

# Micro Risks and (Robust) Pareto Improving Policies\*

Mark Aguiar

Princeton University,  
and NBER

Manuel Amador

University of Minnesota,  
Federal Reserve Bank of  
Minneapolis, and NBER

Cristina Arellano

Federal Reserve Bank  
of Minneapolis

February 2024

## Abstract

We provide sufficient conditions for the feasibility of robust Pareto-improving (RPI) fiscal policies in the class of incomplete markets models of Bewley-Huggett-Aiyagari and when the interest rate on government debt is below the growth rate ( $r < g$ ). We allow for arbitrary heterogeneity in preferences and income risk and a potential wedge between the return to capital and to government bonds. An RPI improves risk sharing and can induce a more efficient level of capital. We show that the elasticities of aggregate savings to changes in interest rates are the crucial ingredients that determine the feasibility of RPIs. We establish that government debt and capital investment associated with an RPI may be complements along the transition, rather than the traditional substitutes. Our analysis shifts the focus of fiscal policy in incomplete markets from explicitly redistributive policies to using government bonds and simple subsidies to robustly improve welfare of all agents at all points in time.

## 1 Introduction

This paper studies Pareto improvements when the risk-free interest rate  $r$  on government bonds is below the growth rate ( $r < g$ ) and fiscal policy consists of non-negative lump-sum transfers, linear taxes or subsidies, and government debt. We do so in the class of incomplete markets models pioneered by Bewley-Huggett-Aiyagari, in which households hold precautionary savings

---

\*We thank our discussants, Mark Huggett, Sarolta Laczó, and Jennifer La’o for comments. We also benefited from comments from Fernando Alvarez, Adrien Auclert, Marco Bassetto, Mike Golosov, Patrick Kehoe, Filipe Martins-da-Rocha, Alan Olivi, Ludwig Straub, Jaume Ventura and Pierre-Olivier Weill. We are particularly grateful for detailed notes on a previous draft provided by Iván Werning that lead to Section 4.2.2. Special thanks to Leonardo Barreto for excellent research assistance. The views expressed herein are those of the authors and not necessarily those of the Federal Reserve Bank of Minneapolis or the Federal Reserve System. *Contact information:* maguiar@princeton.edu; amador.manuel@gmail.com; arellano.cristina@gmail.com

in the form of capital and government bonds, but we allow for an arbitrary amount of ex ante heterogeneity in terms of preferences and income risk. We find scope for Pareto improvements that are robust to details of the household and firm sectors when the aggregate savings schedule is sufficiently elastic with respect to changes in interest rates.

The first step of our analysis is to define a new welfare metric, what we term “Robust Pareto Improvements” (RPI). Relative to an initial equilibrium, an RPI weakly increases every household’s budget set at *every* idiosyncratic state and time. Specifically, all after-tax factor prices, as well as pure profits, if there are any, weakly increase at every date, with at least one factor price strictly increasing at some date. Moreover, lump sum transfers weakly increase, as well.<sup>1</sup> By weakly expanding the budget set of all agents at all dates, these policies necessarily generate a Pareto improvement, and do not require detailed knowledge on preferences or idiosyncratic risk, hence the term “robust.” This welfare criterion rules out tax and transfer schemes that trade-off consumption in one date or state against another, including using the tax system to directly provide insurance or using lump sum taxation to relax the borrowing constraint.<sup>2</sup> While these policies are of course useful, our contribution emphasizes the opportunity to use government debt and simple transfers/subsidies to improve the welfare of all agents at all points in time.

The second step of our analysis establishes when a feasible RPI exists. That is, when is it possible for the government, given its limited fiscal tools, to weakly increase all after-tax factor prices. Given an initial equilibrium, we show that the feasibility of an RPI involves only knowledge of the aggregate savings schedule – that is, total private savings as a function of interest rates and government transfers – and the aggregate production function.

To understand the role of the aggregate savings schedule, we first study an economy without productivity or population growth such that  $r < 0$  in the initial stationary equilibrium. Low interest rates are typical in the Bewley-Huggett-Aiyagari framework because of the precautionary savings motives due to incomplete markets. Within this context, it is natural to conjecture that a policy that increases government debt by some strictly positive amount could be helpful, as the interest rate is low. Issuing government bonds, however, may lead to an increase in interest rates that crowds out capital. Simply issuing debt, therefore, may eventually reduce wages and profits, which hurt households that rely on these sources of income, and hence is not an RPI. The government, however, has additional, albeit costly, policy instruments that could be used to offset

---

<sup>1</sup>As we make clear in the formal analysis, if the borrowing limit is strictly negative, then lump sum transfers must strictly increase to compensate borrowers for any increase in the interest rate. In the Introduction, we consider the case of a zero borrowing limit, and defer the general case to the body of the paper.

<sup>2</sup>For example, our approach rules out the use of lump-sum taxes (even if available) and as a result, the policy cannot exploit the link identified by Woodford (1990) and Aiyagari and McGrattan (1998) between private borrowing constraints and government liquidity. Those policies, however, would require information on the underlying heterogeneities, frictions, and intertemporal tradeoffs of agents, in addition to knowledge about the aggregate savings behavior.

these price declines.

In particular, the government can provide a subsidy on the rental rate of capital that ensures capital remains unchanged, despite the increase in the interest rate on government bonds. This *constant-K* policy guarantees that capital, output, wages, and profits are all the same as in the initial equilibrium. If the government can finance the capital subsidy with just the revenue it receives from bond issuances, and lump-sum transfer any additional surplus, then this policy makes *every* household weakly better off: the return to wealth has increased, after-tax wages and profits have remained constant, and the government is providing a weakly positive lump-sum transfer at all dates.<sup>3</sup>

Initially focusing on the steady state, we derive a simple necessary condition for an RPI to be feasible with a constant- $K$ . Let  $B^0$  denote the outstanding government debt at the new steady state, let  $r^o$  and  $r^0$  denote the original and the new interest rates paid to households, respectively, with  $r^o < r^0 < 0$ , let  $K^o$  denote the initial capital stock, and let the initial stock of debt,  $B^o$ , be zero. For the RPI to be feasible we require that<sup>4</sup>

$$r^0 B^0 = (r^0 - r^o) K^o.$$

The left-hand side is the revenue generated by the government issuance of bonds in steady state, as  $r^0 < 0$ . The right-hand side represents the fiscal cost of the subsidy to capital: the increase in the interest rate,  $r^0 - r^o$ , is the subsidy rate required to keep  $K$  constant, and  $K^o$  is the tax base. The left-hand side captures the level of debt the government is asking households to absorb, while the right-hand side reflects the increase in interest rates necessary to implement it in equilibrium. The key consideration is therefore whether  $B^0$  can be large without a large increase in  $r^0$ ; that is, whether households are willing to increase savings to hold the additional government debt without a large increase in interest rates. This boils down to whether the elasticity of the aggregate demand for savings with respect to the interest rate is sufficiently large, a condition which will reappear in various forms throughout the analysis.

Interestingly, this potential Pareto improvement does not depend on the production technology and is achieved without increases in aggregate consumption or output at any date, as capital and labor remain at their initial levels. Every household, nevertheless, sees its budget set weakly expand at every date and idiosyncratic state, and hence every household perceives that it could increase consumption. In equilibrium, however, the higher interest rate induces some (high-income) households to postpone consumption, allowing others (low-income) to increase theirs, improving risk sharing, despite the absence of a progressive tax and transfer scheme. The

---

<sup>3</sup>Contrast this with the utilitarian metric of Dávila, Hong, Krusell, and Ríos-Rull (2012), which requires that a change in *relative* factor prices improve the lot of the poorest households relative to that of the richest.

<sup>4</sup>Here, we assume initial debt is zero. We relax this in the text.

aggregate saving elasticity being “large enough” is exactly when, in the *aggregate*, households balance the increased desire to save due to the higher interest rate against the increased desire to spend due to the expanded budget set, keeping aggregate consumption from increasing. The willingness to hold government debt rather than consume (in aggregate), despite feeling richer, echoes the result of Samuelson (1958), in which the “social contrivance” of money achieved a better allocation of a fixed endowment.

We extend this insight to the case of general policies, including those that potentially involve changes in capital. We study two cases. First, suppose the economy has “over accumulated” capital such that the marginal product of capital (MPK) is less than the rate of depreciation. That is, capital is above the “Golden Rule” level, which implies that reductions in capital increase resources for consumption. In this case, under some weak regularity conditions, an RPI always exists. The intuition is similar to the canonical analysis of Diamond (1965), given that reducing capital increases resources available for consumption. The heterogeneous agent environment and the stricter RPI metric involves some additional work, but the result intuitively holds in the extended model.

Second, we consider the case when the MPK is greater than the rate of depreciation, which is likely the more realistic scenario. In our environment this scenario is a possibility with  $r < 0$  because we allow for markups in production. The wedge between the MPK and the return on bonds can also be motivated by a liquidity premium, which we discuss in Appendix A. For small perturbations around the initial equilibrium, we show that the discounted sum of each  $t$ 's aggregate saving elasticity to an interest rate change at some date  $\tau$  is the relevant sufficient statistic for RPI feasibility.<sup>5</sup> This discounted sum needs to be large enough, in a manner we make precise. Interestingly, the discount factor is not the risk-free interest rate (which may be negative), but the marginal product of capital net of depreciation. The MPK is the rate at which the economy can trade resources across time, while the rate at which the government trades bonds with households is the risk-free interest rate.

To provide additional insights behind the results, we specialize the analysis to a representative agent (RA) economy with separable utility that features a positive markup. We first ask whether the neoclassical efficient path constitutes an RPI. We show that when this is the case, government debt is useful in “smoothing transfers.” That is, debt reduces the need for the government to use lump sum taxes in the initial periods of the transition to a new steady state. Specifically, the government uses debt to finance an investment subsidy early on, and then services the debt by taxing the additional labor and profit income generated by the larger long-run capital stock. In

---

<sup>5</sup>This is potentially measurable in the data by integrating the discounted impulse response of aggregate household wealth to an exogenous change in the risk-free interest rate or, inversely, the response of interest rates to an exogenous change in government debt held by the public. It is also easily calculated in a calibrated model using the techniques of Auclert, Bardóczy, Rognlie, and Straub (2021), something we discuss in Section 5.

this sense, government debt and capital investment are *complements* rather than the traditional substitutes. We show that in the RA economy a feasible RPI exists if the intertemporal elasticity of substitution is greater than the ratio of capital income to aggregate consumption in the initial economy. The two sides of the inequality show how a willingness to postpone consumption (driven by a large intertemporal elasticity of substitution) and a relatively small aggregate income effect from higher interests (a small share of income due to interest payments) help satisfy the aggregate savings elasticity condition.

After presenting the analytical results, we provide a simulation exercise to complement the analysis. Imposing Epstein and Zin (1989) preferences, using the income process of Krueger, Mitman, and Perri (2016) and the historical data on  $r - g$  in the U.S, we find scope for Robust Pareto Improving policies for a wide range of parameters and debt policies and for policies with and without capital expansions. Our baseline experiment considers a Pareto-improving constant- $K$  fiscal policy that starts at the laissez-faire equilibrium and slowly increases debt to 60% of output, the average observed in U.S. data over the last half-century. A second experiment starts from the 60% level and increases debt to 80% of output, which also generates an RPI. We do find, however, that seigniorage revenue from bonds has limits and features a Laffer curve: more debt increases interest rates and therefore the relative cost for servicing the debt. In our calibration, the upper bound on debt for Pareto improving fiscal policies is about 1.7 times the level of output. Third, we take up the issue of aggregate shocks. We first show how our analytical framework can be extended, and then perform a simple numerical policy experiment to show how an RPI can be implemented in an environment with aggregate risk. Finally, we consider a fiscal policy plan that consists of the same debt path as the first experiment, but with capital increasing towards the Golden Rule. We find that this fiscal plan is also a feasible RPI and generates even larger welfare gains to all households. Debt is an essential part of this fiscal policy, as it provides the revenue that is required early on to finance the subsidies for the capital expansion.

## 1.1 Related Literature

This paper is part of a fast-growing recent literature exploring fiscal policy in environments with persistently low risk-free interest rates. Mehrotra and Sergeyev (2020) use a sample of advanced economies to document that  $r - g$  is often negative, and, they develop a model to study the implications of this finding for debt sustainability. Blanchard (2019)'s presidential address to the American Economics Association gave a major stimulus to the question of debt sustainability under low interest rates. Other recent papers are Bassetto and Cui (2018); Reis (2020); Brunnermeier, Merkel, and Sannikov (2020); Ball and N Gregory Mankiw (2021); Angeletos, Collard, and Dellas (2022); and Barro (2020). Several of these papers focus on aggregate risk and build

on Bohn (1995). Our paper incorporates features of this previous work such as borrowing constraints and the potential role of markups in opening a wedge between the interest rate and the marginal product of capital. However, our focus is on designing Pareto improving policies in the presence of individual heterogeneity and incomplete markets, as in the Bewely-Huggett-Aiyagari tradition, and on the role played by  $r < g$ .

Our work also contributes to the literature studying the effects of fiscal policies in models with heterogeneous agents. Heathcote (2005), Heathcote, Storesletten, and Violante (2017), Dyrda and Pedroni (2020) study taxation in this class of models. Also recently, Bhandari, Evans, Golosov, and Sargent (2020) have explored optimal fiscal and monetary policy within the context of the heterogeneous agent model with nominal rigidities and aggregate shocks.<sup>6</sup> All of these papers focus on a utilitarian welfare criteria, and do not analyze the implications of  $r < g$ . Krueger, Ludwig, and Villalvazo (2021) consider an overlapping generations model in which agents face idiosyncratic risk in the final period of life. They evaluate the tradeoffs for general Pareto weights on different generations of a tax on capital that reduces income risk but potentially exacerbates inter-generational inequality. Boar and Midrigan (2022) studies the optimal shape of non-linear income and wealth taxes in an incomplete markets model for a class of social welfare functions. In contemporaneous work, Kocherlakota (2023) studies the role of public debt bubbles in models of heterogeneous agents that face tail risks, but abstracts from Pareto improvements and environments with capital below the Golden Rule. Di Tella (2020) explores the role of money in a model of risk premia and uninsurable idiosyncratic investment risk. Differently than these papers, we focus on policies that are a Robust Pareto Improvement over a reference (initial) allocation. Our focus on Pareto-improving policies rather than policies that maximize a utilitarian metric has an antecedent in Werning (2007), who explores Pareto-efficient tax policies in a Mirrelesian environment.

There is a large literature on OLG models that explores the Pareto efficiency of competitive equilibria, including classic papers by Allais (1947), Samuelson (1958), Diamond (1965), Cass (1972), and Balasko and Shell (1980). There is a literature examining criteria for Pareto efficiency in stochastic OLG settings, including Abel, N. Gregory Mankiw, Summers, and Zeckhauser (1989), Zilcha (1990), Rangazas and Russell (2005), and Barbie and Kaul (2009). Perhaps more related to our analysis, Hellwig (2021) obtains a condition involving the risk-free interest rate that indicates welfare improvements are possible when reallocations are limited to non-contingent inter-generational transfers. Also related is Abel and Panageas (2022). Bloise and Reichlin (2023) provides the most recent and comprehensive analysis of necessary and sufficient conditions for Pareto efficiency in the stochastic OLG model. They derive a condition for inefficiency that in-

---

<sup>6</sup>Other recent papers that have studied the implications of transfers and government debt in heterogeneous agent models with price rigidities are Oh and Reis (2012) and Hagedorn, Manovskii, and Mitman (2019).

volves the growth-adjusted dominant root of the stochastic discount factor. We relate to the OLG literature on Pareto efficiency and discuss the connections explicitly in Section 4.

The paper proceeds as follows: Section 2 describes the environment; Section 3 formally defines a Robust Pareto Improvement and provides conditions for when an RPI can be implemented in equilibrium; Section 4 provides the main analysis of how RPI fiscal policies work and derives a sufficient statistic for implementability; Section 5 provides numerical examples; and Section 6 concludes.

## 2 Environment

The model hews closely to the canonical environment of Aiyagari (1994). We augment this framework with a government that issues debt, sets a sequence of linear taxes on factor payments, and rebates back to households any fiscal surplus via lump-sum transfers. In many ways, however, our environment is more general. We allow for permanent differences in the income process or preferences across households. The framework also allows for product market markups, driving a wedge between the marginal product of capital and the return on risk-free bonds. For tractability in the benchmark analysis, we assume a zero wealth effect on labor supply, as in the well known "GHH" preferences of Greenwood, Hercowitz, and Human (1988). We generalize to non-GHH preferences in Section 4.2.4.

We suppress exogenous growth in the text, but show in Appendix E how the model extends to growth in the usual straight forward way (given homothetic preferences). As a rule of thumb, the condition  $\beta < 1$  for an interest rate  $r$  in the baseline set-up is replaced with the corresponding  $\beta < 1 + \gamma$ , where  $\gamma$  denotes the constant exogenous growth rate of labor-augmenting productivity.

### 2.1 Households

Each household, from a measure-one continuum and indexed  $i \in [0, 1]$ , draws an idiosyncratic labor productivity  $\theta_i > 0$  at time  $t$ . We do not impose that households face the same stochastic process for idiosyncratic risk. That is, some households may face a permanently lower level of productivity or additional risk. Below we impose a cross-sectional independence restriction that rules out aggregate productivity risk.

If the household provides  $s_i > 0$  units of labor, it receives  $\theta_i s_i$  in labor earnings  $F_C$  is the equilibrium wage rate per efficiency unit of labor. Without loss of generality, we assume firms pay labor taxes.

A household may also receive profit income, which we model as a payment to entrepreneurial

talent, which, like labor productivity, is an endowment that may follow a stochastic process.<sup>7</sup> Let  $c_C^g$  denote household's return to entrepreneurial talent. Define aggregate household profit income as  $y_C = c_C^g \pi_C$  and household's share as  $\lambda_C^g = c_C^g / y_C$ . Household  $g$  faces a potentially stochastic process for  $\lambda_C^g$  that determines its share of aggregate profits, with the restriction that  $\lambda_C^g \geq 0$  and  $\sum_g \lambda_C^g = 1$  for all  $C$ .

At the start of period  $C$ , household  $g$  has  $0_C^g$  units of financial assets, which receive a risk-free return  $(1 + A_C)$  in period  $C$ . Let  $\gamma_C$  denote lump-sum transfers from the government, which are uniform across  $g$ ; the household's budget constraint is

$$z_C^g + 0_{G+1}^g = F d_C^g + \lambda_C^g y_C + (1 + A_C) 0_C^g + \gamma_C$$

where  $z_C^g$  is consumption in period  $C$ .

Households are subject to a (potentially idiosyncratic) borrowing constraint  $0_C^g \leq \bar{0}_C^g$  for all  $C$ . The fact that some households may have a tighter constraint than others captures the possibility that access to financial markets may be heterogeneous. Let  $\bar{0}_C^g$  denote the loosest borrowing constraint faced by household  $g$ .

As we stated above, in our benchmark analysis, we initially restrict attention to "GHH" preferences. In particular, let  $u_C^g(z_C^g) = u^g(z_C^g)$  for some convex function  $u^g$ . We write preferences recursively as  $u_C^g = q^g(z_C^g, \alpha_{G+1}^g)$ , where  $u_C^g$  is household's value and  $\alpha_C^g$  represents a certainty equivalent operator over idiosyncratic shocks  $\epsilon_{G+1}^g, \epsilon_{G+2}^g, \dots$ , conditional on  $\epsilon_C^g$  and the household's stochastic process for its shocks. This notation nests both standard "CRRA" utility as well as the recursive utility of Kreps and Porteus (1978) and Epstein and Zin (1989).

The idiosyncratic state variables for an individual household are  $\epsilon_C^g = (0_C^g, \lambda_C^g)$ , and the aggregate states are the (perfect foresight) sequences for factor prices  $\{w_C, r_C\}$ , aggregate profit income  $\{y_C\}$ , and transfers  $\{\gamma_C\}$ . The household's problem can be written as follows:

$$u_C^g(0_C^g, \lambda_C^g) = \max_{0_C^g, \lambda_C^g} q^g(z_C^g, \alpha_{G+1}^g(0_C^g, \lambda_C^g)) \quad (1)$$

subject to:  $z_C^g + 0_{G+1}^g = F d_C^g + \lambda_C^g y_C + (1 + A_C) 0_C^g + \gamma_C$

We assume that the preference specification is such that all households value more consumption today and in the future (that is,  $u^g$  is strictly increasing in  $z$  given  $\alpha$  and in the continuation

<sup>7</sup>Note that claims to profits (like human capital) are not traded. One could include such claims, with a birth and death process that keeps valuations finite even when  $\pi_C > 1$ . See, for example, Domeij and Ellingsen (2018) and Azinovic, Cole, and Kubler (2023).

<sup>8</sup>Below we assume that the borrowing constraint is always above the natural borrowing limit. See Aiyagari and McGrattan (1998), Heathcote (2005) and Bhandari, Evans, Golosov, and Sargent (2017) for a discussion on the role of such ad-hoc limits in breaking Ricardian equivalence.



values,  $\pi_{G+1}^g$ ).<sup>9</sup> Note that as preferences can vary across households, we can accommodate distinct labor supply elasticities. The framework also nests the classic Aiyagari (1994) model with inelastic labor supply.<sup>10</sup>

Assuming an interior labor supply decision, household's first-order condition with respect to labor is  $E_C^g(\frac{U_C}{U_N}) = F_N d_C^g$  which implies a policy function  $n_C^g(\cdot)$ , where the subscript  $C$  captures the equilibrium wage at period  $C$ .<sup>11</sup>

Similarly, we let  $o_{C+1}^g(\cdot)$  and  $z_{C+1}^g(\cdot)$  denote the optimal saving and consumption policy functions in time  $C$  respectively. The aggregate stock of savings chosen in period  $C$  and carried into period  $C+1$  is

$$Z_{G+1} = \int o_{C+1}^g(o_C^g, l_C^g) dG^g$$

We now state our independence assumption. Let  $f_{G \in [0,1]}$  denote the assignment of productivity across households at time  $G$ . Let

$$\#(F_C, z_C) = \int \int l_C^g dG^g = \int \int l_C^g dG^g$$

We make the assumption that  $\#$  is independent of  $z_C$ . This is a generalization of the typical assumption that  $E$  is common across households and that  $l$  is i.i.d. across  $g$  and  $C$ . The current environment requires only that aggregate labor supply be independent of the distribution; this assumption is weaker than assuming that households are ex ante identical.<sup>12</sup>

## 2.2 Firms

The representative firm has a standard constant-returns technology given by  $y = f(k, n)$ , where  $y$  is capital and  $n$  effective units of labor. We impose that  $f$  is strictly increasing and concave on both arguments, twice differentiable in  $k$ , and satisfies Inada conditions. Firms hire labor and rent capital in competitive markets at rates  $w_C$  and  $r_C$  respectively. Let  $\tau_C^l$  and  $\tau_C^k$  denote linear taxes on factor payments for labor and capital, respectively.

Firms may have market power in the product market. We introduce the potential for mar-

<sup>9</sup>We note that it is possible to generalize this and accommodate some hand-to-mouth households. In that case, we could consider the aggregate  $q^g(G) = q^g(G)$  for some household  $g$  which corresponds to a household that does not value future consumption (it has a discount factor equal to 0). As a result, this household does not save and consumes its entire disposable income every period.

<sup>10</sup>This can be achieved by setting  $E^g = 0$ . In this case, the labor supply decision is not interior and the corresponding first-order condition below does not hold.

<sup>11</sup>As we will see below, the Frisch elasticities of labor supply, encoded in the function  $n_C^g$ , are not important for the analysis (beyond determining the initial equilibrium allocation), as the policies that we explore maintain a constant after-tax wage.

<sup>12</sup>For example, households could belong to one of  $\omega$  types, each with non-trivial measure. Then, within a type, we can assume that the law of large numbers holds, and the aggregate is simply a weighted average across types.

ket power for two primary reasons. One is to ensure our analysis is robust to the presence of markups, which appear to be a feature of the data. Second, it introduces a wedge between the marginal product of capital and the return on household savings. There are several alternative interpretations of why there may be a difference in the marginal product of capital and the return to bonds, even in the absence of risk premia, that we discuss in the next subsection.

For simplicity, we assume that firms charge a price that is a constant markup over marginal cost. Let  $\mu > 1$  be the ratio of price to marginal cost. The representative firm's first-order conditions are

$$\begin{aligned} p_c \cdot d &= \mu (1 + g_c) A_c \\ p_l \cdot d &= \mu (1 + g_l) F_c \end{aligned}$$

where  $c$  and  $l$  represent the aggregate capital and labor demands.

Firm (pre-tax) profits are given by

$$\pi_c = p_c \cdot d - (1 + g_c) A_c - c - (1 + g_l) F_c = \frac{1}{\mu} p_c \cdot d$$

where the last equality follows from constant returns. Profits are taxed by the government at rate  $\tau_c$ , so after-tax profits are  $\pi_c = (1 - \tau_c) \pi_c$ . We can think of the representative firm hiring a bundle of entrepreneurial talent that is in constant aggregate supply at an after-tax price

For some of the analysis that follows, it will be useful to distinguish cases when capital is above or below the "Golden Rule" rule. For a given  $\mu$ , we define the Golden Rule capital level,  $c^*$ , by  $p_c \cdot d = X$ . Recall that when capital is above the Golden Rule, permanent reductions in capital increase resources for aggregate consumption in all periods, while when capital is below the Golden Rule, this is not possible.

## 2.3 Financial Intermediaries

We assume that the capital is owned by competitive financial intermediaries. Intermediaries borrow from the households at rate  $r$  and, in turn, rent capital to firms at  $A_c$  and invest in government bonds at rate  $r$ . Capital depreciates at rate  $\delta$ . Competition in the intermediary market ensures the following equilibrium condition at all times:  $A_c = A_c^1 = A_c - \delta c = X$ . Given the first equality, in what follows, we drop the distinction between  $A_c$  and  $A_c^1$ .

Although the rental rate of capital net of depreciation is equated to the return on financial assets, as noted above, the potential presence of a markup implies that it may differ from the

<sup>13</sup>As is usually the case, making this assumption is not crucial. We could have equivalently assumed that the capital is owned directly by firms, which finance capital purchases with risk-free bonds issued to households.

marginal product of capital. Allowing for this wedge lets us consider environments with low interest rates  $r_t \leq 0$  with capital being above or below the "Golden Rule" level. There are several alternative (or additional) reasons for such a wedge in practice. Uncertainty regarding the return to physical investment would potentially impose a risk premium on the required rate of return to capital. As noted above, we are abstracting from such risk in order to transparently highlight the novel aspects of our analysis.

Even under perfect foresight, there may be additional reasons for a wedge between the marginal product of capital and the risk-free interest rate. One alternative pursued by Ventura (2012) and Farhi and Tirole (2012), is to consider firm-level borrowing constraints. A second alternative is that government bonds provide "liquidity services" relative to the return on physical capital. This latter possibility can be readily introduced by modifying the intermediaries problem. To see this, suppose that a competitive intermediary receives flow return  $X$  from holding physical capital and  $A^1 + d$  from holding government bonds, where  $d$  is the additional (pecuniary) return provided by government bond's "liquidity." The value of  $d$  may depend on the aggregate stock of government bonds (as suggested by Krishnamurthy and Vissing-Jorgensen, 2012a), but is taken as given by an individual intermediary. Equilibrium requires  $X = A^1 + d$ . In appendix A we show how our benchmark results extend to this alternative environment, including when  $d$  declines in the stock of government bonds.

## 2.4 Government

The government's policy consists of a sequence of taxes  $\tau_c^t, \tau_l^t, \tau_g^t$ , as well as a sequence of one-period debt issuances  $b_t$ ,  $g_t$ , and lump-sum transfers  $f_t$ , such that the sequential budget constraint holds at all periods:

$$p_t c_t + q_t^c F d_t + q_t^l A_t c_t + q_t^e e_t + g_{t+1} = (1 + A_t) c_t \quad (2)$$

Note that we allow for the government to potentially dispose of resources freely by writing the constraint as an inequality. We abstract from government purchases, but adding this in would have no bearing on the analysis.<sup>14</sup>

## 2.5 Resource Constraint and Market Clearing

Market clearing in the asset market requires  $e_t = c_t + c_t$ . Market clearing in the labor market requires  $l_t = \#_t$  recall that  $\#_t$  is aggregate efficiency units of labor supplied by households.

<sup>14</sup>As we discuss with initial government debt below, we can consider the initial equilibrium as one with a particular tax structure in which the revenues are not rebated.

Using these, the aggregate resource constraint is

$$Z \int c_t = Z \int \beta c_{t+1} + (1 - \lambda) Z \int c_t$$

**Definition 1 (Equilibrium Definition)** Given an initial distribution of household assets and idiosyncratic shocks  $\{g_0^i, l_0^i, \theta_0^i\}_{i \in [0,1]}$  and a fiscal policy  $\{c_t, \tau_t, \tau_{t+1}^c\}_{t \geq 0}$ , an equilibrium is a sequence of quantities  $\{c_t, \#_t, \alpha_t\}_{t \geq 0}$ , and prices  $\{A_t, A_t^c, F_t, \alpha_t\}_{t \geq 0}$  such that:  $c_t$  and  $\#_t$  are the aggregate stock of savings and the aggregate labor supply consistent with household optimization given prices and transfers,  $\alpha_t$  is the aggregate after-tax profit, and  $\#_t$  are the aggregate capital and labor demands consistent with firm optimization given prices and taxes, the sequential government budget constraint is satisfied, the aggregate resource constraint holds  $A_{t+1} = A_t + X_t$ , and the asset market clears.

We define a stationary equilibrium to be an equilibrium in which all sequences are constant over time.<sup>15</sup>

### 3 Robust Pareto Improvements

In this section we introduce and discuss our welfare metric, "Robust Pareto Improvements" (RPI). We then provide necessary and sufficient conditions for a class of RPI to be implementable as an equilibrium.

#### 3.1 A Robust Welfare Metric

Given idiosyncratic states, a household's welfare is determined by sequences of (after tax) factor prices,  $F_t$  and  $A_t^c$ , aggregate profits,  $\alpha_t$ , and transfers  $\tau_t$ . These are the equilibrium objects that appear in the budget set of the household problem (1). With this in mind, we define what we mean by a "robust" Pareto improvement:

**Definition 2.** Consider two sequences of factor payments  $\{F_t, A_t^c\}_{t \geq 0}$  and transfers  $\{\tau_t\}_{t \geq 0}$ , with  $\delta = \beta$ . We say sequence  $\{F_t, A_t^c, \tau_t\}_{t \geq 0}$  generates a Robust Pareto Improvement (RPI) over sequence

<sup>15</sup>In the analysis that follows, we will assume that such a stationary equilibrium exists. Note that this may require additional assumptions on the stochastic processes for labor productivity and the profit share as well as on their initial cross-sectional distribution. See Aclay (2018), Light (2018), and Achdou et al. (2021) for results on the existence and uniqueness of stationary equilibria in Bewley-Huggett-Aiyagari models.

if it expands budget sets for every agent at every time and every state:

$$F_C \succeq F_C^* \quad c \succeq c^* \quad A_C \succeq A_C^* \quad )_C \succeq )_C^* \quad (A_C \succeq A_C^*)_0 \quad \text{for all } C \in \mathcal{C}$$

with at least one strict inequality.

From the sequential budget set governing the household's problem (1), we see that the consumption possibility set is weakly increasing in  $\bar{w}$  and  $\bar{c}$ . If  $\bar{w} > 0$ , it is also weakly increasing in  $A$ . However, households with negative positions (debt) are worse off if  $\bar{c}$  increases. The fact that  $)_C \succeq )_C^* \quad (A_C \succeq A_C^*)_0$  ensures that additional lump-sum transfers are large enough to make debtors weakly better off, and strictly better off, if  $\bar{c}_C \succeq \bar{c}_C^* \succeq 0$ . From every household's perspective, resources are weakly greater at every  $\omega$  and at every idiosyncratic state, and they are strictly greater for at least a positive measure of households at some

The term "robust" is meant to highlight that limited knowledge is required about idiosyncratic preferences or sources of income. All that is needed to ensure an individual prefers a fiscal policy is that a larger budget set is a good thing for the consumer. In particular, how an individual values intertemporal or inter-state trades plays no role.

It is instructive to clarify how this metric is distinct from some well known alternatives and how it rules out prominent policies studied in the literature that improve outcomes in the context of incomplete markets.

For example, in the classic analysis of government debt in an incomplete markets setting of Aiyagari and McGrattan (1998), a government issues bonds, transfers the proceeds to households, and then levies lump-sum taxes to pay interest on the debt. Unconstrained households can save the transfers in anticipation of the taxes, while those constrained can effectively bring future income forward. This policy effectively relaxes the borrowing constraint (as in Woodford, 1990). From a "period-0" perspective, this may represent a welfare gain to households. However, the fact that future taxes increase violates the conditions for an RPI. Moreover, the introduction of government debt may crowd out capital and reduce the equilibrium wage, representing another violation of RPI. Consider a household that earns only labor income and lacks access to financial markets, which is not an unrealistic description of some households in the data. They may be strictly worse off, in the Aiyagari-McGrattan experiment, but not under an RPI.

Another well-known paper is Davila, Hong, Krusell, and Lippi-Rull (2012). That paper characterizes constrained efficient equilibria under a utilitarian metric. The focus of the analysis is whether alternative consumption or labor supply decisions by households could alter equilibrium factor prices in such a way as to raise the utilitarian objective function. However, it may be the

<sup>16</sup>Uhlig and Braun (2006) contains related results; it illustrates that a tax on capital that raises interest rates can be welfare improving under the utilitarian metric because of improved risk sharing, even if wages decline and the

case, for example, that the efficient equilibrium features a decrease in the interest rate and an increase in the wage, which involves a trade off, that violates the definition of an RPI.

Perhaps the most common metric for evaluating policy is the traditional Pareto criteria, in which every household's expected discounted utility at time zero weakly increases, with a strict increase for at least one. For example, the welfare consequences of government debt is evaluated under the Pareto criteria in Diamond (1965), who highlights both the impact on welfare of both taxes as well as the associated change in factor prices due to the crowding out of capital. Similarly, Samuelson (1975) uses a Pareto criteria to evaluate social security policies that reduce resources while young in exchange for transfers while old. Several other papers explore Pareto improvements in an incomplete markets setting (see, for example, Krueger, Mitman, and Perri, 2016, Hosseini and Shourideh, 2019, and Boerma and McGrattan, 2020). Pareto-improving policies in this setting may involve, for example, better insurance, so that income increases in some states at the expense of others. Again, these tradeoffs may be desirable given a particular set of preferences and beliefs, but do not represent RPIs.

The advantage of the RPI metric is we do not need to take a strong stand on preferences, the nature of idiosyncratic risk, or heterogeneity in either of these across households when evaluating policies. Of course, expanding all budget sets at all dates and times is potentially a high hurdle for policy analysis. This begs the question as to whether and when an RPI is achievable in equilibrium given the limited fiscal tools available to the government.

### 3.2 Restrictions on Fiscal Policy

In this subsection, we derive necessary and sufficient conditions for a fiscal policy to be consistent with the restrictions imposed by equilibrium. That is, given a limited set of taxes, we describe the allocations that the government can implement as equilibria.

We assume that the economy starts at a stationary equilibrium, which potentially may have an amount of government bonds outstanding as well as distortionary taxes. As is clear from the definition of an RPI, we could also start from a non-stationary equilibrium, but in that case all comparisons would be relative to the initial "reference" sequence of factor prices. Let  $w^0$  denote the wage, interest rate, and aggregate profits in the initial stationary equilibrium, and let  $(N^0, K^0)$  denote the associated aggregate labor supply and capital stock. Let  $B^0$  denote government debt in the initial equilibrium, financed by  $\tau^0 = g^0 - g^0$ . For simplicity, we assume that there are zero lump-sum transfers in the initial equilibrium,  $\tau^0 = 0$ , and tax revenue equals  $\tau^0 = g^0$  (that is, the government budget constraint holds with equality).

Starting from this equilibrium, consider that the government unexpectedly announces a new tax revenues are thrown away.

€scal policy. 'at is, in period  $C = 0$ , the government announces a sequence of debt issuances, taxes, and transfers  $\{g_C, \tau_C, t_C\}_{C=0}^{\infty}$ . After the announcement, there is perfect foresight. Given the new policy, households and firms re-optimize. Consider a new equilibrium that arises, with aggregate quantities  $\{c_C, n_C, g_C\}_{C=0}^{\infty}$  and prices  $\{A_C, A_C^F, F_C\}_{C=0}^{\infty}$ . 'e level of period-0 capital and debt, as well as  $A_0$ , are inherited from the initial equilibrium, so  $c_0 = \bar{c}$ ,  $n_0 = \bar{n}$ , and  $A_0 = \bar{A}$ .

We restrict attention to policies that keep after-tax wages and profits unchanged from the initial equilibrium:

**Definition 3.** A "constant wage and profit policy" ensures  $F_C = F$  and  $c_C = \bar{c}$  for all  $C \geq 0$ .

Under a constant wage and profit policy, no agent experiences a change in labor or profit income at each  $C$  and idiosyncratic state  $\theta_C$ . 'is restriction is useful for two reasons. One is that the constant wage ensures the labor market clears at the original employment level, regardless of the elasticity of labor supply.<sup>17</sup> 'e second is that it allows us to keep the interplay of government debt issuance and changes in the interest rate in the foreground.

'e new constant-wage and profit €scal policy impacts the households only through the induced sequence of interest rates and transfers,  $\{r_C, t_C\}_{C=0}^{\infty}$ . For this reason it is expositionally useful to think of interest rates as the target of the €scal policy, as is familiar from the monetary literature. In period 0, each household re-optimizes its consumption-saving policy to incorporate the new sequence of interest rates and transfers, while maintaining the remaining factor incomes constant at  $F$  ( $\bar{c}$ ). Starting from the initial stationary equilibrium in period 0, we define the following functions:

**Definition 4.** Let  $A_{C+1}(r \cdot T)$  denote the aggregate household assets at the end of period  $C$  generated by the households' optimization given  $F_C = F$  and  $c_C = \bar{c}$  for all  $C \geq 0$ . 'e associated aggregate consumption function is

$$C_C(r \cdot T) = F \bar{c} + \bar{c} + (1 + A_C) A_C(r \cdot T) - A_{C+1}(r \cdot T) + \bar{c}$$

'at is, if  $\theta_{C+1}(0 \cdot I \cdot \bar{c})$  and  $\theta_C(0 \cdot I \cdot \bar{c})$  denote households' policy functions in the new equilibrium, then  $A_{C+1} = \theta_{C+1}(0 \cdot I \cdot \bar{c})$  and  $C_C = \theta_C(0 \cdot I \cdot \bar{c})$ . 'ese mappings of the sequence of interest rates and transfers to the sequence of aggregate household assets and consumption summarizes how a €scal policy affects aggregate saving behavior in equilibrium.<sup>18</sup>

'e standard "primal" approach in the Ramsey taxation literature is to restrict attention to

<sup>17</sup>For this, we are using the assumption of zero wealth effect on labor supply.

<sup>18</sup>Our function  $A$  is closely related to the mapping of Auclert, Bardczy, Rognlie, and Straub (2021) as well as the  $C$  function of Auclert, Rognlie, and Straub (2018) and Wolf (2021). All map sequences of policy variables and equilibrium prices into a path of aggregate household saving or spending, starting from an initial distribution of idiosyncratic states.

the set of allocations that can be achieved in a competitive equilibrium by feasible fiscal policies, replacing taxes and prices using equilibrium conditions. We follow a similar approach, with the caveat that we cannot rely on a representative consumer's Euler equation to solve out the interest rate or appeal to Ricardian equivalence for lump-sum transfers. In its place, we include the restriction imposed by household optimality implied by the mapping

We say that the sequence  $(\tau_t, g_t, T_t)$  is feasible if there is a fiscal policy  $(\tau_t, g_t, T_t)_{t=0}^{\infty}$  with  $\tau_t \geq 0$  such that a competitive equilibrium with quantities  $(c_t, n_t)_{t=0}^{\infty}$  and prices  $(A_t, P_t)_{t=0}^{\infty}$  exists, where  $A_0 = A$  and  $P_0 = P$ . We have the following result:

Lemma 1. A sequence of interest rates and transfers  $(r_t, T_t)_{t=0}^{\infty}$ , is feasible if and only if there exists a non-negative sequence  $(g_t)_{t=0}^{\infty}$  and a sequence  $(\tau_t)_{t=0}^{\infty}$ , with  $\tau_t \geq 0$  and  $\tau_t = 0$ , such that for all  $t \geq 0$ :

(i)  $A_{t+1}(1+r_{t+1}) = A_t + g_{t+1} - \tau_{t+1}$ ,

(ii) and

$$A_{t+1} - (1+r_{t+1})A_t = g_{t+1} - \tau_{t+1} - (A_t + X) \tau_{t+1} + (A_t + X) \tau_t \quad (3)$$

Proof. All proofs are in Appendix C.

This lemma provides necessary and sufficient conditions for the sequence  $(\tau_t, g_t, T_t)_{t=0}^{\infty}$  to be consistent with equilibrium. Household optimality is ensured by the definition. Asset market clearing is condition (i) of the lemma. Condition (ii) combines firm optimality and government budget balance, and is discussed next. The aggregate resource constraint (goods market clearing) holds by Walras law. Note that these conditions must be met by any constant wage and profit fiscal policy, not just those that result in an RPI.

The left hand side of condition (3) is the revenue raised from net debt issuance minus any lump sum transfers. The right hand side is the fiscal cost of the subsidies necessary to keep wages and profits constant. In particular, simple accounting implies that government revenue raised by taxing firms is total output minus the firm's after-tax payments to households:

$$\text{Taxes paid by firms} = (c_{t+1}^F)^{\alpha} F_{t+1}^{\beta} (A_t + X) c_t \quad (4)$$

where we use the fact that after-tax wages and profits are unchanged. In the initial equilibrium, we have a similar expression, where tax revenue is used to pay interest on the initial debt. Hence,

$$A^> = (c_{t+1}^F)^{\alpha} F_{t+1}^{\beta} (A^> + X) c^> \quad (5)$$



Using this expression to substitute out  $\hat{p} + F^{\#} >$  in (4) and converting from revenues to subsidies by changing sign, we obtain the right-hand side of (3) as tax subsidies to firms. The inequality follows from the fact that the government is free to dispose of any fiscal surplus it does not choose to lump-sum rebate.

The implications of condition (3) on whether an RPI is feasible will be the focus of the next sections. At this stage, we flag three immediate consequences that will play prominent roles in what follows. First, an increase in  $i_t$  (for a given  $\rho$ ) increases the right-hand side of (3), tightening the constraint. Higher interest rates are costly, as the government needs to subsidize capital to avoid a reduction in firms' demand for the factor. Second, an increase in  $\rho$  (for a given  $A_t$ ) reduces the right hand side of the (3) in the presence of a positive markup  $\mu > X$ , relaxing the constraint. However, increasing  $\rho$  may require an increase in interest rates, to encourage the household sector to save. Finally,  $\dot{A}_t \dot{Y} > 0$  implies  $A_t c_t \dot{Y} > 0$  for  $c_t > 0$ , which is the left-hand side of (3). This captures the fact that negative interest rates are a potentially important source of revenue for a government that borrows.

A convenient feature of Lemma 1 is that the feasibility of sequence of interest rates and transfers is solely determined by aggregates. No additional information is needed, despite the potentially complicated nature of the policies necessary to keep the wage and profits constant and the potentially rich sources of heterogeneity underlying the aggregate saving and consumption functions.

Walras law allows an alternative to Lemma 1 that involves the aggregate resource constraint. Aggregating the households' budget constraints, we have:

$$\begin{aligned} C_t &= F^{\#} > + > + (1 + A_t) A_t c_{t+1} \\ &= ( > \cdot \# >) A_t ( > + >) X > + (1 + A_t) (c_t + \rho) (c_{t+1} + c_{t+1}) \end{aligned}$$

where the second equality follows from asset market clearing and (5). Substituting into (3), we obtain:

Corollary 1. A sequence of interest rates and transfers  $(r_t, T_t)$  is feasible if and only if there exists a non-negative sequence  $\{c_t\}_{t=0}^{\infty}$  with  $c_0 = >$  such that for all  $t \geq 0$ :

$$C_t(r \cdot T) \leq (c_t \cdot \#_0) + (1 - X) c_t c_{t+1} \tag{6}$$

Corollary 1 reduces the question of feasibility to the existence of an investment sequence that finances the aggregate consumption generated by the policy. The next section leverages

<sup>19</sup>A reader may wonder why the initial level of debt,  $>$ , does not explicitly appear here while it did in Lemma 1. We note that  $>$  implicitly appears in the initial asset position of households and thus affects the aggregate consumption function  $C_t(r \cdot T)$ .

Lemma 1 and Corollary 1 to explore the economics of engineering an RPI.

## 4 Robust Pareto Improving Policies

In this section we delve into the details of when and how fiscal policy can engineer a RPI. We begin with a simple policy in which the government issues bonds but keeps capital at a constant level. This exercise will allow us to focus in on how an increase in interest rates generates a welfare gain without changing aggregate resources. We then move to general policies in which the capital stock may evolve over time.

### 4.1 The Constant- $K$ Policy

Let us first consider whether an RPI can be implemented with a constant level of capital. There are many nice features of such a policy. As we will see, one advantage of such a policy is that the feasibility of an RPI applies whether the economy is fully competitive, or has markups,  $\mu > 1$ , and whether the capital stock is above or below the Golden Rule level. A second feature is that it highlights the fact that an RPI is possible without changing total resources (all gains are derived from a better allocation of the same amount of output, while keeping in mind that every household has more resources available for consumption at every state and time). The existence in this case of an RPI is solely due to the inefficiency generated by the incomplete markets.

For simplicity, in this section we assume that the economy is originally at the laissez-faire stationary equilibrium with zero taxes and transfers and  $\tau = 0$ . Now consider a fiscal policy through which the government permanently raises the after-tax return to savings to  $r = \bar{r} > r^*$ , for all  $C > 1$  and sets  $\tau_C$  to its respective lower bound consistent with an RPI,  $\tau_C = (A^0 - \bar{r})\bar{K}$ .

To gain some intuition, let us suppose that the economy converges to a new stationary equilibrium with  $\lim_{C \rightarrow 1} A_C(f(A^0, \bar{K})) = \bar{r} > r^*$ .<sup>20</sup> Assuming that the stationary aggregate savings schedule is upward sloping with respect to the interest rate (as is the case in most of the applications of the Aiyagari model), we have that  $\bar{K} > K^*$ . Letting  $\bar{B} = \bar{K} - K^*$  denote the long run supply of government bonds, we then have that  $\bar{B} > 0$ : the permanent increase in interest rates is associated with a permanent increase in government debt. Condition (3) requires that in the limit:

$$A^0 > (A^0 - \bar{r})(\bar{B} > 0) \tag{7}$$

Given that the right hand side is strictly positive, it is necessary for this RPI to be feasible that

<sup>20</sup>In the original Aiyagari framework, as long as  $\bar{r} > r^*$ , households savings will remain finite in the stationary equilibrium.

$\dot{A}^0 \dot{Y} > 0$ : there must be "seigniorage" from bonds, and this seigniorage revenue must be greater than the right-hand side of (7).

It is helpful to explain the right-hand side of condition (7). 'e increase in the interest rate in the new equilibrium would raise the rental rate of capital and reduce the firm demand for the factor, all else equal. With a constant-policy, the government must subsidize the return from renting capital to avoid this reduction in factor demand. Recall that  $r^0 = A^0 + X$  is the rental rate in the initial equilibrium. In the new equilibrium, the government must set a capital subsidy,  $g_c > 0$ , such that firms pay the same rental rate as in the original equilibrium,  $r^1 = (1 + g_c)(A^0 + X)$ . As firms are paying the same after-tax rental rate, the interest rate  $r^1 > r^0$ , and hence profits, wages, and total output remain unchanged. From the government budget constraint, equation (2), the cost of this subsidy is  $A^0 - A^1 > 0$ . 'e condition then tells us that the stationary revenue from bond issuances must be enough to cover the cost of the capital subsidy plus the cost of the transfer necessary to compensate borrowers for the increase in the rate.

Figure 1: Net Resource Cost with Constant

Note: 'is figure is a graphical depiction of the fiscal tradeo, from condition (7). All elements are normalized by the laissez-faire stationary equilibrium output  $Y^0 > 0$ . 'e downward-sloping line  $r^1 = (1 + g_c)(A^0 + X)$  represents the firm's demand for capital, and the upward-sloping line  $r^1 = (1 + g_c)(A^0 + X)$  depicts aggregate household saving associated with the interest rate and the initial wage  $w^0 > 0$  as well as the transfers generated by any fiscal surplus. 'e intersection is the initial laissez-faire stationary equilibrium. Fiscal costs are represented by  $A^0 - A^1 > 0$ , the area shaded in red, and seigniorage revenue by  $\Delta Y > 0$ , the area shaded in gray. In this example, policy holds capital at the initial laissez-faire capital stock.

Figure 1 depicts the steady-state tradeo, in the canonical capital market equilibrium diagram from Aiyagari (1994). 'e underlying calibration is provided in Section 5, but the qualitative features are fairly general. At each interest rate on the vertical axis, the associated rental rate

of capital is  $\dot{A} = A + X$ . Holding labor supply constant  $\# = \#^>$ , the downward-sloping red line traces out a capital demand equation from the firm's first-order condition  $(\bullet \#^>) = \cdot (A + X)$ , where recall we assume the initial equilibrium has zero taxes.

Similarly, at each candidate  $\bar{A}$  denotes the aggregate steady-state saving of households when the wage is fixed at  $\bar{w}^>$ . These two curves intersect at the laissez-faire equilibrium interest rate  $\bar{A}^>$ , which is the initial equilibrium. Note that in this parameterization  $\bar{A}^> \dot{Y} > 0$ , which is the case of interest. The quantities reflected on the horizontal axis are normalized by  $\bar{y} = (\bar{w}^> \bullet \#^>)$ .

The fiscal policy subsidizes the rental of capital, so that firms are willing to rent  $\bar{r}^>$  at any  $\bar{A}$ . The width of the gray rectangle is  $\bar{A} - \bar{A}^> = \frac{BB}{\bar{w}^>}$ , and its height is the interest rate at the new equilibrium; hence, its area is  $\bar{A}^> (\bar{A} - \bar{A}^>)$ . Starting from zero debt, this area is the left-hand side of (7).

The red rectangle has height  $\bar{A}^> - \bar{A}^0$ , where  $\bar{A}^0$  is the interest rate in the laissez-faire equilibrium. Its width is  $\bar{A}^0 \cdot \bar{y}^>$ , where  $\bar{y}^>$  is the capital stock in the laissez-faire equilibrium. The area of this rectangle is  $\bar{A}^0 (\bar{A}^> - \bar{A}^0) \bar{y}^>$ , which equals the subsidies necessary to keep capital  $\bar{A}$  at  $\bar{A}^0$ . In this example  $\bar{y}^> = 0$ , and hence this is the right-hand side of (7). Condition (7) tells that a necessary condition for the RPI to be feasible is that the area of the gray rectangle exceeds that of the red.

Note that this implies that feasibility is a tighter condition than being on the "upward sloping" portion of the debt Laffer curve. The steady state debt Laffer curve peaks when  $\bar{A}$  is maximized; that is, when the gray rectangle achieves its maximum. After that point, the increase in the interest rate dominates the additional debt issuance and seigniorage revenue declines. However, the fiscal cost,  $\bar{A}^0 (\bar{A}^> - \bar{A}^0)$ , is strictly increasing in  $\bar{A}$  and hence the net revenue (seigniorage minus capital subsidy) for the constant-policy peaks at a level of debt strictly below the peak of the debt Laffer curve.

The diagram restricts attention to the steady state, but contains important insights into the requirements for an RPI to be feasible. The first thing to note is that the level of the initial interest rate matters. That is, households must be willing to hold the economy's wealth at a low interest rate, reflecting a significant demand for precautionary savings. Intuitively, and as we shall see in detail in the calibration of Section 5, this will be the case if households face significant idiosyncratic risk and are patient and risk averse. The large demand for a safe store of value provides a source of seigniorage for the government.

Second, consumers must be willing to hold new debt without a sharp increase in the interest rate. That is, the elasticity of aggregate savings to  $\bar{A}$  must be sufficiently large. The intuition is that the return to saving ( $\bar{A}$ ) cannot increase significantly in response to the issuance of

<sup>21</sup>Recent papers that focus on the debt Laffer curve include Bassetto and Sargent (2020) and Mian, Straub, and Suarez (2022).

<sup>22</sup>For this policy, as we already mentioned, it is necessary that  $\bar{A}^> > 0$ , or else (7) cannot hold.

the increase in the return to capital is the amount of subsidy necessary to keep capital constant. The elasticity of the interest rate to government debt is a primary concern when discussing the crowding out of capital. Here, it determines the amount of fiscal resources that must be dedicated to capital subsidies.

Note that the key elasticity is that of aggregate household savings. This echoes the point made in the previous section that household heterogeneity matters only as it determines the slope of the aggregate savings function. This elasticity can potentially be estimated using aggregate time series, and we survey some of the estimates from the literature in Section 4.2.3.

First, conditional on the initial equilibrium, the feasibility condition is independent of the shape of the aggregate production function or the presence or size of a mark-up. This is because capital and labor do not change under the constant policy. The focus is purely on the shape of the households' aggregate saving function.

This raises an intriguing feature of the Pareto improvement. Aggregate output, consumption, and investment are all held fixed at the initial level,  $y = y^0$  and  $c = c^0$ . Yet every household faces a weakly bigger budget set and a strictly bigger one if  $\theta > 0$ . However, it cannot be the case that aggregate consumption increases. The key is that the higher interest rates induces enough households to reduce their consumption to offset the households that do increase consumption. This is why the elasticity of aggregate saving to the interest rate plays such a crucial role. Heuristically, those with high labor endowment states must be willing to postpone consumption because of the high return on saving. Those with low endowment states on average carry in higher precautionary savings, allowing them to consume more. On net, aggregate consumption remains constant, but it is distributed in a more beneficial way across idiosyncratic states.

The source of the welfare gains in this example has a clear antecedent in Samuelson (1958). In Samuelson's classic OLG analysis, when the real interest rate is below the growth rate, a Pareto improvement is generated if the young delay consuming their endowment in exchange for paper (money), and then trade the paper to the next generation when old. This Pareto improvement is generated even though total output is unchanged by the introduction of money. However, the presence of money does increase the real interest rate above the initial equilibrium. Similarly, in our constant-K experiment, the issuance of government debt increases the return to savings without changing aggregate output. The increase in the real interest rate ensures that private households are willing to hold more government bonds, which they then trade to smooth consumption across states and time, without increasing aggregate consumption. As in Samuelson, the presence of a non-productive asset may improve the allocation of a fixed amount of output across agents.

Samuelson's paper spawned a large literature on Pareto efficiency in OLG environments. None of these papers discuss the elasticity of the aggregate savings schedule, which features

prominently in our analysis. This speaks to the distinct differences that arise in our environment relative to Samuelson and the subsequent OLG literature.

The standard approach to evaluating Pareto efficiency in the OLG literature, for example Balasko and Shell (1980) or Hellwig (2021), is to compare the level of the risk-free interest rate in a competitive equilibrium with the economy's growth rate. At an interior equilibrium<sup>23</sup>, young households are on their Euler equation and indifferent to marginal inter-temporal trades across time at the risk-free interest rate. If the government could transfer resources from young to old at a greater return, then all households would be strictly better off. The government is able to do such a Pareto-improving transfer if the return on risk free bonds is less than the growth rate of the economy, guaranteeing that the original competitive equilibrium is Pareto inefficient. This criteria does not depend on the inter-temporal elasticity of substitution or other preference parameters (other than smoothness), and does not involve our aggregate saving elasticity condition. In our environment, we have a rich set of potential sources of heterogeneity across households, which may imply some agents are not interior on the Euler equation. Moreover, our government has a restricted set of instruments for transferring resources across agents. The limited fiscal tools and the desire for policies to be robust to the nature of idiosyncratic heterogeneity narrow our focus to robust Pareto improvements, which in turn involves additional restrictions on the aggregate savings schedule beyond a low equilibrium interest rate.

Importantly, the source of welfare gains in our environment are distinct from other fiscal schemes in which an agent "pays in" or is taxed. That is, the requirements of an RPI rule out better insurance or reallocation via progressive taxation, tax-and-transfer insurance schemes, or a pay-as-you-go social security system. In the words of Samuelson (1958), the willingness to hold government bonds at low interest rates is a substitute for the "social coercion" of tax and transfer schemes. But in our environment, a debt expansion has the advantage that it can be implemented without the detailed information on private agents' trade-offs that would be required in a tax-and-transfer scheme. We note also that in Samuelson, privately issued zero-interest debt would also serve the purpose of improving upon the equilibrium allocation. In our environment, such a privately issued bubble (which may involve relaxing the ad-hoc borrowing limit) would not yield the government revenues necessary to subsidize capital and prevent the wage (and profits) from falling, and hence is not a path to an RPI.

Figure 1 depicts the steady state tradeoff, faced by a government implementing the constant-policy. We also need to consider policies along the transition. From Lemma 1 condition (3), the

---

<sup>23</sup>Hellwig (2021) also considers the case where the young do not actively save in equilibrium, in which case the implicit interest rate to be compared to the growth rate is read off the marginal rate of substitution of the representative young agent.

transition policy requires (for a constant) that

$$A_{G1} (1 + A_G) A_C \bar{A} > \bullet \quad (8)$$

where we have imposed for additional simplicity that  $\bar{t} = 0$  (and thus  $\bar{c} = (A_C \bar{A}) \bar{c} = 0$ ) and used that  $\bar{c} + \bar{c} = A_C \bar{c}$ . This inequality highlights that it is the response of aggregate savings at all periods to changes in interest rates that determines the feasibility of an RPI with a constant-policy. Smaller short-run elasticities of aggregate savings make the condition (8) harder to satisfy even if it were to hold in the long-run (where the elasticity is potentially higher). The condition also highlights that a gradual approach may have a better chance of working with a constant-policy: if the aggregate savings are very inelastic in the short-run, a permanent increase from the beginning may be infeasible as an RPI, while a gradual increase may work. In the simulation of Section 5 we provide an example of this, and of how debt issuance along the transition ensures (8) holds at all  $t$  despite the relatively small short-run elasticity.

The constant-policy is a useful benchmark to study RPIs because it is robust not only to rich household heterogeneity, but also to production elasticities and markups: the feasibility conditions for RPIs in (8) do not depend on these production-side details. However, if a constant-policy cannot generate a feasible RPI, it may still be feasible for the government to adjust both  $\bar{c}$  as well as  $\bar{c}$  when implementing a policy. We turn to this more general policy problem next.

## 4.2 General Policies

In this subsection, we study more general policies, which may involve changes to the capital stock, that can achieve an RPI. We derive sharper conditions for feasibility and consider in more detail the transition path. As in the constant-policy, we find that the elasticities of aggregate savings continue to be a main determinant for feasibility.

Our guide will be Lemma 1 and Corollary 1. As in the case discussed above, a common theme will be to ensure that aggregate consumption chosen by households does not increase "too much," despite the fact that budget sets expand. Again, the countervailing force will be an aggregate willingness to save induced by an increase in interest rates. We consider two cases in turn, one in which the initial equilibrium has over accumulated capital (that is, capital is beyond the Golden Rule level) and then the converse case. Recall that the Golden Rule capital level, defined by

$$(\bar{c}, \bar{c}) = X$$

#### 4.2.1 Capital above the Golden Rule

If capital is above the Golden Rule, there is a relatively straightforward path to an RPI. The approach builds on Diamond (1965), substituting government bonds as a replacement for the over-accumulated capital. We are in an environment with potentially richer idiosyncratic heterogeneity and have a stricter welfare metric, and hence need to worry about changes in factor prices as capital is reduced. In particular, we cannot trade off lower wages against higher interest rates, or higher consumption when old with lower consumption when young. However, Corollary 1 has already done most of the work regarding the feasibility of an RPI in this environment.

To start, first note that the capital sequence  $\{c_t\}_{t=0}^1$  with  $c_0 = c^*$  and  $c_t = c^*$  for  $C \geq 1$  satisfies the resource constraint (6), with a strict inequality at the original interest rates and transfers  $r = r^*$  and  $T = T^0 = f'(c^*)$ . This is because, at the same interest rates and transfers, the households' problems have not changed, aggregate consumption remains as in the original,  $c_t = (c^*)^{\beta}$  for all  $C \geq 0$ , but initial capital above the Golden Rule implies that a lower investment increases net resources at all dates. From Corollary 1, this strict inequality means there are surplus resources at every date with this new capital sequence.

We can use the language of Lemma 1 to reinterpret this result using the government budget constraint. In particular, for this case, the government issues an amount of bonds  $b_0 = b^*$  in period 0, and then sets  $c_t = c^*$  thereafter. The government policy must guarantee that  $A_t$  and  $c_t$  do not change, which requires an increase in the tax on capital (to reduce the firms' demand for capital), and an increase in the subsidy to labor and profits (to compensate labor and profits for the fall in capital). The strict inequality in (6) translates into a strict inequality in (3): the increase in revenue from the capital tax more than compensates for the cost of the subsidies, and the government runs a strictly positive budget surplus at all times, which it discards.

Although we have uncovered a policy where the government runs a surplus (that it discards), this policy does not yet constitute an RPI as defined (as interest rates and transfers have not changed). A natural enhancement is to lump-sum rebate the surplus, which would then constitute an RPI. The question is whether the asset market can still clear at the original interest rate. If a continuity assumption on how aggregate consumption (or, equivalently, aggregate savings) responds to transfers is satisfied, the answer is yes.

Specifically, consider a sequence of transfers  $\{T_t\}_{t=0}^1$  with  $T_t = T^0$  for all  $C \geq 0$ . Let  $\alpha$  be a positive scalar that governs the magnitude of the change in transfers in the direction

<sup>24</sup>For  $C = 0$ , the resource condition (6) is  $c^* + c^* \dot{Y} = (c^*)^{\beta} + (1 - X)c^*$ , as  $c^* \dot{Y} > c^*$ . For  $C \geq 1$ ,

$$c_t(r^*, T^*) + c_{t+1} = c^* + c^* \dot{Y} = (c^*)^{\beta} + (1 - X)c^* + c^* \dot{Y} - (c^*)^{\beta} + c^* = (c^*)^{\beta} + (1 - X)c^*$$

where the first equality uses the fact that household's problem has not changed; the second equality uses goods market clearing in the original equilibrium; the strict inequality uses the fact that  $(c^*)^{\beta} < c^*$  for  $c^* > c^*$ ; and the final equality uses  $c_t = c^*$  for all  $C \geq 1$ .



The following lemma states that there exists a sequence and magnitude that can be the basis for an RPI:

Lemma 2. Suppose  $\beta > 0$  and consider a sequence  $\{c_t\}_{t=0}^{\infty}$  with  $c_t > 0$  for all  $t \geq 0$  with at least one inequality strict. Suppose there exists an interest rate  $r > 0$  for which the following regularity condition holds:

$$|c_t(r^{t+1} + \beta^t a) - c_{t+1}(r^{t+2} + \beta^{t+1} a)| \rightarrow 0 \text{ for all } t \geq 0 \text{ and } a \geq 0$$

Then there exists a feasible RPI.

The feasibility of an RPI when capital is above the Golden Rule does not require knowledge of the elasticity of savings to interest rates (which was the main consideration of the analysis in the previous section for the constant-policy). The key condition for the excess capital case is that aggregate consumption smoothly varies with transfers, ensuring that the increase in household consumption can be financed with the increased output net of depreciation.

Capital above the Golden Rule as a source of production inefficiency dates back to the classic papers of Diamond (1965) and Cass (1972). More recently, Zilcha (1990), Rangazas and Russell (2005), and Barbie and Kaul (2009) extend the Cass criterion to stochastic settings. The key condition turns on whether the marginal product of capital is low. Production inefficiency is sufficient but not necessary for an equilibrium to be Pareto inefficient. As evidenced by our constant-K analysis, Pareto improvements are possible by reallocating a fixed amount of output.

#### 4.2.2 Capital Below the Golden Rule

We now consider the case of  $\beta < 1$ .<sup>25</sup> With a mark-up wedge between the return to capital and the interest rate, this case is consistent with either a positive or negative

The approach for generating a feasible RPI in this case is distinct from the over-accumulated capital case. In the latter, interest rates do not change, resources are generated from crowding out capital, and these resources are then rebated to consumers as lump-sum transfers. In Appendix B (Lemmas 5 and 6) we show that for the  $\beta < 1$  case, increases in transfers alone are not a feasible path to generate an RPI, and they are not necessary either: when establishing feasibility in this case, it is without loss to set transfers to their lowest possible level  $(A - A^*)/0$ .<sup>26</sup> Hence, we now focus on changes in interest rates.

Corollary 1 tells us then that a pair of sequences of interest rates,  $\{r_t\}_{t=0}^{\infty}$ , and transfers,  $T = \{t_t\}_{t=0}^{\infty}$  is feasible if we can find an associated sequence  $\{c_t\}_{t=0}^{\infty}$  with  $c_0 = \beta$  such that

<sup>25</sup>For expositional reasons, we ignore the knife-edge case  $\beta = 1$  in the text, but discuss it in footnote 27.

<sup>26</sup>In Appendix B we require that aggregate consumption be weakly monotonic in transfers for these results.

the aggregate consumption function satisfies

$$C(r, T) + \beta^{-1} (c_{t+1}) + (1 - \lambda) c_t$$

To build towards the next result, recall that in the initial stationary economy, aggregate consumption is  $c = (c_{t+1}) + X c_t$ . Letting  $b_C = C(r, T) - c$ , Corollary 1 tells us that a fiscal plan is feasible if we can find a sequence  $b_C$  such that for all  $C$

$$b_C = (c_{t+1}) - (c_t) + c_{t+1} - X(c_t)$$

Define  $r$  to be the net marginal return to capital in the initial equilibrium:

$$r = 1 + (c_{t+1}) - X$$

Given that  $r$  is strictly less than the Golden Rule,  $r < 1$ .

Suppose that  $(c_{t+1}) - (c_t) + (1 - X)(c_t) = r b_C$  where  $b_C = c_t - c_{t+1}$ , which will be the case if  $c$  is linear in  $b_C$ . We shall return to the general case of concave production below, but linearity allows us to build intuition towards the more general result.

Given a sequence  $b_C$ , the feasibility condition in (6) boils down to finding a sequence  $b_C$  with  $b_0 = 0$  and  $b_C > 0$  (this latter guaranteeing that capital does not turn negative) such that the resource constraint holds:

$$b_{C,t+1} + b_C = r b_C \tag{9}$$

for all  $C \geq 0$ . Solving forward and evaluating at  $C = 0$ ,<sup>27</sup> a necessary condition for the consumption path  $b_C$  to be feasible is

$$\sum_{C=0}^{\infty} r^C b_C = 0 \tag{10}$$

That is, feasibility requires that the present value of consumption changes discounted at the marginal product of capital be less than the zero.

<sup>27</sup> Specifically, (9) implies

$$b_C = \sum_{B=0}^{\infty} r^B (b_{C+B} - b_{C+B-1}) + b_C = \sum_{B=0}^{\infty} r^B (b_{C+B} - b_{C+B-1}) > 0$$

where the second inequality uses  $b_C > 0$ . Taking the limit as  $r \rightarrow 1$  and evaluating at  $C = 0$  with  $b_0 = 0$  gives us (10). If  $r = 1$ , that is,  $c = c$ , then the condition (10) becomes  $\sum_{C=0}^{\infty} b_C = 0$ . In this case, the marginal net return to investment is zero and hence any increase in consumption must be accomplished by drawing down the initial capital stock.

Condition (10) states the relevant intertemporal price for assessing aggregate feasibility is the marginal product of capital, not the interest rate faced by households. If the government increases aggregate consumption in a period, this must be offset by a decrease somewhere else, where the increase and decrease are evaluated in present value terms using  $1 + r$ . As we show below, condition (10) can be rewritten in terms of savings elasticities, connecting this result to our constant- policy discussion.

However, before doing this, we extend this condition to the case of a general concave production function, marginal changes in interest rates, and obtain a sufficiency result. To do so, we need a strict inequality in the present value resource condition and a continuity condition. We first state the general result and then provide intuition for how we use these conditions:

Proposition 1. Assume  $\beta = 0$  and  $r > \bar{r}$ . Consider a sequence  $(C_t, A_t)$ , with  $A_t = A$  for all  $t \geq 1$  with at least one inequality strict. Suppose that there exist savings  $s_t$  and  $\lambda_t \geq 0$  such that

$$\sum_{t=0}^{\infty} \lambda_t [C_t(r + ba \cdot T) - a] = 0 \quad \text{for all } a \in (0, \bar{r}) \quad (i)$$

and there exists  $\delta \in (0, 1)$  for which the following regularity condition holds

$$j[C_t(r + ba \cdot T) - a] > \delta \quad \text{for all } C_t = 0 \text{ and } a \in (0, \bar{r}) \quad (ii)$$

Then there exists a feasible RPI.

Recall that for the constant- policy, aggregate consumption could not be higher than the initial consumption level in any period. With more general policies, consumption can deviate from the initial level in any direction. Proposition 1 says that the present value of these changes in consumption must be bounded above by zero. As with the constant policy, the key insight is that an increase in the interest rate must induce households to save in aggregate rather than increase consumption.

The present value discount factor is still the net return to physical capital, not the market interest rate. With  $\beta < 1$ , these will be different. Hence condition (i) of the proposition presents a simple and somewhat surprising separation between demand considerations (preferences) and supply (technology). The aggregate response of consumption to an interest rate change is determined by the initial distribution of wealth, household preferences, their idiosyncratic risk, and the interest rate that households face on their savings, which is captured by the role of technology is embedded in the discount factor used to sum over time. With a markup, the discount rate of the government to evaluate the feasibility of an RPI does not coincide with the market interest rate faced by households.

As with Lemma 2, the result holds a particular "direction" of change  $\epsilon$  fixed, and parameterizes distance in that direction by  $\alpha$ , although in this case it is the interest rate sequence rather than transfers that changes. Condition (i) imposes a strictly negative upper bound on the present value of the changes in consumption. The strict inequality implies "extra" resources not used for consumption. The proof of the proposition uses this surplus to offset the second order implications of  $(\epsilon, \alpha) \rightarrow (\epsilon + \delta, \alpha)$  "missed" by the first-order term:  $b_C$

The Lipschitz continuity condition (ii) is used to ensure that we remain in the neighborhood of the initial equilibrium for a small change in interest rates at all times. This allows us to continuously govern the extent of changes in consumption and capital with the parameter  $\alpha$ , replacing an upper bound on the second order terms.

To gain more insight, let us narrow attention to just one interest rate change, say at time 1. That is,  $b = f(0, 0, \dots, A_1, 0, \dots, 0)$ , for  $A_1 > 0$ . Note that  $m_C(r^* + \alpha b, T) = 3C_D(r^* + \alpha b, T) - 3a$  evaluated at  $\alpha = 0$  and  $A_1 = 1$ . If this derivative exists and it is bounded in the neighborhood of  $\alpha = 0$  for all  $C > 0$ , then condition (ii) is satisfied, and for condition (i) it is sufficient that

$$\lim_{C \rightarrow 0} \frac{m_C}{m_A} < 0 \quad (11)$$

If we can find such  $g$ , then we have an implementable RPI.

From Definition 4 and letting  $\beta = 1 + A^*$ , we have for  $C > 1$

$$\frac{m_C}{m_A} = \begin{cases} \frac{m_C}{m_A} > \frac{m_C}{m_A} & \text{for } C < g \\ \frac{m_C}{m_A} > \frac{m_C}{m_A} + \beta & \text{for } C = g \end{cases}$$

where  $m_C$  for  $C > 1$  is defined in the same way as for aggregate consumption. As  $\beta > 0$  by definition, we have  $m_C > m_A$

Taking the discounted sum, (11) can be written

$$\lim_{C \rightarrow 0} \sum_{t=1}^{\infty} \frac{m_C}{m_A} \beta^{t-1} < 0 \quad (12)$$

Define the elasticity of aggregate household savings at  $C = g$  with respect to  $A_1$  as

$$b_{C,g} = \frac{m_C'}{m_A} > \quad (13)$$

<sup>28</sup>To see this, let  $(a) = C_D(r^* + \alpha b, T) > 0$ . Let  $\epsilon(0) = \lim_{\alpha \rightarrow 0} (a)$ . Condition (11) says  $\epsilon(0) < 0$ , for some  $\alpha > 0$ , which in turn implies that for  $0 < \epsilon < \epsilon(0)$  there is an  $\alpha > 0$  such that for all  $\delta > 0$  we have  $(a) < \epsilon + \delta < 0$ , which is condition (i).

We can now state a corollary to Proposition 1:

Corollary 2. Assume  $\bar{c} = 0$ , and  $\beta > \bar{y}^c$ . Assume in addition that  $A_C$  is differentiable with respect to  $\beta$  for some  $\beta > 1$ , and  $b_{C,g}$  is defined by (13). If

$$\frac{\beta - \bar{y}^c}{\beta} > \sum_{g=1}^{\infty} (\beta^{-g}) b_{C,g} \quad (14)$$

then an RPI is feasible.

This condition states that the present discounted value of savings elasticities, scaled by the gap between  $\beta$  and  $\bar{y}^c$ , must be greater than one. In the constant case, an elasticity condition has to hold at every  $C$  as implied by equation (8). Corollary 2 states that, with the ability to move resources across time via investment, the elasticity condition only needs to hold in a present value sense.

The fact that a large elasticity of savings is useful in making an RPI feasible is based on the same intuition as Figure 1, but now it is the present value of a sequence of elasticities. The sequence  $b_{C,g}$  is related to the "sequence-space Jacobian" of Auclert, Barry, Rognlie, and Straub (2021), a point we discuss in the context of the calibrated model of Section 5.

The term  $\beta - \bar{y}^c$  represents the difference in the intertemporal price at which the government trades with "technology" versus at which it trades with households. This difference is governed by the markup. In particular, if the initial equilibrium has  $\bar{g} = 0$ , then

$$\beta - \bar{y}^c = \beta (\bar{g} - \beta) (A^* + X) = (\beta - 1)(A^* + X)$$

Thus, a larger markup aids in satisfying the feasibility condition.

We caution once more against concluding that a markup naturally implies a feasible RPI. An RPI is inconsistent with reducing pure profits or with providing subsidies to inputs financed with a lump-sum tax. The role of the markup here is that the feasibility condition recognizes that a government can transfer resources intertemporally at rate  $\beta$  while the households (in aggregate) choose not to do so due to market power. As shown in Appendix A, if  $\bar{d}$  differs from the return on bonds due to a liquidity premium, Corollary 2 holds with the term  $\beta - \bar{y}^c$  in (14) replaced by  $\bar{d}$ . In this case, the government can exploit the convenience yield generated by government

<sup>29</sup>In particular, condition (8) will hold to a first-order for all  $C$  if we can find a sequence  $\beta_C = A_C - \bar{c} > 0, C = 1, 2, \dots$ , such that

$$\sum_{g=1}^{\infty} \frac{1}{\beta_C^{1+g}} b_{C,g} > 1 \quad \forall C$$

for all  $C_j > 0$ . Thus, we have an infinite sequence of conditions, one for each  $C$ , rather than the single expression at  $C = 1$  for the case in which  $\beta$  is not constant.

bonds, rather than the presence of a markup, to finance the RPI.<sup>30</sup>

We have shown that both a markup and a negative risk-free interest rate help make an RPI feasible. What if neither is present? That is, what if  $(\beta - \beta^*) > 0$  and  $X = A^* > 0$ . In this case, the government lacks the resources to implement an RPI:

Proposition 2. Consider starting from a laissez-faire equilibrium with  $\beta > 0$ ,  $\beta^* = 1$ , and  $(\beta - \beta^*) > 0$ . Suppose that  $\limsup_{C_1} A(\beta, T) = 1$  implies  $\limsup_{C_1} C(\beta, T) = 1$  for any non-negative sequence  $\{C_t\}$ . Then, there is no feasible RPI.

Let us briefly comment on the main assumption for this result: it requires that household consumption be unbounded as household wealth increases without bound. This is a natural assumption,<sup>31</sup> and is, for example, satisfied in the standard Aiyagari environment,<sup>32</sup> as the presence of  $A^* > 0$  or a markup (or some combination of the two) is a necessary requirement for an RPI.

#### 4.2.3 The Elasticity of Aggregate Savings

The existence of an RPI, at least locally to the initial equilibrium, depends on the weighted sum of aggregate savings elasticities given by (14). These are the impulse responses of aggregate savings  $C$   $g$  periods after (or before, if negative) a one-time exogenous shock to the interest rate at  $g$ . The key statistic is then a weighted sum of these responses, where the weight is given by the net marginal product of capital, and scaled by the difference between the marginal product of capital and the risk-free interest rate.<sup>33</sup> Conceivably, this statistic could be estimated using a vector autoregression, assuming one could identify a policy-induced change in the risk-free interest rate (or, equivalently, an exogenous change in government debt).

Testing the sensitivity of interest rates to changes in government debt or deficits was an active area of empirical research in the 1980s and 1990s.<sup>34</sup> Perhaps surprisingly, there are a number of empirical studies that conclude the Ricardian equivalence benchmark of no change in the interest rate, in the spirit of Barro (1974), is a reasonable description of the data. Nevertheless, there are other empirical estimates that conclude otherwise, and our reading of this literature is that there is no clear consensus.<sup>35</sup>

<sup>30</sup>Bassetto and Cui (2024) study the optimal fiscal policy in the presence of a liquidity premium and establish conditions under which the government does or does not completely eliminate the liquidity premium in the Ramsey solution.

<sup>31</sup>Recall that we have ruled out lump sum taxes, and hence a situation in which infinite private wealth is offset by an infinite household tax liability.

<sup>32</sup>For the argument, see Chamberlain and Wilson (2000), Lemma 2.

<sup>33</sup>Recall that if  $(\beta - \beta^*) > 0$ , then  $\beta > \beta^*$  and we know an RPI is feasible from Lemma 2.

<sup>34</sup>See the surveys and associated references of Barth, Iden, and Russek (1984), Bernheim (1987), Elmendorf and Gregory Mankiw (1999), and Gale and Orszag (2003); and Engen and Hubbard (2005).

<sup>35</sup>Note that the impact of government borrowing on interest rates is distinct from the elasticity estimated from quantitative easing (QE) episodes, in which the government trades short for long maturity government bonds or

In the Bewley-Huggett-Aiyagari literature, there are a few theoretical results. For example, for the case of CRRA utility, Benhabib, Bisin, and Zhu (2015) show that the household saving function's sensitivity to the risk-free interest rate is increasing in the intertemporal elasticity of substitution (IES). A similar result is proved by Achdou et al. (2021). See Farhi, Olivi, and Werning (2022) for a general analysis of consumer behavior under incomplete markets. Thus, the derivative with respect to  $r$  is governed by the IES, with a larger IES indicating a more elastic response, at least for the very wealthy. At the other end of the asset domain, Achdou et al. (2021) show that, for those at the lowest income realization and approaching the borrowing constraint, the sensitivity of savings to  $r$  also depends positively on the IES.

These results pertain to individual savings behavior at the extremes of the asset distribution. For a representative agent (RA) economy, this is enough. In Section 4.3, we explore such an environment to gain some analytical insights. More generally, one needs to turn to computational examples, which we do in Section 5.

#### 4.2.4 Income Effects on Labor Supply

It is straightforward to generalize the results of the previous sections to more general preferences, including those that feature income effects on labor supply.

To this goal, consider the class of iso-elastic preferences over consumption and leisure that are consistent with balanced growth. That is, for  $W_i \in (0, 1)$ , let

$$G^i(2 \cdot \frac{r}{1+i}) = \begin{cases} \frac{\log(2^{1-i}(1+i)^i)^{1-W_i}}{1-W_i} & \text{for } W_i < 1 \\ -W_i(1-i) \log 2 + i \log(1+i) & \text{for } W_i = 1 \end{cases}$$

In this case, the intra-period first order condition for the consumption/leisure choice is

$$\frac{i}{1+i} \frac{2}{1+i} = \log F(1+i)$$

Aggregating over agents, setting  $\frac{i}{1+i}$  and normalizing  $\frac{R}{1+i} = 1$ , we have that

$$i^{\wedge} c = F^>(1+i)$$

A slightly modified version of Corollary 1 holds, where we just need to adjust the resource government bonds for private assets. See, for example, Krishnamurthy and Vissing-Jorgensen (2012b) and Koijen, Koulischer, Nguyen, and Yogo (2021). Related is Krishnamurthy and Vissing-Jorgensen (2012a). These studies estimate the elasticity of the "convenience yield" of certain bonds relative to other assets, an extension we discuss in Appendix A.

constraint to be

$$C(r, T) = (c^0 + (1 - \alpha) c^1)^{\frac{1}{\alpha}}$$

for  $\beta = 1 - \alpha C(r, T) F$ .

Our analysis of the constant-K policy remains unaltered, given that  $\beta$  is unchanged and aggregate output is constant (assuming that the resource constraint holds with equality), implying that aggregate consumption equals  $\beta$  and thus, labor supply is unchanged.

The perturbation results in subsection 4.2.2 also hold with these preferences. Specifically, Proposition 1 holds. To see this, note that to a first order  $\beta = \beta + \beta \beta$ , with  $\beta$  proportional to  $\beta$  for  $F_C = F$ . Thus, equation (9) holds with  $\beta$  scaled by a constant proportion. Hence, inequality (10) remains unchanged.

Given that the wage does not change in our policy analysis, the parameter  $\beta$  effectively controls the strength of the income effect on labor supply, generating a linear relationship between aggregate consumption and aggregate labor supply. This aggregation implies our results carry through independently of the strength of this income effect and the distribution of wealth.

### 4.3 A Representative Agent Economy

In this subsection, we use a Representative Agent (RA) economy to shade in some details behind the previous section's results. The RA economy is a special case of our environment with no idiosyncratic risk, no differences in preferences across households, and all profits shared equally. The RA economy's analytical tractability allows us to shed light on how government debt is used to "smooth" transfers, which may be necessary given that the short-run elasticity of aggregate savings will be smaller than the long-run elasticity. Moreover, the fact that an RPI is feasible in an RA economy establishes that the markup can open the door to implementable RPIs, even when after-tax profits remain bounded below by the level in the initial equilibrium and the government cannot resort to lump sum taxation.

#### 4.3.1 The Aggregate Consumption Function

We assume the RA preferences are given by standard separable utility  $V(C)$  and that the economy starts from a laissez-faire equilibrium  $\beta = \beta > 0$ .<sup>36</sup> Note that in an RA economy with separable utility, in the steady state we have  $\beta = 1 - V'(\beta)$ .

For the RA economy, the aggregate consumption function  $C(r, T)$  satisfies the Euler equation

<sup>36</sup>We assume that labor supply is exogenous and equal to  $\beta$ , which, as before, for our purposes will be equivalent to endogenous labor supply with zero wealth effect.



$D^0 C^0 = V(1 + A_{G+1})D^0 C_{G+1}^0$  and the present value budget constraint

$$0_0 + \sum_{G=0}^{\infty} \beta^G (c_{G+1} - \beta c_G) = 0 \quad (15)$$

where  $0_0 = \beta = \beta$ ,  $\beta = \beta$ , and  $\beta = \beta$ . We restrict attention to sequences of  $\beta$  such that the present discounted value of transfers is bounded. The timing of the transfers does not matter for the household consumption allocation, only the discounted present value does.<sup>37</sup>

#### 4.3.2 Transfer Smoothing: Implementing the First Best

It is instructive to explore whether the first best allocation constitute a feasible RPI and whether government debt plays a role. Recall that the economy may be distorted by a markup, and the question we address in this subsection is whether, and how, the markup distortion could be removed without recourse to lump sum taxation. In what follows, the target allocation is the familiar efficient one from the neoclassical growth model absent the markup distortion, and thus we omit the derivation.

Starting from  $\beta$ , let  $\{c_t, g_t\}_0$  denote the path of capital that would be chosen by a social planner with unlimited fiscal instruments to maximize the RA's welfare. Let  $\{c_t, g_t\}_0$  denote the associated optimal consumption allocation. Let  $\{\beta_t, g_t\}_0$  be the sequence of interest rates that decentralizes this consumption sequence, that is,  $\beta_t = (1 + A_c) \beta + (\beta_t - \beta) X$ ,  $c_t > 1$ , and with  $\beta_0 = 1 + A_c$ . We know that the RA's Euler equation will be satisfied,  $D^0(c_t) = V'_{G+1} D^0(c_{G+1})$ .

As  $\beta$  is below the efficient steady state due to the markup distortion, and dynamics in the neoclassical growth model are monotone, we know that  $\beta_t$  is an increasing sequence. This implies that  $c_t$  is decreasing over time. However,  $\beta = \lim_{G \rightarrow \infty} \beta_G = 1 + A_c$ , at every  $t$  the new interest rate sequence remains weakly greater than the initial interest rate (a requirement for an RPI). The question is how can the government implement this sequence without lump-sum taxation.

First, let us focus on the case without the use of government debt. In this first-best allocation, the government budget constraint must hold with equality, given that no resources are wasted. From Lemma 1, a binding government budget constraint implies that

$$\beta c_t = (\beta_t - \beta) (A_c + X) c_{t-1} + (\beta_t - \beta) (A_c + X) \beta^0$$

<sup>37</sup>As is well known, a RA may result from complete markets or Gorman aggregation. We do not specify the underlying household heterogeneity, but assume that no household is made worse off by an increase in the interest rate. In particular,  $Q = 0$ , which we assume does not bind for the exercises under consideration.

for  $C = 1$  and  $\dot{c}_0 = 0$ . As  $f_c$  is an increasing sequence and is concave, the sequence of transfers  $f_c$  is increasing over time for  $C = 1$ . Hence, if  $\dot{c}_1 > 0$ , the sequence  $(f_c)_{c=0}^{\infty}$  constitutes a feasible RPI.

However, if  $\dot{c}_1 < 0$ , it still may be possible to implement the first best using government bonds. In particular, the first best can be implemented as an RPI as long as

$$0 < \sum_{c=0}^{\infty} \beta^c \dot{c}_c = \sum_{c=0}^{\infty} \beta^c (f_c - \dot{c}_c) > 0 \quad (A^* + X) > 0$$

If this inequality holds, but  $\dot{c}_c < 0$  for some interval  $C \in [0, \bar{C}]$  the government can issue bonds during the initial periods to cover the shortfall and avoid negative transfers. As long as the present value is weakly positive, the government budget constraint will be satisfied. This is what we mean by the role of government debt in "transfer smoothing." Note that there is no guarantee that the inequality will hold. This highlights that the first best may not be achievable without resorting to lump-sum taxation, even with the ability to smooth transfers using debt.

The takeaway from this exercise is that to implement the first best, the government must subsidize investment to build up the capital stock. This requires a higher interest rate for households and a subsidy to firms. The short-run elasticity of aggregate savings is smaller than the long-run elasticity (which is infinite with separable utility), requiring an overshooting of the interest rate in the short-run relative to the steady state. The government can smooth this cost by using government bonds. In this manner, capital investment and government debt are complements rather than substitutes along the transition.

### 4.3.3 The Intertemporal Elasticity of Substitution

One advantage of the RA example is the close link between the elasticities of aggregate savings appearing in Corollary 2 and the intertemporal elasticity of substitution (IES) of the representative consumer. Consider the same perturbation used in the corollary; namely, a single period which  $\beta_t > \beta^*$ , with every other period seeing  $\beta_t = \beta^*$ . As  $\beta^* = 1 - \delta$  in the RA economy, consumption is constant before and after with a one-time increase between  $c_{t-1}$  and  $c_t$ .

As in the main analysis, the key behavioral response is whether the private household is willing to postpone consumption due to the increase in interest rate. In the RA case, this is governed by the IES. In the appendix we prove that, for the representative agent case, our sufficient condition in Corollary 2 holds for some  $\epsilon$  if the IES is large enough:

Lemma 3. Let  $Z = D^0(c) / (D^0(c) - \delta c)$  denote the IES evaluated at the initial consumption

level. If  $\eta > 1$  and

$$Z_i \frac{A_i}{\eta} > 1$$

then there exists an implementable RPI.

The necessity of  $\eta > 1$  follows from the RA assumption, as the markup represents the only inefficiency that can be potentially corrected. The role of the IES reflects that a more elastic response to a change in interest rate makes an RPI easier to implement. The term on the right-hand side of the inequality reflects the wealth effect of higher interest rates. In particular, it is the fraction of initial consumption financed with asset income, or one minus the share of net income paid to labor and profits. The larger the asset share becomes, the more the interest rate increase induces the consumer to raise consumption. Note that this ratio is strictly less than one, and hence an IES greater than one is sufficient to satisfy the condition.

The above confirms that the elasticity of aggregate savings (and consumption) to a change in the interest rate is the gatekeeper of a feasible RPI. In the RA economy, this boils down to the tradeoff, between the IES and the share of income paid to financial assets. In the heterogeneous household model, we cannot map the sequence of elasticities to a single preference parameter. For that model, we turn to calibrated simulations.

## 5 Simulations

In this section, we present simulation results for various policy experiments. The policy experiments will highlight a main insight from the analytical results; namely, that the feasibility of an RPI depends on an aggregate elasticity and not the particular characteristics of idiosyncratic preferences. We also consider an exercise with aggregate risk. As a prelude, we also extend Corollary 1 to the case of aggregate uncertainty.

The primitives and calibration of the quantitative model are fairly standard, and we defer details to Appendix D, which also discusses the computational algorithm. We flag a few salient features of the calibration in the text. Preferences are Epstein-Zin, for which we set the IES parameter  $\eta$  to one. We calibrate the discount factor and the coefficient of relative risk aversion as follows. We target a steady state with 60% debt-to-output and capital-to-output of 2.5, where the debt corresponds to the US average over the period 1966-2021 and the capital ratio is taken from Aiyagari and McGrattan (1998) and Krueger, Mitman, and Perri (2016). We treat this steady state as the result of a constant-K policy starting from a laissez-faire economy. The difference between the average one-year treasury rate and average nominal GDP growth in the United States between 1962 and 2021 is -1.4%, which will be the target for the return on bonds in our steady

state.<sup>38</sup> The resulting parameter values are a discount factor  $\beta = 0.993$  and a coefficient of risk aversion  $\gamma = 5.5$ . The markup parameter  $\mu$  is set to 1.4, which is within the range of estimates in Basu (2019).<sup>39</sup> We also take a parsimonious approach to allocating profits by assuming a distinct class of entrepreneurs who are endowed with managerial talent and consume profit distributions in a hand-to-mouth manner. While stark, this approach offers several advantages including that it approximates that a significant share of entrepreneurial rents accrues to a small portion of the population and that profits do not affect factor prices, so we can solve the economy without taking a stand on the idiosyncratic details of the entrepreneurial class. Finally, and related to the previous point, the analysis is invariant to the extent to which profits are offset by fixed costs versus representing pure rents.

## 5.1 Robust Pareto Improvements: Feasibility

To explore the feasibility of an RPI in the calibrated model, we begin with the sufficient condition in Corollary 2. In particular, we solve for the laissez-faire stationary equilibrium for a range of IES and markups, holding other parameters as in the benchmark calibration. Starting from each initial equilibrium, we compute the sequence  $\{A_t\}$  using the methods of Auclert, Baatzy, Rognlie, and Straub (2021). For this exercise, we use 50 years. We then sum the sequence of elasticities, discounted by the initial equilibrium's  $\beta^t = 1 + (\beta - 1)^t$  and then test whether inequality (14) is satisfied.<sup>40</sup>

We present the results in Figure 2. On the vertical axis is the markup and the horizontal axis represents the IES. Each point represents a different parameterization, and hence a different initial equilibrium as well as a different  $\{A_t\}$  sequence. The star represents the benchmark calibration of  $\mu = 1.4$  and  $Z = 1$ .

There are three regions of interest. The bottom-left light-gray area contains parameterizations in which  $\beta > \beta^*$ ; that is, capital is above the Golden Rule in the laissez-faire equilibrium. From Lemma 2, an RPI exists by substituting bonds for capital. The complement of the light gray area represents economies in which  $\beta < \beta^*$ . This region is divided into a white region and a darker gray area. The darker region in the top right contains parameterizations for which condition (14) is satisfied; that is, an RPI is feasible. The white area represents parameterizations for which the inequality (14) does not hold.

<sup>38</sup>This estimate is consistent with the ones in Blanchard (2019) and Mehrotra and Sergeyev (2020).

<sup>39</sup>The aggregate markup may also reflect smaller markups at different stages of production in a vertical supply chain, as in Ball and N Gregory Mankiw (2021). In fact, 1.4 is close to the number they use in their numerical exercises. As noted, part of the wedge between the interest rate and the marginal product of capital could be the liquidity premium. Cui and Radde (2020), for example, find an average liquidity premium of roughly 1%. Our wedge  $(1 - \beta)(A + X) = (1.4 - 1)(1.4 + 0.1) = 344\%$ . Hence the liquidity premium could be 1/3 of the total wedge.

<sup>40</sup>We sum these elasticities over 1000 periods. See Appendix D.4 for details on the computation.

To gain some intuition for the various regions, fix an IES and consider moving along the vertical axis as we increase the markup. For a given higher markup implies a lower initial capital stock.<sup>41</sup> For low markups, capital is over accumulated and hence an RPI is feasible. Once  $\mu > \bar{\mu}$ , we enter the white region. In this area, the gap between  $\mu$  and  $\bar{\mu}$  is too small for (14) to be satisfied. For this intermediate range, the low-hanging fruit ordered by  $\mu$  is no longer present, while the distortion of capital is not sufficiently large to generate enough revenue for an RPI. This region reinforces the point that the presence of a markup on its own is not sufficient for a feasible RPI. As  $\mu$  increases, the gap between  $\mu$  and  $\bar{\mu}$  increases and an RPI becomes feasible, and hence we enter the dark gray region.

Now fix  $\mu$  and vary the IES. There are two competing effects of a higher IES. One is that households' saving behavior becomes more sensitive to changes in the interest rate. This favors the feasibility of an RPI. The second is that the initial equilibrium  $\bar{K}$  increases in the IES. This reflects that a higher IES generates less precautionary savings in the initial equilibrium, and hence less capital and a higher initial interest rate.<sup>42</sup> This latter effect on the level of interest rates makes (14) less likely to hold. We see these competing effects in the figure. Fixing  $\mu$  for example, at low IES initial precautionary savings are large enough that  $\mu > \bar{\mu}$ . For intermediate IES,  $\mu < \bar{\mu}$  but the elasticity of aggregate savings is too small to support a marginal RPI. Finally, for large IES  $\bar{K}$  is sufficiently elastic that an RPI is feasible.

These results imply that in the calibrated model there is scope for robust Pareto improvement, but are silent about the policy implementations and what they would mean for welfare. In the next subsection, we explore in detail specific policy plans using global solutions.

## 5.2 Baseline Constant-K Policy

We now describe transitions as the government implements its fiscal policy. The economy transitions from a counter-factual laissez-faire stationary equilibrium to the benchmark stationary equilibrium with fiscal policy. In the first policy plan we consider, the government increases debt from zero to 60% of output, while keeping capital as well as after-tax wages and profits constant. Our posited path of public debt is depicted at the top left panel of Figure 3. By construction, capital is held fixed at the laissez-faire level, as depicted in the top middle panel of Figure 3. Given the policy of constant capital, output and aggregate consumption (reported in the lower middle

<sup>41</sup>For the class of economies we consider, a change in the markup does not change the stationary equilibrium interest rate. This reflects several assumptions: the homotheticity of preferences, a zero borrowing limit; and the fact that profits are consumed hand-to-mouth by a separate class of agents.

<sup>42</sup>The fact that the laissez-faire interest rate varies with the IES while holding risk aversion constant stems from the fact that precautionary savings depend on more than the extent of risk aversion. Kimball and Weil (1992) show that with Kreps-Porteus preferences, the strength of the precautionary savings motive is determined by attitudes towards both risk and intertemporal substitution.

Figure 2: RPI Feasibility

Notes: The two shaded areas represent values of mark-up and IESZ where an RPI is feasible. In the white area, condition (14) is violated. In the dark shaded area, capital is below the Golden Rule and the condition in Corollary 2 is satisfied. In the light shaded area, capital is above the Golden Rule and therefore an RPI is feasible by Lemma 2. The rest of parameters are fixed as in the baseline. The star represents the benchmark parameterization of  $\theta = 1.0$  and  $\sigma = 1.4$ .

panel) do not change. Given this path of debt and capital, the equilibrium interest rate path clears the asset market and the associated transfers satisfy the government budget constraint.

The top right panel of Figure 3 plots the path for government transfers and seigniorage revenue from debt issuance  $\tau_{t+1} = (1 + A_t) \tau_t$  both relative to output. Transfers are larger on impact (about 5% of output) remain positive throughout the transition, and settle to a small positive level of about 0.1% of output in the steady state. The difference between transfers and seigniorage revenue is equal to the tax revenues, which are negative owing to the capital subsidies.

The bottom left panel in Figure 3 plots the path for the interest rate. Interest rates rise with public debt to induce households to hold a greater stock of aggregate wealth. Note that interest rates overshoot during the transition, as the short-run elasticity of savings to interest rates is lower than its long-run level. We find a short-run impact elasticity  $\frac{\partial A_t}{\partial r_t} \left( \frac{A_t}{A^*} \right) \left( \frac{r_t}{r^*} \right)$  of 4.6, while a higher long-run elasticity  $\frac{\partial A_1}{\partial r_1} \left( \frac{A_1}{A^*} \right) \left( \frac{r_1}{r^*} \right)$ , of 75. The sharp spike in interest rates in the short run makes the policy especially expensive, however, the short-run cost is more than offset by the funds raised by debt issuance. In the long run, the policy continues to be especially expensive because although the elasticity is higher, interest rates remain elevated in the new steady state. Our long-run elasticity implies that a 23% increase in aggregate assets is associated with a 30 basis points increase in interest rates.

The bottom right panel plots the dispersion of household consumption relative to the laissez-faire dispersion. Consumption dispersion decreases by about 10% upon the introduction of the fiscal policy plan, as households with low assets and low productivity benefit from government transfers that support their consumption. As transfers fall over time, consumption dispersion

increases but remains about 2% below the one in the laissez-faire economy. The smaller long-run consumption dispersion reflects that households on average hold a greater stock of precautionary savings, given the elevated interest rate.

Figure 3: Constant- Policy Transition

The transition paths of positive transfers and higher interest rates imply that our baseline constant- fiscal policy is Pareto improving. We now evaluate the magnitude of the welfare gains. Table 1, Column 1 reports welfare for various households upon the announcement of the policy. Welfare is measured in consumption equivalence units relative to the laissez-faire economy. Across the distribution of households for assets and productivity, the economy with fiscal policy delivers higher welfare for every household. The table reports five measures of welfare gains: the mean gain; the minimum gain; and the mean gains for the bottom 10%, the 40-60th percentiles, and the top 10% of the asset distribution. The mean welfare gains are computed by integrating over idiosyncratic states, conditional on belonging to the respective asset bin, weighted by the invariant distribution of the laissez-faire economy.

The mean welfare gain is 2.6%, and the minimum gain is 2.1%. Looking across the wealth distribution, welfare gains are greatest for the poorest households. While all households receive the same transfer, the poorer households benefit relatively more in percentage terms. However, gains are not monotonic in wealth. The top decile of asset holders experience a greater welfare gain than those in the middle of the asset distribution. This reflects the fact that the benefits of a higher interest rate are increasing in wealth. At some point in the distribution, this effect dominates the uniform transfer, generating a non-monotonicity in percentage welfare gain as a

function of initial wealth.

Table 1: Changes in Welfare

Policies	Constant- (1)	High Initial Debt (2)	Agg. Shocks (3)	Capital Expansion (4)
Welfare Gains at Announcement (%)				
Overall Mean	2.6	0.7	2.7	5.2
Overall Minimum	2.1	0.5	2.1	4.5
Poor ( 10 pct)	3.7	1.0	3.6	5.3
Middle Wealth ( 40-60 pct)	2.3	0.6	2.4	4.8
Rich (i 90 pct)	2.8	0.8	2.8	6.7

The preceding experiment explored the path from zero debt to levels observed in recent US history. A natural question is whether there are feasible improvements beyond the 60% debt-to-output scenario, and how the results depend on the initial debt level.

Consider an initial steady-state equilibrium with 60% debt to output. Factor taxes are set to zero, and the revenue earned from the negative rates on bonds are lump-sum rebated back to households. Relative to the final steady state in the previous experiment, there is no subsidy to capital in our initial indebted economy. Aside from this, all parameters from the benchmark experiment are held fixed.

From this steady state, the government announces a fiscal policy plan that increases debt by 5% on impact reaching 80% in the long run. The debt path is shown in the top left panel of Figure 4. We continue to consider a constant-K policy, where the government sets capital taxes appropriately such that capital remains at the initial level, as shown in the top middle panel of the figure. Aggregate consumption, therefore, remains unchanged.

The fiscal policy plan in this experiment constitutes a robust Pareto improvement. It leads to an increase in interest rates and transfers, as shown in Figure 4. As before, the policy improves risk sharing, as seen in the bottom right panel of the figure. The welfare gains for this case are in Column 2 in Table 1. Welfare gains are smaller, about a fourth, relative to the previous experiments, mainly because the increase in debt is smaller.

This experiment raises the question of how far debt can go while keeping capital constant without requiring additional tax revenue. In Appendix D.3, we explore the limits of debt in generating RPIs. Starting from our baseline constant-K economy, we show that up until debt levels of roughly 1.7 times the level of output, seigniorage exceeds fiscal costs at the steady state, implying positive lump-sum transfers to households. Beyond this level of debt, the increase in interest rates makes weakly positive transfers infeasible.<sup>43</sup>

<sup>43</sup>Bassetto and Sargent (2020) argue in an OLG framework that the peak of the debt Laffer curve may occur while  $\lambda$  is strictly below the growth rate, making  $\lambda > 1$  an unreliable guide for fiscal expansions.



Figure 4: Constant-K Policy Transition with Higher Initial Debt

### 5.3 Aggregate Shocks

We now study the feasibility of RPIs from government debt expansions in a case with aggregate productivity shocks. We will show that the main insights from the constant-K policy remain unchanged with aggregate uncertainty.

We first extend the feasibility conditions of Section 3 to an environment with aggregate productivity shocks. Let  $\mathbf{B}_t$  denote the aggregate state at time  $t$ ; its aggregate state evolves according to a Markov chain, and we let  $\mathbf{B}^C = (\mathbf{B}_0, \dots, \mathbf{B}_C)$  denote histories through time  $C$ . Production at history  $\mathbf{B}^C$  is given by  $(\mathbf{B}_C, \alpha(\mathbf{B}^{C-1}), \beta(\mathbf{B}^C))$ , where  $\alpha$  is chosen the previous period. Let  $\{r_t(\mathbf{B}^{C-1})\}_t$  be a sequence of risk-free rates as functions of histories;  $\{A_t(\mathbf{B}^C)\}_t$  be the corresponding returns to capital, and  $\{T_t(\mathbf{B}^C)\}_t$  be lump-sum transfers. Households now solve a portfolio problem, choosing how many units of bonds and capital to hold after every history. For simplicity, we set the borrowing limit on bonds to  $b_t = 0$ , and restrict households to hold only non-negative levels of capital. Consider an initial laissez-faire equilibrium denoted by superscript  $0$  and as before, we consider policies that leave after-tax wages and profits unchanged (path by path). An RPI relative to the laissez-faire equilibrium is generated by a triplet  $(\mathbf{r}, \mathbf{A}, \mathbf{T})$ , with  $A_0 = A_0^0$  and  $A_{t+1}(\mathbf{B}^C) = A_{t+1}^0(\mathbf{B}^C)$ ,  $A_t(\mathbf{B}^C) = A_t^*(\mathbf{B}^C)$ , and  $T_t(\mathbf{B}^C) = 0$ , for all  $\mathbf{B}^C \in \mathcal{B}^C$ , with a strict inequality at some history  $\mathbf{B}^C$ .

We can extend Corollary 1 with one additional restriction. In particular, let  $\mathbf{k}(\mathbf{B}^C; \mathbf{r}, \mathbf{A}, \mathbf{T})$  denote the aggregate policy correspondence for households desired holding of physical capital

at state  $\mathcal{B}^C$  given prices and transfers. As before,  $\mathcal{K}(\mathcal{B}^C; r \cdot r \cdot \cdot T)$  denote the desired aggregate consumption at state  $\mathcal{B}^C$ . We have:

Lemma 4. A triplet of risk-free rates, capital returns, and transfers  $(r \cdot r \cdot \cdot T)$  is feasible if and only if there exists a sequence  $(\mathcal{B}^C)_{C \geq 0}$  with  $\mathcal{K}(\mathcal{B}^C; r \cdot r \cdot \cdot T) \geq \mathcal{K}(\mathcal{B}^{C-1}; r \cdot r \cdot \cdot T)$  for all  $\mathcal{B}^C \in \mathcal{C}$ , with  $\mathcal{B}^0 = \mathcal{B}^*$ , such that for all  $\mathcal{C} \geq 0$  and  $\mathcal{B}^C$ :

$$\mathcal{K}(\mathcal{B}^C; r \cdot r \cdot \cdot T) = (\mathcal{B}^C - \mathcal{K}(\mathcal{B}^{C-1}; r \cdot r \cdot \cdot T)) + (1 - \lambda) \mathcal{K}(\mathcal{B}^{C-1}; r \cdot r \cdot \cdot T)$$

The key difference between this lemma and Corollary 1 is the restriction that the capital sequence is consistent with household optimization. In the deterministic setting, bonds and capital are perfect substitutes, and this allows a degree of freedom of how household wealth can be allocated. With aggregate risk, there is a portfolio problem behind the households' capital choices that must be respected in achieving an RPI. Note that as before, heterogeneity enters through the production function, but now also through the portfolio choice policy correspondence.

An issue in a stochastic environment is that while  $\mathcal{K}(\mathcal{B}^C; r \cdot r \cdot \cdot T) \geq \mathcal{K}(\mathcal{B}^{C-1}; r \cdot r \cdot \cdot T)$  may hold on average, it may not hold at every history. The extension of the Balasko-Shell criterion for Pareto efficiency to stochastic settings has been taken up comprehensively by Bloise and Reichlin (2023). This literature shows that the fact that  $\mathcal{K}(\mathcal{B}^C; r \cdot r \cdot \cdot T) \geq \mathcal{K}(\mathcal{B}^{C-1}; r \cdot r \cdot \cdot T)$  may be positive at certain histories does not imply Pareto efficiency. As long as such episodes are not too frequent or persistent, in a sense made precise in the paper, the competitive equilibrium is Pareto inefficient. Bloise and Reichlin (2023) also discuss the related, but distinct, question of when a government can rollover its debt indefinitely.

We now consider a simple quantitative example with aggregate risk to illustrate how the benchmark insights extend to the case with risky capital. Abusing notation, we let the production function at time  $C$  be  $y_C = \theta_C \cdot \mathcal{K}^{\alpha}$  where  $\theta_C$  is the realized aggregate productivity. We assume that this aggregate productivity follows a simple process. In particular, in period 0,  $\theta_0$  is known, but in period 1 the economy faces an aggregate shock, with productivity increasing or decreasing by 5%,  $\theta_1 = \theta_0 \cdot (1 \pm \delta)$ , both outcomes with an equal probability of 0.5. In subsequent periods, aggregate productivity evolves deterministically according to  $\theta_C = (1 - \delta) \theta_{C-1} + \delta \theta_C$ , with  $\delta = 0.05$ . We let  $\mathcal{B}^C$  index the history of the realizations of this aggregate shock up to time  $C$ . The evolution of productivity for both shock realizations is depicted in the top right panel of Figure 5; the solid lines in the figure correspond to the paths for the boom and the dash lines correspond to the recession.

For simplicity, we let the initial distribution of wealth be equal to the stationary distribution that would have arisen absent aggregate shocks. The equilibrium of the laissez-faire economy subject to the shocks determines the evolution of the capital stock for each of the productivity paths; which we depict in the top middle panel of Figure 5. We then evaluate the same debt policy

plan as in the benchmark of Section 5.2, with debt increasing from zero to 60% of steady-state output. As in our baseline constant-K policy,  $\epsilon$ scal policy, through an appropriate choice of taxes, maintains the equilibrium capital paths equal to the ones in the laissez-faire economy. Note that with aggregate uncertainty, the net returns on capital are no longer equal to the return on the risk-free bonds. These returns differ in our example only in period 1. Transfers then must satisfy the following government budget constraint:

$$q_1(B^c) = (1 + A_c(B^{c-1})) d(B^{c-1}) - d(B^c) (A_c(B^c) - A_c^*(B^c)) d(B^c) \quad (16)$$

for all  $B^c$ .

The bottom left panel of Figure 5 plots the equilibrium returns to capital net of depreciation. The solid line depicts the boom path, and the dashed line the bust. We also depict period-1 risk-free rate as the star, keeping in mind that for  $t \geq 1$  the risk-free equals the respective net return on capital, as all aggregate risk has been resolved. Returns are initially higher in the boom because of the higher productivity. The period-1 return to bonds lies between the ex post return on capital in the two states. As  $\epsilon$  mean reverts, the additional capital accumulated early in the boom (see top middle panel) drives the return to capital below its bust counterpart.

The lower middle panel depicts the difference in net returns relative to the laissez-faire economy. Consistent with the requirements of an RPI, these differences are positive at every date along each path. Transfers, depicted in the last panel, are also always positive. Hence, the proposed sequences generate an RPI. Note that in the recession, output falls by 5%, and the risk-free interest rate is -1.3%. Hence  $\epsilon_c \geq 0$  for  $C = 1$ . Despite this, and consistent with the above discussion about fluctuations in 6, the government has enough resources from debt issuance to generate an RPI.

The magnitude of the welfare gains from this policy are similar to that under the constant-K policy in the deterministic benchmark. As reported in Column 3 of Table 1, the mean welfare gain computed in period 0 after the announcement of the policy is 2.7% and the minimum gain is 2.1%. This simple experiment illustrates that RPIs are feasible in environments with aggregate risk.

## 5.4 Capital Expansion

We now consider a  $\epsilon$ scal policy plan that engineers an increase in capital in our deterministic environment that reaches the Golden Rule level in the new steady state. In particular, with this policy, capital relative to output increases from 2.5 to 3.0<sup>44</sup>. We assume that the government also pursues the same path of debt issuance as in the previous baseline constant-K experiment. We

<sup>44</sup>Recall that  $\beta = U_c$  and  $X = 0.10$ , and hence given  $\beta = 0.3$ , the Golden Rule is achieved at  $\beta = 3.0$ .

Figure 5: Policy Transition with Aggregate Shocks

End that transfers are positive throughout, and hence the Escal plan is a feasible RPI. Column 4 of Table 1 reports the welfare gains for this experiment. Welfare increases for all households upon the Escal policy announcement. In this case, Escal policies benefit the rich households more than poor households, with gains upon impact of 6.7% and 5.3%, respectively. The gains in this policy experiment are much larger than for the constant policy, because they reflect not only better risk sharing but also a higher level of capital and consumption in the long run.

Figure 6 plots the variables of interest during this transition. Along the path, we normalize quantities by the initial laissez-faire income, keeping in mind that contemporaneous income is increasing with capital. The first two panels of the figure's top row present the posited paths of debt and capital. The top right panel illustrates that government transfers are positive throughout the transition. They fall in the middle of the transition and increase towards the end. Transfers increase towards the end because interest rates are declining and capital is increasing, easing the Escal burden.

As in the constant- policy experiment, seigniorage revenue from borrowing falls during the transition but settles at a lower level, owing to the higher interest rates. As seen in the bottom left panel of the figure, interest rates rise more with a Escal policy that crowds in capital, because households need to be induced to hold the additional capital as well as debt.

The bottom middle panel shows that aggregate consumption falls early in the transition, as the economy increases investment in new capital, and settles above the laissez-faire level in the new steady state with higher capital. However, throughout the transition, the dispersion of household

## Figure 6: Policy Transition with Capital Expansion

consumption remains uniformly below the level in the laissez-faire economy. As seen in the bottom right panel, the standard deviation drops about 9%, and increases to about 4% lower than in the laissez-faire.

This policy experiment assumed that the government issued debt during the transition. In the analysis of Section 4.3, we showed that debt issuance may be useful along the transition to a higher capital stock to smooth transfers if the short-run elasticity of household savings is significantly lower than the long-run elasticity. This configuration made debt issuance a complement to capital accumulation. We can use the quantitative model to explore this property in greater depth.

Specifically, in Figure 7, we analyze an alternative fiscal policy that implements the same path of capital, but with zero debt issuance. The crucial result here is that without debt issuances, the government needs to lump-sum tax households early in the transition, violating the RPI condition. The large increase in the interest rate necessary to induce households to hold more wealth (the bottom left panel) implies large fiscal costs from capital rental subsidies. This experiment illustrates the transfer smoothing role of debt: public debt may be a necessary tool that complements a capital expansion.

Figure 7: Policy Transition with Capital Expansion and No Debt

## 6 Conclusion

Many governments around the world are rapidly expanding their public debt in the context of low interest rates. Our analysis points to a force that increases the benefits of such expansions. The analysis provided simple conditions for fiscal feasibility, complementing the typical dynamic inefficiency condition of Samuelson (1958) and Diamond (1965). We find that the elasticities of aggregate savings to changes in interest rates are the crucial statistics that determine feasibility. As long as the aggregate household savings schedule is sufficiently elastic and/or the markup is large, robust Pareto improvements are feasible. In calibrated examples using U.S. data on household heterogeneity and historical data on interest rates and growth rates, we find scope for Pareto improving policies for a wide range of debt and tax policies.

The government uses seigniorage debt revenue to provide transfers to households and to subsidize factor prices. These policies are welfare improving for all households because they improve risk sharing without resorting to explicit redistribution. There is growing interest in using fiscal as well as monetary policy to tackle inequality and the lack of insurance markets. Many of these policy proposals feature some sort of explicit or implicit tradeoff, either across agents or across time for a given agent, making them potentially difficult to implement politically. Our contribution highlights a path to welfare improvements that does not involve such tradeoffs, as well as provides explicit conditions for its feasibility.

## References

- Abel, Andrew B., N. Gregory Mankiw, Lawrence H. Summers, and Richard J. Zeckhauser (1989). "Assessing Dynamic Efficiency: Theory and Evidence". In *Review of Economic Studies*, pp. 1-19.
- Abel, Andrew B. and Stavros Panageas (2022). "Running Primary Deficits Forever in a Dynamically Efficient Economy: Feasibility and Optimality". In: Working Paper Series 30554. DOI: 10.3386/w30554.
- Achdou, Yves, Jiequn Han, Jean-Michel Lasry, Pierre-Louis Lions, and Benjamin Moll (2021). "Income and Wealth Distribution in Macroeconomics: A Continuous-Time Approach". *Review of Economic Studies*
- Ackgöz, Ömer T. (2018). "On the existence and uniqueness of stationary equilibrium in Bewley economies with production". In *Journal of Economic Theory*, pp. 18-55.
- Aiyagari, S. R. (1994). "Uninsured Idiosyncratic Risk and Aggregate Savings". *Quarterly Journal of Economics*, pp. 659-684.
- Aiyagari, S. R. and Ellen R McGrattan (1998). "The optimum quantity of debt". *Journal of Monetary Economics*, pp. 447-469.
- Allais, Maurice (1947) *Economie et intérêt* Impremiere National.
- Angeletos, George-Marios, Fabrice Collard, and Harris Dellas (2022). "Public Debt as Private Liquidity: Optimal Policy". In: MIT Working Paper
- Auclert, Adrien, Bence Borotzky, Matthew Rognlie, and Ludwig Straub (2021). "Using the sequence-space Jacobian to solve and estimate heterogeneous-agent models". *Econometrica*, pp. 2375-2408.
- Auclert, Adrien, Matthew Rognlie, and Ludwig Straub (2016) *The Intertemporal Keynesian Cross* Working Paper 25020. National Bureau of Economic Research.
- Azinovic, Marlon, Harold Cole, and Felix Kubler (2023) *Asset Pricing in a Low Rate Environment* Tech. rep. Working Paper.
- Balasko, Yves and Karl Shell (1980). "The overlapping-generations model, I: The case of pure exchange without money". In *Journal of Economic Theory*, pp. 281-306.
- Ball, Laurence M and N Gregory Mankiw (2021) *Market Power in Neoclassical Growth Models* National Bureau of Economic Research Working Paper 28538,
- Barbie, Martin and Ashok Kaul (2009). "The Zilcha Criteria for Dynamic Inefficiency Reconsidered". In: *Economic Theory*, pp. 339-348.
- Barro, Robert J (1974). "Are Government Bonds Net Wealth?". *Journal of Political Economy*, pp. 1095-1117.
- | (2020). *r minus g* Harvard University Working Paper.
- Barth, James R., George Iden, and Frank S. Russek (1984). "Do Federal Deficits really Matter?" In: *Contemporary Economic Policy*, pp. 79-95.
- Basselo, Marco and Wei Cui (2018). "The Escal theory of the price level in a world of low interest rates". In: *Journal of Economic Dynamics and Control*, pp. 5-22.
- | (2024). "A Ramsey Theory of Financial Distortions". In: *Journal of Political Economy*

- Bassetto, Marco and Thomas J. Sargent (2020). "Shotgun Wedding: Fiscal and Monetary Policy". In: *Annual Review of Economics* 12.1, pp. 659{690.
- Basu, Susanto (2019). "Are Price-Cost Markups Rising in the United States? A Discussion of the Evidence". In: *Journal of Economic Perspectives* 33, pp. 3{22.
- Benhabib, Jess, Alberto Bisin, and Shenghao Zhu (2015). "The wealth distribution in Bewley economies with capital income risk". In: *Journal of Economic Theory* 159, pp. 489{515.
- Bernheim, B Douglas (1987). "Ricardian Equivalence: An Evaluation of Theory and Evidence". In: *NBER Macroeconomics Annual* 2, pp. 263{304.
- Bhandari, Anmol, David Evans, Mikhail Golosov, and Thomas J. Sargent (2017). "Public debt in economies with heterogeneous agents". *Journal of Monetary Economics* 81, pp. 39{51.
- Boix, C. (2020). "Inequality, Business Cycles and Monetary-Fiscal Policy". University of Minnesota Working Paper
- Blanchard, Olivier (2019). "Public Debt and Low Interest Rates". *American Economic Review* 109.4, pp. 1197{1229.
- Bloise, Gaetano and Pietro Reichlin (2023). "Low safe interest rates: A case for dynamic inefficiency?" In: *Review of Economic Dynamics* 16, pp. 633{656.
- Boar, Corina and Virgiliu Midrigan (2022). "Efficient Redistribution". *Journal of Monetary Economics*
- Boerma, Job and Ellen R McGrattan (2020). "Identifying efficient tax reform". Tech. rep. Working Paper.
- Bohn, Henning (1995). "The Sustainability of Budget Deficits in a Stochastic Economy". *Journal of Money, Credit and Banking* 27.1, p. 257.
- Brunnermeier, Markus K, Sebastian A Merkel, and Yuliy Sannikov (2020). "Debt as safe asset: Mining the bubble". In: *Princeton University Working Paper*
- Cass, David (1972). "On capital overaccumulation in the aggregative, neoclassical model of economic growth: A complete characterization". *Journal of Economic Theory* 4, pp. 200{223.
- Chamberlain, Gary and Charles A. Wilson (2000). "Optimal Intertemporal Consumption under Uncertainty". In: *Review of Economic Dynamics* 3, pp. 365{395.
- Cui, Wei and Soren Radde (2020). "Search-based endogenous asset liquidity and the macroeconomy". In: *Journal of the European Economic Association* 18.5, pp. 2221{2269.
- Davila, Julio, Jay H. Hong, Per Krusell, and Victor Ros-Rull (2012). "Constrained Efficiency in the Neoclassical Growth Model With Uninsurable Idiosyncratic Shocks". *Econometrica* 80.6, pp. 2431{2467.
- Di Tella, Sebastian (2020). "Risk premia and the real effects of money". *American Economic Review* 110.7, pp. 1995{2040.
- Diamond, Peter A. (1965). "National Debt in a Neoclassical Growth Model". *American Economic Review* 55.5, pp. 1126{1150.
- Domeij, David and Tore Ellingsen (2018). "Rational bubbles and public debt policy: A quantitative analysis". In: *Journal of Monetary Economics* 96, pp. 109{123.
- Dyrda, Sebastian and Marcelo Pedroni (2020). "Optimal Fiscal Policy in a Model with Uninsurable Idiosyncratic Shocks". In: *University of Toronto*



- Elmendorf, Douglas W. and N. Gregory Mankiw (1999). "Government debt". *Handbook of Macroeconomics*. Ed. by J. B. Taylor and M. Woodford. Vol. 1. Handbook of Macroeconomics. Elsevier. Chap. 25, pp. 1615-1669.
- Engen, Eric M. and R. Glenn Hubbard (2005). "Federal Government Debt and Interest Rates". In: *NBER Macroeconomics Annual 2004*, Volume 19. MIT Press, pp. 83-160.
- Epstein, Larry G. and Stanley E. Zin (1989). "Substitution, Risk Aversion, and the Temporal Behavior of Consumption and Asset Returns: A Theoretical Framework". *Econometrica* 57.4, p. 937.
- Farhi, Emmanuel, Alan Olivieri, and Jean Werning (2022). "Price Theory for Incomplete Markets". National Bureau of Economic Research Working Paper 30037.
- Farhi, Emmanuel and Jean Tirole (2012). "Bubbly liquidity". *The Review of Economic Studies* 79.2, pp. 678-706.
- Gale, William G. and Peter R. Orszag (2003). "Economic effects of sustained budget deficits". In: *National Tax Journal* 56.3, pp. 487-502.
- Greenwood, Jeremy, Zvi Hercowitz, and Gregory W. Human (1988). "Investment, Capacity Utilization, and the Real Business Cycle". *The American Economic Review* 78.3, pp. 402-417.
- Hagedorn, Marcus, Iourii Manovskii, and Kurt Mitman (2019). "The Escalator Multiplier". National Bureau of Economic Research Working Paper 25571.
- Heathcote, Jonathan (2005). "Fiscal Policy with Heterogeneous Agents and Incomplete Markets". In: *The Review of Economic Studies* 72.1, pp. 161-188.
- Heathcote, Jonathan, Kjetil Storesletten, and Giovanni L. Violante (2017). "Optimal Tax Progressivity: An Analytical Framework". *Quarterly Journal of Economics* 132.4, pp. 1693-1754.
- Hellwig, Martin F. (2021). "Safe Assets, Risky Assets, and Dynamic Inefficiency in Overlapping-Generations Economies". *Journal of Economic Theory* 201, pp. 383-568.
- Hosseini, Roozbeh and Ali Shourideh (2019). "Retirement Financing: An Optimal Reform Approach". In: *Econometrica* 87.4, pp. 1205-1265.
- Kimball, Miles and Philippe Weil (1992). "Precautionary Savings and Consumption Smoothing across Time and Possibilities". National Bureau of Economic Research Working Paper 3976.
- Kocherlakota, Narayana R. (2023). "Public Debt Bubbles in Heterogeneous Agent Models with Tail Risk". In: *International Economic Review* 64.2, pp. 491-509.
- Koijen, Ralph S.J., François Koolischer, Benjamin Nguyen, and Motohiro Yogo (2021). "Inspecting the mechanism of quantitative easing in the euro area". *Journal of Financial Economics* 140.1, pp. 1-20.
- Kreps, David M. and Evan L. Porteus (1978). "Temporal Resolution of Uncertainty and Dynamic Choice Theory". In: *Econometrica* 46.1, p. 185.
- Krishnamurthy, Arvind and Anne Vissing-Jorgensen (2012a). "The Aggregate Demand for Treasury Debt". In: *Journal of Political Economy* 120.2, pp. 233-267.
- (2012b). "The Effects of Quantitative Easing on Interest Rates: Channels and Implications for Policy". In: *Brookings Papers on Economic Activity: Fall*, 2012, pp. 1-55.
- Krueger, Dirk, Alexander Ludwig, and Sergio Villalvazo (2021). "Optimal Taxes on Capital in the OLG Model with Uninsurable Idiosyncratic Income". *Risk* Working Paper.

- Krueger, Dirk, Kurt Mitman, and Fabrizio Perri (2016). "Macroeconomics and Household Heterogeneity". In: *Handbook of Macroeconomics*. Elsevier, pp. 843{921.
- Light, Bar (2018). "Uniqueness of equilibrium in a Bewley-Aiyagari model". *Economic Theory* 69.2, pp. 435{450.
- Mehrotra, Neil and Dmitriy Sergeyev (2020). "Debt Sustainability in a Low Interest Rate World". Federal Reserve Bank of New York Working Paper.
- Mian, Atif, Ludwig Straub, and Amir Su€ (2022). "Goldilocks Theory of Fiscal De€ts". Tech. rep. Working Paper.
- Oh, Hyunseung and Ricardo Reis (2012). "Targeted transfers and the €scal response to the great recession". In: *Journal of Monetary Economics* 69, S50{S64.
- Rangazas, Peter and Steven Russell (2005). "The Zilcha Criterion for Dynamic Inefficiency". In: *Economic Theory* 26.3, pp. 701{716.
- Reis, Ricardo (2020). "The constraint on Public Debt when  $\gamma > 1$ ". London School of Economics Working Paper.
- Samuelson, Paul A. (1958). "An Exact Consumption-Loan Model of Interest with or without the Social Contrivance of Money". In: *Journal of Political Economy* 66.6, pp. 467{482.
- | (1975). "Optimum Social Security in a Life-Cycle Growth Model". *International Economic Review* 16.3, p. 539.
- Uhlig, Harald and R Anton Braun (2006). "The welfare enhancing effects of a selfish government in the presence of uninsurable, idiosyncratic risk". In: *Journal of Economic Theory* 127.2, pp. 738{758.
- Ventura, Jaume (2012). "Bubbles and capital flows". *Journal of Economic Theory* 147.2, pp. 738{758.
- Werning, Ivan (2007). "Pareto Efficient Income Taxation". MIT Working Paper.
- Wolf, Christian (2021). "Interest Rate Cuts vs. Stimulus Payments: An Equivalence Result". Working Paper.
- Woodford, Michael (1990). "Public Debt as Private Liquidity". *The American Economic Review* 80.2, pp. 382{388.
- Zilcha, Itzhak (1990). "Dynamic efficiency in overlapping generations models with stochastic production". In: *Journal of Economic Theory* 52.2, pp. 364{379.

# Online Appendix to Micro Risks and (Robust) Pareto Improving Policies

## A Liquidity Premium on Government Bonds

In this appendix, we provide an alternative perspective on the wedge between the return to physical capital and the interest rate on government bonds. We set  $\lambda = 1$  and instead appeal to a large body of work documenting that government bonds carry a "convenience yield" or a "liquidity premium," as documented in Krishnamurthy and Vissing-Jorgensen (2012a). In particular, government bonds pay a lower yield than comparable AAA corporate bonds or other non-government safe assets.

We model this as an intermediation technology that uses government debt as an input. Suppose that for every  $\lambda$  units of government debt held on its balance sheet, the representative intermediary generated  $\lambda$  units of the numeraire good. The arguments of  $\lambda$  can be any of the aggregate state variables, including the stock of government debt or total output. However, the technology is seen as constant returns to scale from the perspective of an individual competitive intermediary.<sup>45</sup>

The representative intermediary earns  $r^d$  for every unit of capital held and  $r^b + d$  for every unit of government debt, where  $r^b$  is the interest paid on government bonds. Competition in the intermediation sector yields the following arbitrage conditions:

$$r^d = r^b + d = r^c$$

where  $r^c$  is the interest rate paid to households on deposits.

In what follows, we re-trace the relevant steps of the benchmark analysis. As we proceed, we do not restate the technical assumptions made for each respective result.

### A.1 Revisiting Lemma 1 and Corollary 1

As in the benchmark model, the amount raised in any period by the government via factor taxes is

$$(1 - \tau) F^k - (A^k + X)$$

where zero markups imply  $\tau = 0$ . In the initial equilibrium, the tax revenues are used to pay for the initial debt,

$$A^k = (A^k - d^k) = (1 - \tau) F^k - (A^k + X)$$

<sup>45</sup>is intermediation technology for government bonds has antecedents in monetary models, where money is used to reduce transaction costs as in Kimbrough (1986) and Schmitz and Uribe (2004). Note, however, that  $\lambda$  is a function of aggregate variables; for example, the aggregate quantity of government debt. This implies that the "liquidity service" of an individual bond held by an intermediary depends on how large is the total stock of bonds held by the intermediation sector as a whole. In this sense, there is a systemic component to the intermediation technology.

The change in tax revenues for  $C = 0$  is therefore:

$$(r \cdot \#) - (r \cdot \#) = (A_C + X) C + (A^* + X) \cdot + (A^* - d^*) \cdot$$

The equivalent of (3) is therefore

$$G_1 \left( 1 + \frac{A_C - d_C}{C} \right) C - (r \cdot \#) - (r \cdot \#) = (A^* + X) \cdot + (A_C + X) C - \frac{(A^* - d^*)}{A^*} \cdot$$

The only difference between this expression and (3) is that the rate of government debt differs from the return on capital by  $d_C$ .

Liquidity services are part of aggregate output (which are included in the interest households earn on deposits). Hence, income accounting implies

$$(r \cdot \#) + d^* \cdot = F \cdot \# + (A^* + X) \cdot + A^* \cdot$$

Following the same steps as in the proof of Corollary 1, we obtain

$$C_C = (r \cdot \#) + (1 - X) C + G_1 + d_C C$$

This is the same as in the benchmark, once we recognize liquidity services as part of aggregate output. Note that while increasing government debt generates resources, it may also raise the equilibrium interest rate, requiring the government to intervene in factor markets as in the baseline. This suggests that the elasticity of aggregate savings also plays a role, as shown below.

## A.2 Revisiting Corollary 2

Taking the last inequality and subtracting consumption in the initial equilibrium, we obtain

$$b_C = (r \cdot \#) - (r \cdot \#) + (1 - X) b_C = b_{G_1} + d_C C - d^* \cdot$$

which is the same as in the benchmark given the additional liquidity services.

It is useful to consider a perturbation from a laissez-faire initial equilibrium in which all taxes are zero and  $r = 0$ . This provides a reference that is undistorted by fiscal policy, and hence there are no welfare gains from correcting initial tax distortions. This implies:

$$(r \cdot \#) = A^* + X \cdot$$

or  $r' = r$ , where we recall that  $r = 1 + (r \cdot \#) - X$  and  $r' = 1 + A^*$ .

The counterpart of equation (9) is

$$b_{G_1} + b_C - d_C C = r' b_C$$

Note that as  $r = 0$ ,  $d_C C$  is the change in liquidity services. Hence, the counterpart to equation

(10) is

$$\sum_{G=0}^{\infty} \frac{c}{1+r} b_C - d_C c = 0''$$

'is requires that the present value of consumption innovations net of liquidity services is less than zero. Now suppose we have a small innovation to the interest rate at time 0. Following the same steps as in the benchmark analysis, we have

$$\sum_{G=0}^{\infty} \frac{c}{1+r} = 1'' > \sum_{G=0}^{\infty} \frac{c}{1+r} = \frac{m_A c}{m_A} + \sum_{G=0}^{\infty} \frac{g}{1+r} > \\ = \sum_{G=0}^{\infty} \frac{g}{1+r} > \bullet$$

where the second line uses  $\sum_{G=0}^{\infty} \frac{c}{1+r} = 1''$ , as there is no markup.

Assuming regularity conditions for  $d_C$  and small changes to  $c$  we can approximate

$$d_C c = d^>(c) + (d_C - d^>) c = d^> c$$

where the last equality uses  $d^> = 0$  and  $d^>$  is the marginal product of liquidity services in the initial equilibrium.

At the margin, the returns to physical capital net of depreciation and to government bonds inclusive of liquidity services are equated in the initial equilibrium. To a first order, it therefore is irrelevant whether changes in household wealth are backed by changes in  $c$  or  $c$ . For expositional purposes, suppose changes in household wealth are equivalent to changes in government bonds

$$\frac{c}{A_g} = \frac{A_c}{A_g}$$

and therefore for small changes we have<sup>46</sup>

$$d_C c = d^> \frac{m_A c}{m_A} A_g''$$

'e sufficient condition for a feasible RPI becomes

$$\sum_{G=0}^{\infty} \frac{g}{1+r} > d^> \sum_{G=0}^{\infty} \frac{c}{1+r} = 1''$$

Rearranging, and using our definition of  $d^>$  from the benchmark, the counterpart of equation (14) becomes

$$\frac{d^>}{1+r} \sum_{G=0}^{\infty} \frac{c}{1+r} (1-g) b_C > 1''$$

<sup>46</sup>If  $\epsilon > 0$ , then  $d^>$  in the following expression would be replaced by  $1 + \frac{m d m}{d^>}$ . The latter term is the elasticity of the convenience yield to changes in government bonds. It is this elasticity that is the focus of event studies surrounding quantitative easing (QE) episodes, such as Krishnamurthy and Vissing-Jorgensen (2012b) and Kojien, Koulischer, Nguyen, and Yogo (2021).

'is is similar to the benchmark's equation (14), but with the liquidity premium replacing the wedge between the marginal product of capital and the interest rate. In the benchmark, the government could exploit that wedge, which existed because of a markup. In this alternative, the government can generate liquidity services by issuing debt. 'e larger the marginal product of bonds in generating liquidity services, the easier it is to satisfy feasibility.<sup>47</sup> We obtain the result that the roles of  $\beta$  and  $b_{c,j}$  in the infinite sum is exactly the same as in the benchmark.

## B Transfers When Capital is Below the Golden Rule

Consider the following notion of monotonicity of aggregate consumption with respect to transfers:

**Definition 5.** We say  $C = f(C, G_0)$  is weakly increasing in  $T$  if  $T^0 \leq T$  implies  $C(r, T^0) \leq C(r, T)$ , where the inequality holds for all in the respective sequences. If  $T$  for which there is a  $c$  such that  $c^0 \leq c$  implies  $C(r, T^0) < C(r, T)$  and that there is a  $B$  such that  $C_B(r, T^0) < C_B(r, T)$ , we say  $C$  is strictly increasing in  $T$ .

'is is a natural property, in that holding constant all interest rates, one would naturally expect an increase in lump sum transfers would induce households (in aggregate) to consume more.<sup>48</sup>

'e following result says that if consumption is weakly increasing in transfers, then we can ignore the role of transfers when looking for an RPI (as long as an interest rate have changed). 'at is, transfers are not necessary for evaluating feasibility:

**Lemma 5 (Transfers are not necessary)** Suppose that  $C$  is weakly increasing in  $T$ . Let  $(r, T)$  be a feasible RPI where for some  $c \in A$ .  $C(r, T^0)$  where  $T^0 = f(A, A^0)$  is also a feasible RPI.

**Proof.** Note that an RPI requires that  $c \in A$  and  $c = (A, A^0)$ . 'e fact that the  $C$  is weakly increasing implies that  $C(r, T) \leq C(r, T^0)$ , as  $T \geq T^0$ . 'e sequence of  $c$  that implements the  $(r, T)$  then also implements  $(r, T^0)$ . Given that  $c \in A$  for some  $c$  it follows that  $(r, T^0)$  is a feasible RPI.

'e following result says that if consumption is strictly increasing in transfers, than an RPI is not feasible without a change in an the interest rate. 'at is, transfers alone are not sufficient:

**Lemma 6 (Transfer are not sufficient)** Suppose that  $C$  is strictly increasing in  $T$ . If  $\beta > \bar{\beta}$ , then there is no feasible RPI in which  $r > \bar{r}$ .

<sup>47</sup>If Ricardian equivalence held, then a version of the Friedman rule would apply; that is, the government should issue debt until the marginal return to liquidity services is driven to zero. Here, issuing debt is not neutral, and hence will change allocations and factor prices and potentially violate the requirements of an RPI.

<sup>48</sup>For an individual agent in incomplete markets it is possible to construct examples where individual consumption falls given an increase in future transfers. However, we are counting on heterogeneity to guarantee that such individual behavior does not aggregate. Wolf (2021) presents examples of permanent income and hand to mouth households where these assumptions hold. See also Farhi, Olivi, and Werning, 2022 for general comparative statics results for incomplete market economies.

Proof. Suppose there is a feasible RPI  $\{c^t, n^t\}_{t=0}^{\infty}$ . There must be a non-negative sequence  $\{g_t\}_{t=0}^{\infty}$  such that

$$C_D(r^{t+1}) + g_{t+1} = (1 + g_t) C_D(r^t) + (1 - X) c^t$$

Exploiting the concavity of technology, and that  $r^{t+1} = (1 + g_t) r^t$ , we have that

$$g_{t+1} > (1 + g_t) g_t - (C_D(r^{t+1}) - C_D(r^t))$$

Note that  $g_t \geq 1$ , together with  $C_D$  increasing in  $r$ , implies that  $C_D(r^t) > C_D(r^{t-1})$  for all  $t$ .

Let  $B$  be the first time where  $C_D(r^B) > C_D(r^{B-1})$  (such a time exists, given that  $C_D$  is strictly increasing in  $r$ ). Then, the above implies that  $B > 1$ . Now note that

$$B < g_B > (1 + g_{B-1}) g_{B-1} - (C_D(r^B) - C_D(r^{B-1}))$$

Given that  $g_t \geq 1$ , it follows then that  $C_D(r^t) > C_D(r^{t-1})$  for  $t$  large enough, a contradiction.

When we focus on the case where the economy operates below the Golden Rule, the above result tells us that in a feasible RPI (under a reasonable assumption) an interest rate must change at some date. The reason is that with only increases in transfers, aggregate consumption will be higher at all times with the RPI than originally, an impossibility given the resource constraint and  $\bar{y} < \bar{c}$ .

## C Proofs

### C.1 Proof of Lemma 1

Towards sufficiency, suppose that the conditions of the lemma hold. Then, for  $\bar{c} > 0$ , set  $\{g_t\}_{t=0}^{\infty}$  such that

$$\frac{\bar{c} (1 + g_t)^t}{(1 + g_t)^t} = F >$$

This ensures the labor market clears at  $F_C = F >$  and  $\bar{c}_t = \bar{c}$ , where GHH preferences ensure that the households are willing to supply  $\bar{c}$  at wage  $F >$ . Note that as  $\bar{c}$  is given,  $g_0$  is the same as the initial equilibrium. Similarly, the government taxes or subsidizes profits so that

$$c = (1 + g_t) e_C = (1 + g_t) (1 - X) \bar{c} = \bar{c}$$

This determines  $g_t$ .

Finally, the government must ensure that the representative firm's choice of capital is consistent with the risk-free interest rate:

$$(1 + g_t) \bar{c} = (1 + g_t) A_C = (1 + g_t) (A_C + X) \bar{c}$$

which then determines  $g_t$ .

The sequence of tax rates defined above ensure that firms optimize and markets clear for labor and capital. By definition of  $A_C$  and condition (i) of the lemma, the market for assets also clears given  $A_C$ . The final equilibrium condition involves government revenues and transfers. The

total government revenue (before transfers) of this tax policy is given by

$$\begin{aligned}
 \text{Revenue} &= \tau_c F^c + \tau_l A_c + \tau_e e_c \\
 &= (1 + \tau_c) F^c + (1 + \tau_l) A_c + (1 - \tau_e) e_c = F^c + A_c + b_c \\
 &= \frac{\tau_c (c^c)^\alpha + (c^c)^\alpha}{\tau_c} F^c + A_c + \frac{(1 - \tau_e) (c^c)^\alpha}{\tau_c} \\
 &= (c^c)^\alpha F^c + A_c
 \end{aligned}$$

where the third line uses  $(1 + \tau_c) b_c = \tau_c$ ; the firm's first-order condition for labor and capital; and  $b_c = (1 - \tau_e) \tau_c$ . The last line follows from Euler's theorem. Note that national income accounting implies

$$(c^c)^\alpha = F^c + A^c + A^x$$

Hence, we can replace  $F^c + A^c = (c^c)^\alpha - A^x$  and  $A^c = A + X$  to obtain

$$\text{Revenue} = (c^c)^\alpha - (c^c)^\alpha (A + X) + (A + X) + A^x \quad (\text{C.17})$$

As transfers equals revenue plus net debt issuance, we have

$$T_c = (c^c)^\alpha - (c^c)^\alpha (A + X) + (A + X) + A^x + \tau_{d1} (1 + A) c^c$$

where the inequality allows for free disposal of government surpluses. This is condition (3), and thus ensures that the government has a non-negative surplus at equilibrium. Given the proposed taxes, transfers, and debt issuances, this establishes that given the sequences in the premise, we can construct a tax plan that implements an equilibrium.

Necessity of condition (i) in the lemma follows from the market clearing condition in the definition of equilibrium. The necessity of condition (ii) follows from firm optimization and the government budget constraint.

## C.2 Proof of Corollary 1

Using

$$(c^c)^\alpha = F^c + A^c + A^x$$

we have

$$C_c = (c^c)^\alpha (A + X) + A^x + (1 + A) A_c + \tau_{d1} c^c$$

Using  $A_c = c^c + d$  this is equivalent to

$$C_c + \tau_{d1} = (c^c)^\alpha (A + X) + A^x + (1 + A)(c^c + d) + \tau_{d1} c^c$$

Substituting into (6) and re-arranging gives (3).



### C.3 Proof of Lemma 2

For a given  $\epsilon$ , let  $T^0 = T^> + \epsilon a$  be the new transfer sequence. From the continuity condition, we have

$$C_d(r^> \cdot T^0) > j C_d(r^> \cdot T^0) > j \epsilon a$$

For  $C = 0$ , we have

$$\begin{aligned} C_d(r^> \cdot T^0) + \epsilon &> \epsilon a + \epsilon \\ &= (\epsilon \cdot \#) X \epsilon + \epsilon a \\ &= (\epsilon \cdot \#) + (1 - X) \epsilon + (\epsilon a + \epsilon) \end{aligned}$$

and hence the condition in Corollary 1 holds for  $\forall a \in (\epsilon, \epsilon)$  as  $\epsilon > j \epsilon$ .

For  $C = 1$ , it is sufficient if

$$\epsilon a + \epsilon > (\epsilon \cdot \#) X \epsilon$$

or, using  $\epsilon = (\epsilon \cdot \#) X \epsilon$ ,

$$\epsilon a > (\epsilon \cdot \#) X \epsilon - 1 (\epsilon \cdot \#) X \epsilon$$

Letting  $a_2 = \epsilon - 1 (\epsilon \cdot \#) X \epsilon - 1 (\epsilon \cdot \#) X \epsilon > 0$ , this condition is satisfied if  $\forall a > a_2$ .

Collecting, for  $\forall a \in \min\{a_1, a_2\}$ , the transfer scheme  $T^0 = T^> + \epsilon a$  is implementability and represents an RPI.

### C.4 Proof of Proposition 1

For a given  $\epsilon$ , let  $r^0 = r^> + \epsilon a$ . Note that  $(r^0, T^>)$  is an RPI. Let us propose the following sequence of  $\{c_C^1\}_{C=0}^{\infty}$ :

$$c_C = r^> + \epsilon \sum_{B=0}^{C-1} X^B (C_B(r^0, T^>) > j + a) \quad \text{for } C \geq 1$$

with  $c_0 = r^>$ . We will check that such sequence implements  $(r^0, T^>)$  for a small enough  $\epsilon$ .

Note that

$$\begin{aligned} j c_C > j &= \sum_{B=0}^{C-1} X^B j C_B(r^0, T^>) > j + a \\ &= \sum_{B=0}^{C-1} X^B a + a = \frac{1 - X^C}{1 - X} a + a = \frac{1}{1 - X} a \end{aligned}$$

where the second line uses property (ii). Then, there exists  $\bar{C}$  such that  $c_C > 0$  for all  $C \geq \bar{C}$  and  $\forall a_1$ .

Let

$$\bar{c} = \sup_{j \in \{1, \dots, n\}} \{j : j \leq a_1\}$$

As  $\bar{c}$  is continuous and this is a compact domain,  $\bar{c}$  is finite. Note that for  $a \in \mathbb{R}$ , Taylor's theorem implies that

$$(c \# \bar{c}) + (1 - X) c = (c \# \bar{c}) + (1 - X) \bar{c} + \frac{1}{2} (c - \bar{c})^2$$

for some  $\theta$  between  $\bar{c}$  and  $c$ . Using that  $(c \# \bar{c}) + (1 - X) \bar{c} = \bar{c}$  and that  $j \in \{1, \dots, n\}$ , we have that

$$(c \# \bar{c}) + (1 - X) c \geq \bar{c} + \frac{1}{2} (c - \bar{c})^2$$

Then, a sufficient condition for (6) from Corollary 1 is

$$C_0(r^0 T) + \frac{1}{2} (c - \bar{c})^2 \geq C_0(r^0 T) + \frac{1}{2} (c - \bar{c})^2, \text{ for all } C \geq 1$$

For the first inequality, using the proposed, we have that

$$\sum_{B=0}^X (C_B(r^0 T) - \bar{c}) + a \geq 0$$

which holds given (i).

For the second inequality, using the proposed, we have

$$\sum_{B=0}^X (C_B(r^0 T) - \bar{c}) + a \geq \sum_{B=0}^X (C_B(r^0 T) - \bar{c}) + \frac{1}{2} (c - \bar{c})^2 \geq 0$$

Given that  $\bar{c} \geq 0$ , there exists  $\alpha_2 \geq 0$  such that

$$\bar{c} \geq \frac{1}{2} \alpha_2$$

for all  $a \geq \alpha_2$ .

Let  $\bar{a} = \min\{\alpha_1, \alpha_2\}$ . Then  $(r^0 T)$  for any  $a \geq \bar{a}$  is a feasible RPI.

## C.5 Proof of Corollary 2

Divide both sides of equation 12 by  $\bar{c}$ , factor out  $\bar{c}$ , and use the definition of  $\bar{c}$  to obtain (14). As shown in the text, this implies (11) is satisfied, which in turn is sufficient for (i) in Proposition 1. Condition (ii) holds by the differentiability of  $C_C$  which is implied by the differentiability of  $A_C$

stated in the premise.

### C.6 Proof of Lemma 3

As in the benchmark model's Corollary 2, consider a policy that sets  $A^g = A^* + A_g$  for some  $g > 0$  and  $A_g < 0$ . Recall that in the representative agent environment,  $V^g = 1 + A^* = 1 + V$ . From the Euler equation, we have

$$c^g = \begin{cases} \frac{2}{3} \text{ for } C^g > 1 \\ \frac{2}{3} \text{ for } C^g \leq 1 \end{cases}$$

where  $c^g$  and  $c^g$  satisfy the Euler equation at time  $g = 1$ :

$$D^g(c^g) = V(1 + A_g)D^g(c^g)$$

For small changes around the initial equilibrium consumption, we can differentiate this to obtain:

$$D^g(c^g) \frac{32}{3A_g} = V D^g(c^g) + D^g(c^g) \frac{32}{3A_g}$$

where we use the fact that  $1 + A^* = 1 + V$ . Rearranging, we have

$$\frac{32}{3A_g} \frac{32}{3A_g} = Z V \quad (C.18)$$

where  $Z = D^g(c^g) / (D^g(c^g) + c^g)$ .

Using  $V = 1 + A^*$ , the budget constraint requires:

$$\sum_{g=0}^{\infty} V^g c^g + \frac{1}{1+A_g} \sum_{g=0}^{\infty} V^{g+1} c^g = 1 + V + (F^g - \#^g + c^g) \sum_{g=0}^{\infty} V^g c^g + \frac{1}{1+A_g} \sum_{g=0}^{\infty} V^{g+1} c^g$$

Differentiating and using  $V^g = F^g - \#^g + c^g + A^* V^g$ , we obtain:

$$\frac{32}{3A_g} + V^g \frac{32}{3A_g} \frac{32}{3A_g} = V^{g+1} A^* Z$$

Combining this with (C.18), we obtain:

$$\begin{aligned} \frac{32}{3A_g} &= V^{g+1} A^* Z \\ \frac{32}{3A_g} &= \frac{32}{3A_g} + V Z \end{aligned}$$

is implies

$$\begin{aligned} \sum_{g=0}^{\infty} \frac{c^g}{m^g} &= \sum_{g=0}^{\infty} \frac{c^{2g}}{3^g} + \sum_{g=0}^{\infty} \frac{c^{2g}}{3^g} \frac{3^g}{3^g} \\ &= \frac{1}{1-c} \sum_{g=0}^{\infty} c^{2g} > Z > + \frac{1}{1-c} \sum_{g=0}^{\infty} c^{2g} \end{aligned}$$

Letting  $g \rightarrow \infty$ , equation (11) is satisfied if  $Z > 0$ , or  $Z_j > \frac{A^g}{c}$ , which is the condition in the lemma.

## C.7 Proof of Proposition 2

Towards a contradiction, suppose there is a feasible RPII  $\chi$ . Given that we start from the laissez-faire allocation, this requires that  $\bar{\alpha}$  is non-negative. From the feasibility condition in Corollary 1, we have that there exists a sequence  $\theta$  such that

$$\begin{aligned} F^g > \theta^g + \theta^g + (1+A)A_d(r \cdot T) A_{G1}(r \cdot T) + c \left( \frac{c \theta^g}{Z} + (1-X) \right) c_{G1} \\ & \left( \frac{c \theta^g}{Z} + (1-X) \right) > \theta^g + \theta^g + (1+A)A_d(r \cdot T) A_{G1}(r \cdot T) + c \left( \frac{c \theta^g}{Z} + (1-X) \right) c_{G1} \end{aligned}$$

where the last inequality follows from concavity of  $f$ . Using that  $\theta^g = 1 + A^g$  as  $\theta^0 = 1$ , we have

$$\begin{aligned} F^g > \theta^g + \theta^g + (1+A)A_d(r \cdot T) A_{G1}(r \cdot T) > \theta^g + (1+A^g)(c_{G1} \theta^g) (c_{G1} \theta^g) \\ A^g > \theta^g + (1+A)A_d(r \cdot T) A_{G1}(r \cdot T) + c (1+A^g)(c_{G1} \theta^g) (c_{G1} \theta^g) \end{aligned}$$

And thus

$$A_{G1}(r \cdot T) > c_{G1} (1+A)(A_d(r \cdot T) \theta^g + (A_c - A^g) c_{G1} \theta^g)$$

Note that starting from the laissez-faire implies that  $\bar{\alpha} = \theta^g$ , and thus  $A_{G1}(r \cdot T) > c_{G1}$  is always non-negative, and turns strictly positive whenever  $A_c > A^g$  or  $c_{G1} \theta^g > 0$ . Hence, we have that

$$A_{G1}(r \cdot T) > c_{G1} (1+A)(A_d(r \cdot T) \theta^g + (A_c - A^g) c_{G1} \theta^g)$$

and given that  $A^g > 0$  ( $\theta^g > 0$  and  $\theta^g = 1$ ), it follows that  $A_{G1}(r \cdot T) > c_{G1}$  must necessarily go to infinity as  $c$  increases. Finite technology implies that  $c$  must remain bounded, and thus  $A_{G1}(r \cdot T) > c_{G1}$ . The assumption in the proposition then implies that for any  $\theta$  there exists a  $B$  such that  $A_{G1}(r \cdot T) > B$ . For  $\theta$  sufficiently large, the resource constraint must be violated, generating the contradiction.

## C.8 Proof of Lemma 4

‘e proof follows similar steps as the proof of Corollary 1. Factor taxes are defined in the same manner as in the proof of that corollary. ‘e government budget constraint is:

$$\begin{aligned} & \int d(B^t) \left( B^t \cdot d(B^{t-1}) \cdot \#_C^{\geq}(B^t) \right) - \left( B^t \cdot \#_C^{\geq}(B^{t-1}) \cdot \#_C^{\geq}(B^t) \right) \\ & A_C^{\geq}(B^t) d(B^{t-1}) + A_C^{\geq}(B^t) \#_C^{\geq}(B^{t-1}) + G_{-1}(B^t) - (1 + A_C(B^{t-1})) d(B^{t-1}) \end{aligned}$$

‘e aggregated household budget set is:

$$d(B^t) = F_C^{\geq}(B^t) \#_C^{\geq}(B^t) + \#_C^{\geq}(B^t) + (1 + A_C(B^t) \chi) d(B^{t-1}) - G_{-1}(B^t) + (1 + A_C(B^{t-1})) d(B^{t-1}) - G_{-1}(B^t) + d(B^t)$$

We have

$$\left( B^t \cdot \#_C^{\geq}(B^{t-1}) \cdot \#_C^{\geq}(B^t) \right) = F_C^{\geq}(B^t) \#_C^{\geq}(B^t) + \#_C^{\geq}(B^t) + A_C^{\geq}(B^t) \#_C^{\geq}(B^{t-1})$$

Using this to substitute for  $F_C^{\geq}(B^t) \#_C^{\geq}(B^t) + \#_C^{\geq}(B^t)$  in the HH budget constraint, and then use the resulting expression to substitute for  $\int d(B^t)$  in the government budget constraint, we obtain the expression in the lemma:

$$d(B^t) - \left( B^t \cdot d(B^{t-1}) \cdot \#_C^{\geq}(B^t) \right) + (1 - \chi) d(B^{t-1}) - G_{-1}(B^t) = 0$$

‘is condition ensures that the government budget constraint and aggregate market clearing hold, given a sequence of functions  $d(B^t)$  and  $G_{-1}(B^t)$ . A necessary and sufficient condition for equilibrium is that the aggregate household policy for consumption  $d(B^t; r \cdot r^T)$  satisfies the above resource condition and the sequence  $\{d(B^t)\}_{t=0}^{\infty}$  is  $2 \leq K(B^t; r \cdot r^T)$ .

## D Simulation

### D.1 Preferences and Technology

‘e utility function we consider for households is of the Epstein-Zin form

$$U_C = \left( (1 - \beta) G_C^{1-\gamma} Z + \beta E_t \left[ U_C^{1-\gamma} \right]^{\frac{1}{1-\gamma}} \right)^{\frac{1}{1-\gamma}} \quad (D.19)$$

where  $\beta$  is the discount factor,  $\gamma$  is the elasticity of intertemporal substitution,  $\gamma$  is the risk aversion coefficient, and  $G$  is the composite of consumption and labor.  $G_C = Z_{8C} = \frac{1}{8C} a$ . ‘e parameter  $a$  controls the Frisch elasticity of the labor supply. We set some of the preference parameters to conventional values in the literature and others as part of the calibration. ‘e elasticities of intertemporal substitution and of labor supply are set to the common parameter values of 1 and 0.2, respectively. ‘e discount factor and coefficient of risk aversion are set as part of the calibration exercise described below. We set the borrowing constraint to zero for all households.

An important part of the parametrization is the stochastic structure for idiosyncratic shocks. We adopt the structure and estimates from Krueger, Mitman, and Perri (2016), which use micro data on a %er-tax labor earnings from the PSID. Idiosyncratic productivity shocks contain a persistent and a transitory component, and their process is as follows:  $\log Y_{8C,t} = \log Y_{8C,t-1} + \eta_{8C,t}$  and

$y_{t+1} = d + \rho y_t + \epsilon_t$  with persistence  $\rho$  and innovations of the persistent and transitory shocks  $(\epsilon_t)$ , and associated variances given by  $\sigma^2(\epsilon_t)$ . We set the three parameters controlling this process  $(d, \rho, \sigma^2(\epsilon_t))$  to .9695, .0320, and .0435, respectively, to reflect the estimated earnings risk in Krueger, Mitman, and Perri (2016) for employed individuals and the endogenous labor supply decision in our model. We discretize this process into 10 points, based on the Rouwenhurst method.

As mentioned in the text, we take a parsimonious approach to allocating profits to households and assume a distinct class of entrepreneurs who are endowed with managerial talent and consume profit distributions in a hand-to-mouth manner.

The technology specification is Cobb-Douglas,  $(Y, K, L) = U \#^{1-U}$ . We use standard values for the coefficient  $U$  and for the depreciation rate of capital. The values are  $U = 0.3$  and  $\delta = 0.1$ . The markup parameter  $\mu$  is set to 14.

We calibrate the discount factor and the coefficient of relative risk aversion as follows. We target a steady state with 60% debt-to-output and capital-to-output of 2.5, where the debt corresponds to the US average over the period 1966-2021 and the capital ratio is taken from Aiyagari and McGrattan (1998). We treat this steady state as the result of a constant-K policy starting from a laissez-faire economy. The average interest rate relative to growth in the US over the sample period is -1.4%, which will be the target for the return on bonds in our steady state. The resulting parameter values are a discount factor  $\beta = 0.993$  and a coefficient of risk aversion  $\gamma = 5.5$ .

## D.2 Constant-K Simulation

Our "baseline fiscal policy" is the one which keeps capital constant starting from the laissez-faire.

Table 2: Baseline Constant-Policy and Laissez-Faire Economies

	Data	Constant- Policy	Laissez-Faire
Aggregates			
Public Debt (% output)	60	60	0
Interest Rates(%)	-1.4	-1.4	-1.7
Capital (rel. output)	2.5	2.6	2.6
Wealth Distribution			
Q1 Wealth Share	1	1	1
Q2 Wealth Share	4	4	4
Q3 Wealth Share	11	11	10
Q4 Wealth Share	23	23	23
Q5 Wealth Share	59	59	59

Table 2 presents some moments in the stationary equilibrium of the economy with baseline constant- fiscal policy and the laissez-faire economy. The levels of public debt, interest rates, and capital in the economy with the baseline fiscal policy match the data moments by construction. The table shows that an increase in debt to output of 60% raises interest rates by 0.3 percentage points. We also present some moments on the wealth distribution in the steady states (namely the wealth share of each asset quintile) and compare them with data as reported in Krueger,

<sup>49</sup>The economy is dynamically efficient, also by construction. To see this,  $\beta = U = 0.3^{2.5} = 0.12$ , which is greater than the depreciation rate of 0.1.

Mitman, and Perri (2016). Our model economies generate skewed distributions of wealth, with most of the wealth being held by the top quintile of the distribution, although they are not quite as skewed as the data. In addition in our model economies, a small fraction of agents, about 2%, are at their borrowing constraint at any period.

### D.3 Debt Laffer Curve

We revisit the logic of Figure 1. In particular, long-run seigniorage is given by  $\tau$ , while the costs are captured by  $A_0$ . In Figure D.1, we plot these two components for stationary equilibria with different levels of debt to output for the constant-policy studied in subsection 5.2. At each debt level, tax policy is set to deliver laissez-faire wages and profits. As can be seen, up until debt levels of roughly 1.7 times the level of output, seigniorage exceeds fiscal costs, implying positive lump-sum transfers to households. Beyond this level of debt, the increase in interest rates makes weakly positive transfers infeasible.

Note that these two curves intersect while seigniorage is still increasing in debt. Eventually,  $A_0$  becomes close enough to zero that seigniorage begins to decline in debt. The peak of this Laffer curve occurs at debt levels roughly four times output. Feasible Pareto-improving levels of debt consistent with a constant-policy, however, are much lower than this peak.

While Figure D.1 establishes only that the policy is feasible in the new steady state, the analysis of transition dynamics in the baseline case above suggests that feasibility in the steady state is the critical metric. Along the transition, the government is a net issuer of bonds. As long as the revenue from the net issuances dominates any overshooting of the interest rate, feasibility rests on long-run considerations.

Figure D.1: Steady-State Seigniorage and Tax Revenue across Debt

## D.4 Computational Algorithm

This appendix describes the computational algorithm we use in solving the model. The code is available at

[https://github.com/manuelamador/micro risks\\_pareto\\_improving\\_policies](https://github.com/manuelamador/micro risks_pareto_improving_policies).

Our procedure consists of three steps. First, we compute the initial and final stationary equilibria. The initial one is the laissez-faire equilibrium and the final one has fiscal policy active. A second step computes the transition of this economy. Finally, we compute the aggregate savings elasticities associated with an initial laissez-faire equilibrium and operationalize Corollary 2.

## D.5 Stationary Equilibrium

The computations of the policy and value functions rely on an endogenous grid method, modified for the presence of Epstein-Zin preferences. In particular, we use the value function, equation (D.19), together with the first order condition with respect to consumption:

$$(1 - \beta) G_{8C}^{1Z} = \beta V_{E_{t+8\mathbb{E}1}}^{1W} \frac{W_{t+8\mathbb{E}1}^{1Z}}{W_t^{1Z}} E_{t+8\mathbb{E}1} + \frac{\beta + \beta \mathbb{E}1}{\beta \mathbb{E}1} \beta$$

The envelope condition implies

$$\frac{\beta + \beta \mathbb{E}1}{\beta \mathbb{E}1} = (1 - \beta) G_{t+8\mathbb{E}1}^{1Z} G_{8\mathbb{E}1}^{1Z}$$

Taken together, we obtain the following Euler equation:

$$G_{8C}^{1Z} = \beta V_{E_{t+8\mathbb{E}1}}^{1W} \frac{W_{t+8\mathbb{E}1}^{1Z}}{W_t^{1Z}} E_{t+8\mathbb{E}1} + \frac{\beta + \beta \mathbb{E}1}{\beta \mathbb{E}1} \beta G_{t+8\mathbb{E}1}^{1Z} G_{8\mathbb{E}1}^{1Z} \quad (D.20)$$

We let  $G_{8C}^{1Z} = G_{8C}^{1Z}$

**Initial.** To compute the initial laissez-faire stationary equilibrium, we proceed as follows. Given a guess for the initial interest rate  $r^0$ , we obtain the wage level consistent with the technology  $F^0$ . We then solve the household problem given wages and interest rates  $(w^0, r^0)$  (and set  $\beta = 0$ ). We do this as follows. Given the wage, the labor supply is easily obtained from the GHH preferences. We then iterate backwards using an endogenous grid method based on (D.20) and the value function (D.19). That is, we start with a guess for  $r_{t+8\mathbb{E}1}$  and  $G_{t+8\mathbb{E}1}^{1Z}$  and use the Euler equation and the value function to compute the values of  $r_t$  and  $G_{8C}^{1Z}$  that are consistent with the guess and the borrowing constraint, using a linear interpolation. We iterate until  $r_t$  and  $G_{8C}^{1Z}$  have converged to some tolerance.

Having solved the households problem, we use the stationary policy function to obtain a transition function for the distribution of households (as in Young, 2010), and compute the implied stationary distribution,  $\mu^0(\cdot|I)$ . To obtain the stationary general equilibrium, we repeat this for different values of  $r^0$  until the aggregate of household savings in the stationary state is consistent (for a given tolerance) with the capital stock given  $\bar{n}$  and the implied total labor supply  $\bar{L}$ .



**Final.** The final stationary equilibrium computation follows a similar approach as the initial one. In this case, we know that the wage, and the labor supply remain equal to the values in the initial equilibrium. For a given guess of the interest rate  $R^1$ , a target level of government debt  $B^1$  and a long-run level of capital  $K^1$ , we use the government budget constraint to obtain the implied transfers,  $T^1$ , that make the government budget constraint hold with equality in the stationary equilibrium (using inequality (3) with equality). We then solve the household problem given  $w^o, R^1$  and  $T^1$ . As in the initial stationary equilibrium, we iterate on  $R^1$  (and obtaining a new  $T^1$ ) until the aggregate of the household savings equal the sum of  $K^1$  and  $B^1$ .

## D.6 Transition

At time 0, the government announces a sequence of fiscal policies that implements a sequence of capital and debt  $\{K_t, B_t\}_{t=0}^H$ . We will assume that at period  $H$ , the economy is in the final stationary equilibrium, with  $K_H = K^1$  and  $B_H = B^1$ .

We use Lemma 1 to compute the transition as follows. We start with a guess of interest rates  $\{R_0, R_1, R_2, \dots, R_H\}$  with  $R_0 = R^o$  and  $R_H = R^1$ . Given this guess, we can use (3) with equality to obtain the sequence of implied transfers,  $T_t$ . Starting from the value function  $V^1$  and the additional state  $\eta^1$  set at the values of the final equilibrium, we use the Euler equation and the value function to iterate backwards and construct a sequence of  $V_t$  and  $\eta_t$ . With these sequences, we compute the policy functions and the transition function for the distribution of households. We then, starting from  $\Lambda^o$ , iterate forward the evolution of the distribution. With this, we compute the aggregate of the household savings at each time,  $A_t$ . We then look for a root: a sequence  $\{R_t\}$  such that  $A_t = B_t + K_t$  for all  $t \leq H$  (up to some tolerance), as required by Lemma 1.<sup>50</sup>

## D.7 Transition with Aggregate Shocks

We extend our algorithm to incorporate aggregate uncertainty that is resolved in period 1. We start the economy at time  $t = 0$  from the same initial stationary laissez-faire equilibrium as in the previous examples. Agents understand that at  $t = 1$  the economy is hit with an aggregate productivity shock,  $Z_1 \in \{Z^h, Z^l\}$  with equal probability, and that the productivity reverts to the initial level over time.

We first recover the path of aggregate capital that will arise in the laissez-faire economy after introducing the shock. Given that the borrowing limit is 0 (and there are not short-selling constraints), we do not need to solve a portfolio problem, as households will only invest in capital. We guess and iterate on two paths on capital returns that generate market clearing given the household optimization and aggregation. We assume that the economy is back at the initial steady state levels after  $H$  periods (that is, the capital paths have converged back to where they started). The procedure to compute this is similar to what we did in the benchmark exercise. From this step, we recover the laissez-faire sequences of capital, one for each of the two shock paths,  $\{K_0^h, K_1^h, \dots, K_H^h\}$  and  $\{K_0^l, K_1^l, \dots, K_H^l\}$ . Note that given our shock structure,  $K_0^h = K_0^l = K_0$ , and  $K_1^h = K_1^l$ .

<sup>50</sup>For this part, we use a quasi-newton method based on the Jacobian of the aggregate asset function at the initial equilibrium. The computation of the jacobian is discussed in the next subsection.

The government announces a sequence of fiscal policies that implement a sequence of capital and debt  $\{K_t^j, B_t\}_{t=0}^H$  for  $j = \{h, l\}$  where  $K$  remains as in the laissez-faire transition. The paths of  $B$  are assumed equal to the path in the benchmark exercise and independent of the shock. As before, we assume that at period  $H$ , the economy is in the final stationary equilibrium, with  $K_H = K^1$  and  $B_H = B^1$ .

We compute the transitions as follows. We start with guesses for capital returns and risk-free rates. Given the shock structure, the returns on capital net of depreciation  $R_t^k$  and bonds  $R_t$  are equal to each other for  $t > 1$ . This means that we need to guess the paths for the returns to capital  $\{R_1^{j,k}, R_2^{j,k}, \dots, R_H^{j,k}\}$  for  $j = \{h, l\}$  and the risk-free rate from period 0 to 1,  $R_1$ , with  $R_0^j = R^0$  and  $R_H^j = R^1$  for  $j = \{h, l\}$ . Given these guesses, we can use (16) to obtain the sequence of implied transfers,  $T_t^j$ . As above, starting from the value function  $V^1$  and composite  $x^1$  set at the values of the final equilibrium, we use the Euler equation and the value function to iterate backwards up to  $t = 1$  and construct sequences of  $V_t^j$  and  $x_t^j$  for each shock path  $j = \{h, l\}$ . Note that from  $t = 1$  on, each path does not face uncertainty, and therefore the households do not choose a portfolio between capital and bonds.

The problem at  $t = 0$ , however, contains a portfolio problem, which we solve using a change of variables and by generalizing the endogenous grid method. Let  $\theta_i$  be household's  $i$  share of total savings  $a_{i,1}$  allocated to risky capital and  $(1 - \theta_i)$  be the share allocated to bonds. At  $t = 0$  households choose total savings  $a_{i,1}$  and the portfolio  $\theta_i$  to satisfy an Euler equation and a portfolio equation:

$$x_{i,0}^{1/\zeta} = \beta \mathbb{E}_z V_{i,1}^{1/\zeta} \gamma^{\frac{w-1}{1-w}} \mathbb{E}_{z,j} V_{i,1}^{1/\zeta} \gamma^{\zeta} ((1 - \theta_i)R_1 + \theta_i R_1^{k,j}) x_{i,1}^{1/\zeta} \\ R_1 \mathbb{E}_{z,j} V_{i,1}^{1/\zeta} \gamma^{\zeta} x_{i,1}^{1/\zeta} + \bar{\lambda}_i = \mathbb{E}_{z,j} V_{i,1}^{1/\zeta} \gamma^{\zeta} (R_1^{k,j}) x_{i,1}^{1/\zeta} + \underline{\lambda}_i$$

where  $\bar{\lambda}_i$  is the multiplier of the constraint that  $\theta_i > 0$  and  $\underline{\lambda}_i$  is the multiplier of the constraint that  $\theta_i < 1$ .

In our backward iteration, we arrive at  $t = 1$ , with  $x_{i,1}$  and  $V_{i,1}$ . We first solve for  $\theta_i$  using the portfolio equation above by taking into account that  $x_{i,1}$  and  $V_{i,1}$  depend on total cash-on-hand  $\omega_{i,1}$  which is the portfolio return,  $\omega_{i,t} = (\theta_i R_1^{k,j} + (1 - \theta_i)R_1)a_1$ . Effectively, we perform a change of variables and solve for optimal  $\theta_i$  to satisfy the portfolio equation, using interpolation. We then iterate back to period 0 and solve for optimal savings, taking into account that optimal  $\theta_i$  depends on  $a_1$ , using the Euler equation.

We now have all the sequences of  $V_{it}^j$  and  $x_{it}^j$  for all households. With these sequences, we compute the policy functions and the transition function for the distribution of households. We then, starting from  $\Lambda^0$ , iterate forward the evolution of the distribution. With this, we compute the aggregate of the household savings at each time,  $A_t^j$  for  $j = \{h, l\}$  and also compute the capital demand in period 0,  $K_1$ . We then look for a root: sequences  $\{R_t^{k,j}\}$  and  $R_1$  such that  $A_t^j = B_t^j + K_t^j$  for all  $t = H$  and  $j = \{h, l\}$  and  $K_1 = K^1$  (up to some tolerance).

## D.8 Elasticities

The computation of the elasticities we fixed a horizon,  $H$ , and set a value  $\tau < H$  to be the date where the interest rate changes. We then solve for the sequence of  $V$  and  $\eta$  associated with a

sequence of interest rates such that  $R_t = R^0$  for  $t \notin \tau$  and  $R_\tau = R^0 + \Delta$ , by iterating backwards from  $t = \tau$  and starting with the laissez-faire equilibrium values. We iterate forward the distribution and compute the implied aggregate savings at each date from  $t = 1$  to  $H$ ,  $A_t^{up}$ . We do the same for a sequence of interest rates such that  $R_t = R^0$  for  $t \notin \tau$  and  $R_\tau = R^0 - \Delta$ , and obtain the sequence of aggregate savings,  $A_t^{down}$ . We then compute the (two-sided) numerical derivative,  $(A_t^{up} - A_t^{down})/(2\Delta)$  at each time up to  $H$ , and use these to construct the elasticities  $\xi_{t,\tau}$ .<sup>51</sup>

## E The Growth Economy

In this appendix, we show how the key expressions of Section 2 are modified by the presence of exogenous labor-augmenting technological growth. The derivations are standard and are included for completeness.

Assume technology is given by

$$Y_t = F(K_t, (1+g)^t L_t),$$

where  $g > 0$  is the constant rate of growth of labor-augmenting technology. Letting a tilde denote variables divided by  $(1+g)^t$ , constant returns implies

$$\tilde{Y}_t = (1+g)^{-t} Y_t = F(\tilde{K}_t, \tilde{L}_t).$$

The representative firm's first-order conditions are (dropping  $t$  subscripts)

$$\begin{aligned} F_k(\tilde{K}, \tilde{L}) &= \mu(1 + \tau^k)r^k \\ F_l(\tilde{K}, \tilde{L}) &= \mu(1 + \tau^n)\tilde{w}. \end{aligned}$$

We also have  $\tilde{\mathbf{H}} = (1 - \tau^n)(\mu - 1)F(\tilde{K}, \tilde{L})/\mu$ .

Given the absence of a wealth effect on labor supply, we assume that the disutility of working grows at rate  $g$  as well (dropping  $i$  and  $t$  indicators):

$$x(c, n) = c - (1+g)^t v(n),$$

giving us

$$\tilde{x}(\tilde{c}, n) = (1+g)^{-t} x(c, n) = \tilde{c} - v(n).$$

We also assume that the borrowing constraint is scaled by  $(1+g)^t$ .

We can write the household's problem as

$$\begin{aligned} V_t(a, z, \theta) &= \max_{a^0, n, c} \phi(x(c, n), h(V_{t+1}(a^0, z^0, \theta^0))) \\ \text{s.t. } c + a^0 &= w_t z n + \theta \Pi_t + (1 + r_t)a + T_t \\ a^0 &= (1 + g)^{t+1} \underline{a}, \end{aligned}$$

where we have altered the last constraint to account for growth and  $h$  is a certainty equivalent

<sup>51</sup>This is what Auclert, Bardóczy, Rognlie, and Straub, 2021 refer to as the “direct method”.

operator. The constraint set can be rewritten as

$$\begin{aligned} \tilde{c} + (1 + g)\tilde{a}^0 &= \tilde{w}_t z n + \theta \tilde{\mathbf{H}}_t + (1 + r_t)\tilde{a} + \tilde{T}_t \\ \tilde{a}^0 &= \underline{a}. \end{aligned}$$

Thus, if  $(c, n, a^0)$  is feasible at time  $t$ , then  $(\tilde{c}, n, \tilde{a}^0)$  satisfies the normalized constraint set and vice versa. If we assume  $\phi$  is constant-returns in  $x$  and  $h$  is homogeneous of degree 1, if  $V_t(a, z, \theta)$  satisfies the consumer's Bellman equation, then  $\tilde{V}_t(\tilde{a}, z, \theta) = (1 + g)^{-t} V_t(a, z, \theta)$  satisfies

$$\tilde{V}_t(\tilde{a}, z, \theta) = \max_{\tilde{c}, n, \tilde{a}^0} \phi(\tilde{x}(\tilde{c}, n), (1 + g)h(\tilde{V}_{t+1}(\tilde{a}^0, z^0, \theta^0))),$$

subject to the normalized constraint set, and vice versa.<sup>52</sup>

Note that for an interior optimum for  $n$ , the first-order condition can be expressed as follows:

$$v^0(n) = z\tilde{w}.$$

Hence, labor supply is constant as long as  $\tilde{w}$  remains constant.

The government's budget constraint can be rewritten in normalized form:

$$\tilde{T}_t = \tau_t^n \tilde{w}_t N_t + \tau_t^k r_t^k \tilde{K}_t + \tau_t^\pi \tilde{\mathbf{H}}_t / (1 - \tau_t^\pi) + (1 + g)\tilde{B}_{t+1} - (1 + r_t)\tilde{B}_t.$$

Let  $\tilde{X}_t = \tau_t^n \tilde{w}^o N^o + \tau_t^k r_t^k \tilde{K}_t + \tau_t^\pi \tilde{\mathbf{H}}^o / (1 - \tau_t^\pi)$  denote normalized tax revenue before transfers when keeping after tax normalized wages and profits constant. Following the same steps as the proof of Lemma 1, we have

$$\tilde{X}_t = F(\tilde{K}_t, N^o) - F(\tilde{K}^o, N^o) - (r_t + \delta)\tilde{K}_t + (r^o + \delta)\tilde{K}^o.$$

Condition (iii) of Lemma 1 (equation (3)) becomes

$$(1 + g)\tilde{B}_{t+1} - (1 + r_t)\tilde{B}_t - \tilde{T}_t = F(\tilde{K}^o, N^o) - F(\tilde{K}_t, N^o) - (r^o + \delta)\tilde{K}^o + (r_t + \delta)\tilde{K}_t.$$

Condition (ii) becomes  $\tilde{T}_t = (r_t - r^o)\tilde{a}$ , and condition (i) remains unchanged. Note that in a steady state (that is, relevant aggregates grow at rate  $g$ ), Condition (iii) becomes

$$(g - r_{ss})\tilde{B}_{ss} - \tilde{T}_{ss} = F(\tilde{K}^o, N^o) - F(\tilde{K}_{ss}, N^o) - (r^o + \delta)\tilde{K}^o + (r_t + \delta)\tilde{K}_{ss}.$$

Hence, debt increases government revenues in the steady state as long as  $g > r_{ss}$ . Expressions in Claims 1 and 2 are adjusted in a similar fashion to obtain normalized equivalents.

## References

Aiyagari, S. R. and Ellen R McGrattan (1998). "The optimum quantity of debt". In: *Journal of Monetary Economics* 42.3, pp. 447–469.

<sup>52</sup>For the simulations, we use  $\phi(x, h) = ((1 - \beta)x^{1-\chi} + \beta h^{1-\chi})^{1/\chi}$ . In this case, we can define  $\tilde{\beta} = \beta(1 + g)^{1-\chi}$  and write  $\tilde{\phi}(\tilde{x}, h) = ((1 - \tilde{\beta})\tilde{x}^{1-\chi} + \tilde{\beta}h^{1-\chi})^{1/\chi}$ , where  $\chi = (1 - \beta)/(1 - \tilde{\beta})$ . This is well defined as long as  $\tilde{\beta} < 1$ . Growth can be accommodated by re-scaling the discount factor, as expected with homogeneous preferences.

- Auclert, Adrien, Bence Bardóczy, Matthew Rognlie, and Ludwig Straub (2021). “Using the sequence-space Jacobian to solve and estimate heterogeneous-agent models”. In: *Econometrica* 89.5, pp. 2375–2408.
- Farhi, Emmanuel, Alan Olivi, and Iván Werning (2022). *Price Theory for Incomplete Markets*. National Bureau of Economic Research Working Paper 30037.
- Kimbrough, Kent P (1986). “The optimum quantity of money rule in the theory of public finance”. In: *Journal of Monetary Economics* 18.3, pp. 277–284.
- Koijen, Ralph S.J., François Koulischer, Benoît Nguyen, and Motohiro Yogo (2021). “Inspecting the mechanism of quantitative easing in the euro area”. In: *Journal of Financial Economics* 140.1, pp. 1–20.
- Krishnamurthy, Arvind and Annette Vissing-Jorgensen (2012a). “The Aggregate Demand for Treasury Debt”. In: *Journal of Political Economy* 120.2, pp. 233–267.
- (2012b). “The Effects of Quantitative Easing on Interest Rates: Channels and Implications for Policy”. In: *Brookings Papers on Economic Activity: Fall 2011*, p. 215.
- Krueger, Dirk, Kurt Mitman, and Fabrizio Perri (2016). “Macroeconomics and Household Heterogeneity”. In: *Handbook of Macroeconomics*. Elsevier, pp. 843–921.
- Schmitt-Grohé, Stephanie and Martin Uribe (2004). “Optimal fiscal and monetary policy under sticky prices”. In: *Journal of Economic Theory* 114.2, pp. 198–230.
- Wolf, Christian (2021). *Interest Rate Cuts vs. Stimulus Payments: An Equivalence Result*. MIT Working Paper.
- Young, Eric R. (2010). “Solving the incomplete markets model with aggregate uncertainty using the Krusell–Smith algorithm and non-stochastic simulations”. In: *Journal of Economic Dynamics and Control* 34.1. Computational Suite of Models with Heterogeneous Agents: Incomplete Markets and Aggregate Uncertainty, pp. 36–41.