ex-ante Welfare at Node A
VI. Conclusion
[To be inserted]

Appendix 1: Evolution of Yield Curve

Figure A1: Yield Curve during Restructurings
(i) Greece 2011-2012
- Announcement of restructuring - Jul. 21 2011
- Completion of exchange, Mar. 28 2012
- Inverse yield - Jun. 14 2010

(ii) Cyprus
- Announcement of restructuring May 15 2013
- Completion of exchange July 1 2013
- Inverse yield - Mar. 28 2012 (RHS)

Sources: Asonuma and Trebesch (2014), Bloomberg, Datastream, IMF (2013), Markit, press release from the authorities

Figure A2: Yield Curve after Restructurings
(i) Greece 2011-2012
- Completion of exchange, Mar. 28 2012
- Normal curve after restructuring, Mar. 19 2013

(ii) Argentina 2001-2005
- Completion of exchange, Jun. 7 2005
- Normal curve after restructuring, Jun. 16 2005

Source: Asonuma and Trebesch (2014), Bloomberg, Datastream, IMF (2013), Markit, press release from the authorities
Appendix 2: Proofs of Proposition 1

As indicated in Section 4.3, we rely on two assumptions; (i) correlation between the risk premium and expected return over two periods is smaller than the multiple of correlation between the risk premium and expected return on period 1 or 2, i.e., \( \rho \cdot \gamma > \chi \); (ii) probability of high fiscal revenue in period 1 and 2 is independent, i.e., \( \pi_1 \) is orthogonal to \( \pi_2 \) (we relax this assumption in simulation exercise in Section 5.2.).

The term premia satisfy the following properties:

1. **The conditional term premia at node B is positive.**

   **Proof.** The conditional term premia at node B is shown as follows:

   \[
   E_1 \left[ \frac{p_{S,B} \cdot p_{S,B}^{'}}{p_{L,B}} | y_1 = \bar{y} \right] = \frac{\gamma \cdot \rho \cdot \pi_2}{\chi \cdot \pi_2} - 1 = \frac{\gamma \cdot \rho}{\chi} - 1 = \frac{\gamma \cdot \rho - \chi}{\chi} > 0
   \]

   where \( \rho \geq \rho \cdot \gamma > \chi \).

2. **The conditional term premia at node C is negative under moderate or low probability of high fiscal revenue realization at period 2 (\( \pi_2 < \pi_C \)).**

   **Proof.** We assume that \( \pi_C \) satisfy that term premia is zero i.e.,

   \[
   E_1 \left[ \frac{p_{S,C} \cdot p_{S,C}^{'}}{p_{L,C}} | y_1 = 0 \right] = 0
   \]

   \[
   \Leftrightarrow \gamma \left((1 - I_{REST}) + I_{REST} \cdot \pi_C \cdot m \cdot (1 - \alpha)\right) \cdot \rho \cdot \pi_C = \chi \left((1 - I_{REST}) \cdot \pi_C + I_{REST} \cdot \pi_C \cdot (1 - \alpha)\right)
   \]

   Conditional term premia can be written as:

   \[
   E_1 \left[ \frac{p_{S,C} \cdot p_{S,C}^{'}}{p_{L,C}} | y_1 = 0 \right] = \frac{\gamma \left((1 - I_{REST}) + I_{REST} \cdot \pi_2 \cdot m \cdot (1 - \alpha)\right) \cdot \rho \cdot \pi_2}{\chi \left((1 - I_{REST}) \cdot \pi_2 + I_{REST} \cdot \pi_2 \cdot (1 - \alpha)\right)} - 1
   \]

   Differentiating conditional term premia, we obtain:

   \[
   \frac{\partial E_1 \left[ \frac{p_{S,C} \cdot p_{S,C}^{'}}{p_{L,C}} | y_1 = 0 \right]}{\partial \pi_2} = \frac{I_{REST} \cdot m \cdot (1 - \alpha)}{\chi \left((1 - I_{REST}) + I_{REST} \cdot (1 - \alpha)\right)} \geq 0
   \]
where equality holds when $I_{REST} = 0$ or $\alpha = 1$ i.e., sovereign does not choose to restructure its debt and there is full face value haircut. Obviously conditional term premia at node C is monotonically increasing. Therefore, if probability of high fiscal revenue realization is moderate or low ($\pi_2 < \tilde{\pi}_C$), conditional term premia at node C is negative i.e., $E_1 \left[ \frac{ps\cdot p'S}{pl} | y_1 = 0, \pi_2 \right] < E_1 \left[ \frac{ps\cdot p'S}{pl} | y_1 = 0, \tilde{\pi}_C \right] = 0$.

3. The ex-ante term premia at node A is positive when the probability of high fiscal revenue realization is high enough ($\pi_2 > \tilde{\pi}_A$) and negative otherwise ($\pi_2 < \tilde{\pi}_A$).

**Proof.** We assume that $\tilde{\pi}_A$ satisfy that term premia is zero i.e., $E_0 \left[ \frac{ps\cdot p'_S}{pl} | \pi_2 = \tilde{\pi}_A \right] = 0$

$$\Leftrightarrow \gamma(\pi_1 + (1 - \pi_1)\{(1 - l_{REST}) + l_{REST} \cdot \tilde{\pi}_A \cdot m \cdot (1 - \alpha)\}) \cdot \rho \cdot \tilde{\pi}_A = \chi[\pi_1 \cdot \tilde{\pi}_A + (1 - \pi_1)\{(1 - l_{REST}) \cdot \tilde{\pi}_A + l_{REST} \cdot \tilde{\pi}_A \cdot (1 - \alpha)\}]$$

Ex-ante term premia can be written as:

$$E_0 \left[ \frac{pS \cdot p'_S}{pl} \right] = \frac{\gamma(\pi_1 + (1 - \pi_1)\{(1 - l_{REST}) + l_{REST} \cdot \pi_2 \cdot m \cdot (1 - \alpha)\}) \cdot \rho \cdot \pi_2}{\chi[\pi_1 \cdot \pi_2 + (1 - \pi_1)\{(1 - l_{REST}) \cdot \pi_2 + l_{REST} \cdot \pi_2 \cdot (1 - \alpha)\}]} - 1$$

Differentiating conditional term premia, we obtain:

$$\frac{\partial E_0 \left[ \frac{pS \cdot p'_S}{pl} \right]}{\partial \pi_2} = \frac{l_{REST} \cdot m \cdot (1 - \alpha)}{\chi[\pi_1 + (1 - \pi_1)\{(1 - l_{REST}) + l_{REST} \cdot (1 - \alpha)\}]} \geq 0$$

where equality holds when $l_{REST} = 0$ or $\alpha = 1$ i.e., sovereign does not choose to restructure its debt and there is a full face-value haircut. As in the previous case, ex-ante term premia is also monotonically increasing. When probability of high fiscal revenue realization is high ($\pi_2 > \tilde{\pi}_A$), ex-ante term premia is positive i.e., $E_0 \left[ \frac{ps\cdot p'_S}{pl} | \pi_2 = \tilde{\pi}_A \right] > 0$. Otherwise ($\pi_2 < \tilde{\pi}_A$), ex-ante term premia is negative i.e., $E_0 \left[ \frac{ps\cdot p'_S}{pl} | \pi_2 = \tilde{\pi}_A \right] < 0$.
4. The threshold for ex-ante term premia to be negative is lower than the threshold for conditional term premia at node C ($\tilde{\pi}_A < \tilde{\pi}_C$).

**Proof.** Ex-ante term premia can be written as:

$$E_0 \left[ \frac{p_S \cdot p_S'}{p_L} \right] = \frac{\chi \cdot \pi_1 \cdot \pi_2}{TP_1} \left[ \frac{Y \cdot \rho \cdot \pi_2 - 1}{\chi \cdot \pi_2} \right] + \frac{\chi((1 - I_{REST})\pi_2 + I_{REST} \cdot \pi_2(1 - \alpha))}{TP_1} \left[ \frac{Y \left( \frac{1 - I_{REST}}{\pi_2} \cdot m \cdot (1 - \alpha) \right) \rho \cdot \pi_2}{\chi((1 - I_{REST}) \cdot \pi_2 + I_{REST} \cdot \pi_2(1 - \alpha)) - 1} \right]$$

$$= \frac{TP_A}{TP} E_1 \left[ \frac{p_{SB} \cdot p_{SB}'}{p_{LB}} \right]_{y_1 = \bar{y}} + \frac{1 - TP_A}{TP} E_1 \left[ \frac{p_{SC} \cdot p_{SC}'}{p_{LC}} \right]_{y_1 = 0}$$

(A8)

We assume that $\tilde{\pi}_A \geq \tilde{\pi}_C$ (the threshold for ex-ante term premia to be negative is at least higher than the threshold for conditional term premia at node C) and Property 3 holds.

First, let us start with a case $\tilde{\pi}_A = \tilde{\pi}_C$. The left-hand side of equation (A8) at $\pi_2 = \tilde{\pi}_A$ is

$$E_0 \left[ \frac{p_S \cdot p_S'}{p_L} \right]_{\pi_2 = \tilde{\pi}_A} = 0.$$ The right-hand side of equation (A8) at $\pi_2 = \tilde{\pi}_A$ is

$$\frac{TP_A}{TP} E_1 \left[ \frac{p_{SB} \cdot p_{SB}'}{p_{LB}} \right]_{y_1 = 0} \geq 0 \text{ and } \frac{1 - TP_A}{TP} E_1 \left[ \frac{p_{SC} \cdot p_{SC}'}{p_{LC}} \right]_{y_1 = 0} > 0.$$ Thus, it contradicts.

Next, we consider a case $\tilde{\pi}_A > \tilde{\pi}_C$. The left-hand side of equation (A8) at $\pi_2 = \tilde{\pi}_A$ is

$$E_0 \left[ \frac{p_S \cdot p_S'}{p_L} \right]_{\pi_2 = \tilde{\pi}_A} = 0.$$ The right-hand side of equation (A8) at $\pi_2 = \tilde{\pi}_A$ is

$$\frac{TP_A}{TP} E_1 \left[ \frac{p_{SB} \cdot p_{SB}'}{p_{LB}} \right]_{y_1 = 0} \geq 0 \text{ and } \frac{1 - TP_A}{TP} E_1 \left[ \frac{p_{SC} \cdot p_{SC}'}{p_{LC}} \right]_{y_1 = 0} > 0.$$ Thus, it contradicts.

Therefore, $\tilde{\pi}_A < \tilde{\pi}_C$, i.e., the threshold for ex-ante term premia to be negative is lower than the threshold for conditional term premia at node C.
Appendix 3: Model parameters

Table A1: Model Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Risk premium on short-term bonds in period 0</td>
<td>γ = 0.7</td>
</tr>
<tr>
<td>Risk premium on short-term bonds in period 1</td>
<td>ρ = 0.7</td>
</tr>
<tr>
<td>Risk premium on long-term bonds in period 0</td>
<td>χ = 0.375</td>
</tr>
<tr>
<td>Discount rate</td>
<td>m = 0.95</td>
</tr>
<tr>
<td>Curvature of restructuring probability</td>
<td>θ = 25</td>
</tr>
<tr>
<td>Mean restructuring probability</td>
<td>φ = 0.5</td>
</tr>
<tr>
<td>Baseline nominal haircut</td>
<td>α = 0.5</td>
</tr>
</tbody>
</table>
| Probability of fiscal revenue conditional on previous realization | \[
\begin{pmatrix}
    p + \delta & p - \delta \\
    (1 - p) - \delta & (1 - p) + \delta
\end{pmatrix}
\]
| Probability of high fiscal revenue in period 2        | p = [0,1] |
| Persistency and mean reversion of fiscal revenue      | δ = [-0.5,0.5] |

Appendix 4: Baseline -Term Premia

Figure A3: Ex-ante Term Premia at Node A
Appendix 5: Evolution of Yield Curve in Non-Restructuring cases

Figure A4: Yield Curve for Non-Restructuring Cases

(i) Pakistan 2011-2012
(ii) Russia 2008-2009

Source: Markit
Reference


