Customer Acquisition, Rising Concentration and Productivity Growth Slowdown*

Shihan Shen†

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Abstract

The cost of marketing and advertising has declined enormously due to the advance of digital technologies. This paper studies the macroeconomic consequences of lower marketing cost, and finds that it is a critical driving force of several striking macroeconomic trends, including rising market concentration and productivity growth slowdown since the 1990s. I develop an endogenous growth model with product market search frictions. Firms invest in innovation and marketing to build customer base, which is a long-term asset. Then I exogenously feed in the observed large drop of marketing cost into the quantitative model and find that it accounts for 83% of the rise in market concentration, measured by the largest firm's market share. Cheaper marketing generates a positive level effect and a negative growth effect on productivity. These two effects together explain around 1/3 of the decline in productivity growth rate and successfully captures its "first-rise-then-fall" pattern over time. Finally, I conduct a welfare analysis and find that firms tend to over-invest in marketing compared to the socially optimal allocation, which implies that welfare can be improved by a marketing tax.

Keywords: marketing and advertising, customer base, market concentration, productivity growth, innovation

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†UCLA Economics. Email: sshen92@ucla.edu.

1 Introduction

With the advance of digital technologies in the past few decades, the cost of marketing and advertising has declined enormously due to a dramatic change in the composition of advertising delivery. The resulting rise of marketing/advertising intensity reshapes the way firms build customer base and further affects firm dynamics. This paper studies the macro consequences of lower marketing cost and finds that it well explains two recent macroeconomic trends: rising market concentration, measured by market share of the largest firms, and slowdown in aggregate productivity growth. To my knowledge, this paper is the first study to connect macroeconomic trends to the evolution of marketing landscape and customer base accumulation.

To do this, I develop an endogenous growth model with product market search frictions, where firms choose how much to innovate and how much to advertise. The informational frictions between firms and buyers render customer relationships long-term in nature, and turn the customer base into a form of capital for firms. Then I conduct an experiment in which I exogenously feed in the observed large drop of marketing cost into the quantitative model to study how this may have contributed to changes in the aggregate economy.

I find that the decline of marketing cost is a critical driving force of the recent rise in aggregate market concentration. From 2016 to 2019, the average market share of the largest firm within 3-digit SIC industries has increased from 24% to almost 30%. Quantitative analysis shows that cheaper marketing can account for 83% of this change. There are always concerns among economists and policymakers that the considerable rise in concentration may depress long-run economic performance by squeezing out competitors[1]. After all, during the same period of rising concentration, the TFP growth rate dropped from around 1.5% to

[1]Policymakers are concerned about the negative effects of superstar firms on competition. For instance, the Executive Order on Promoting Competition in the American Economy in 2020 addresses concerns on the potential unfair competition between big firms and small businesses. (For more details, check link). The recent bill of "American Innovation and Choice Online Act" (S. 2992) considers limiting the "big tech" companies (e.g., Facebook, Google, Amazon) from using their website to promote their own products over similar products by other companies (For details, check link).

0.6%, despite a temporary boom in the late 1990s. My paper contributes to this debate by studying how lower marketing cost, as a key mechanism of increasing concentration, affects long-run productivity growth and economic welfare.

I show that on the transitional dynamics, the declining marketing cost generates a positive "level effect" on productivity due to higher output quantity brought about by more efficient matching, and a negative "growth effect" driven by lower innovation intensity due to a shift towards marketing investment. The combination of these two effects explains around 1/3 of the decline in productivity growth and successfully captures the "first-rise-then-fall" pattern in the TFP growth rate over time. In a welfare analysis, I find that as marketing becomes cheaper, the level effect dominates the growth effect and results in an increase of total welfare. However, compared to the socially optimal allocation, firms tend to over-invest in marketing because one firm's marketing generates a negative externality on other firm's customer acquisition.

This paper highlights an essential part of firm operations: building and maintaining customer base. In fact, the rise of many large firms' market share is coincided with their huge and fast-growing customer base. Almost all the superstar firms emerged in the past 20 years, such as Amazon, Uber, etc., own a large number of users or subscribers[2]. Conventional wisdom focuses on the contribution of innovation to attract customers. Specifically, firms invest in R&D to develop better product quality, which increases their relative market size by diverting demand away from firms selling low-quality goods. However, in addition to R&D, firms also invest substantially in marketing, advertising and the like, to propagate their products and build customer base.

Between 1996 and 2019, firms spend as much as 8% of GDP on marketing related activities, with advertising alone taking up around 2.2%. This is comparable to the share of R&D

[2]For instance, from 2014 to 2021, Amazon's sales share in the U.S. had doubled from 28.1% to 56.7%, and in the meantime, their Prime member user base expanded by four times, from 40 million to 157.4 million. Take Uber as another example. Founded in 2009, Uber grew from a small startup into a giant that owns 71% of sales in the ride-sharing market in 2022. Their customer base today has reached 118 million, which is almost 1/3 of total U.S. population.

expenditures over GDP (2.4% to 3%) in the same period. The heavy spending on marketing and advertising provide evidence of frictions in product markets, which require firms to spend resources on customer acquisition. The importance of marketing is also elaborated in the industrial organization and marketing literature. Argente et al. (2021) shows that firms build market share by adding new customers through advertising rather than manipulating markups. Thomas (2001) also points out that many firms are taking a customer-oriented approach as compared to a product-oriented approach in the new economy where more and more firms sell services and customer relationship becomes critical.

The landscape of marketing has changed dramatically in recent years: the development of digital technologies greatly reshapes the delivery of advertising. From late 1990s, online marketing gradually becomes prevalent and accounts for almost half of total advertising in today's world. This raises marketing efficiency and thus reduces the cost of advertising. Using the composition of U.S. advertising spending and the price index of information and communications technologies (ICT) investment, I find that the average advertising price index has plummeted by more than 40% from 1996 to 2019. Meanwhile, aggregate firm marketing expenditures have increased enormously. After taking into account the decline of price index, real advertising spending of U.S. firms has risen by 5 times over the past 20 years. Interestingly, the gap of marketing expenditures between large and small firms (by sales) has widened, with the large firms increasing their marketing investment and small firms downsizing it. This implies that the technological change in marketing might potentially explain the enlarging gap in customer base and market share across firm size.

To capture the effect of marketing cost on customer base accumulation, firm dynamics and further on the aggregate economy, I develop an endogenous growth model where a continuum of firms sell goods to customers in a market hampered by informational frictions. My model features two key aspects: (i) firms innovate to raise quality through R&D investment, and (ii) firms invest in marketing and advertising to become more visible to customers who search for new products to buy. Search frictions in the product market renders the customer base a valuable long-term asset to firms. Those in the customer base of a firm will buy again at the firm if their lifetime utility of continuing the customer relationship outweighs that of searching.

Marketing investment increases product exposure to buyers and enables the firm to match with more customers in the search process. Upon meeting, a customer discovers the quality, price, as well as her preference on the good, and decides whether to buy based on the revealed information. Holding other factors constant, the better the product quality, the more likely a customer makes the purchase.

Marketing and R&D investments are complementary, because by performing R&D, a firm improves product quality, which increases a customer's probability of purchase and thus raises the return to advertising. In general, firms with a larger customer base invest more in both R&D and marketing. Due to the scale effect of innovation, the cost of innovation does not scale up with firm customer base, but profits do because non-rivalrous technologies/ideas can be replicated at close-to-zero marginal cost (Haskel and Westlake (2017), De Ridder (2019)). Hence, large firms tend to invest more in innovation than their smaller counterparts to take advantage of their large customer base. By complementarity, large firms also invest more intensively in marketing.

Then I use the model to analyze impact of declining marketing cost on firm dynamics. As marketing becomes less costly, every firm has incentives to invest more in marketing. With the increasing presence of advertisement, it becomes easier for customers to learn the existence or characteristics of a product, which generates more firm-customer matches for all firms. However, as large firms invest more in R&D and are more likely to provide better quality, they are able to convert a larger proportion of matched customers into actual buyers relative to their smaller counterparts. Therefore, although marketing becomes equally cheaper for all firms, large firms disproportionately obtain more new buyers in the search process, which enlarges the difference in customer base across firm size. As customer base directly affects firm decisions on R&D and marketing investment, the gap between large and small firms is further amplified in a positive feedback loop, and eventually leads to a surge of market concentration and the rise of superstar firms.

Despite reallocating economic activities towards a smaller set of firms, rising concentration caused by lower marketing cost is not necessarily harmful to the aggregate productivity. In fact, cheaper marketing generates mixed effects on productivity growth by differently affecting production and innovation incentives for large and small firms. As marketing becomes cheaper, the aggregate level of marketing increases, which alleviates search frictions and facilitates more matches and consumption. The resulting rise in aggregate output quantity has a positive effect on productivity growth.

Nevertheless, due to enlarging difference in customer base, large firms increase their marketing investment by disproportionately more than the smaller ones, which implies that small firms now have a lower share of advertisement and are thus less likely to be encountered by customers than before. Despite a rise in aggregate marketing, the capacity of consumption is limited by the total measure of customers (population), creating a congestion effect among firms. Consequently, the customer base of small firms might even shrink due to fewer matches with new customers, which further reduces their long-term incentives to innovate and/or advertise.

On the other hand, large firms expand their customer base by obtaining more new customers during the search process. However, in an economy with high aggregate level of marketing, the option value of search is also larger because customers are now more likely to get matched. Therefore, in order to retain their existing customers, firms now have to offer more competitive prices, which reduces profits earned on their current customer base. In this sense, declining marketing cost reduces firms' market power to charge high prices or markups, and thus lowers the expected return of innovation investment. This constitutes a force towards lower productivity. These offsetting factors exert an ambiguous overall effect on aggregate productivity growth and this is why the quantitative analysis is of paramount importance.

I calibrate the model on the balanced growth path by matching various moments for the U.S. economy in the late 1990s (1996-1999), using data on all U.S. listed firms from Compustat as well as aggregate moments. With this initial calibration I describe firms' innovation and marketing strategies to develop intuition about the model. I then calibrate the reduction in marketing cost to fully capture the five-fold increase in aggregate real marketing investment observed in the data.

Using the calibrated model, I conduct an experiment where I exogenously feed in the observed drop of marketing cost to infer how this may have changed the economy from late 1990s to today. First, I quantify the aggregate impact of marketing shock on business concentration and productivity growth on the balanced growth path. I find that my model can explain 83% of the rise in the largest firm's market share within 3-digit SIC industries, and around 1/3 of the decline in aggregate productivity growth. To understand the role of changing customer base in rising concentration, I conduct a counterfactual exercise to decompose the effect on concentration into changes in quantity and changes in prices. It shows that the market concentration is mainly driven by the enlarging gap in customer base across firm size rather than large firms charging higher markups, supporting the findings in Baqaee and Farhi (2020) and De Loecker et al. (2020).

Second, I demonstrate the dynamics of transition from the low to high marketing equilibrium. Over the transition path, market leader's sales share increases rapidly at the beginning and then gradually grows to its new steady state. The aggregate productivity growth experienced an initial surge of 37 basis point for 5 years and then it declines by 68 basis point over time to reach the new balanced growth path. This is consistent with the "first-rise-then-fall" pattern in data. A decomposition analysis indicates that the temporary boom at the beginning is driven by the increase in output quantity due to more matches in the product market. This level effect gradually fades out as the economy converges to the new steady state. Additionally, marketing also affects firm R&D incentives and, therefore, has an indirect effect on growth (the "growth effect"). With an increasing customer base, large firms tend to increase their R&D investment, although the lower prices on existing customers generate opposite incentives. Small firms decrease their R&D investment due to both deteriorate marketing share and lower prices. In the calibrated model, the average innovation growth rate decreases from 1.51% in the first period to 1.19% in the new steady state, implying that the negative effect of small firms and declining price dominates the rise of customer base in large firms.

Last but not the least, I study the consequences of lower marketing cost on welfare. Following the classic literature Nelson (1974), Butters (1977), Grossman and Shapiro (1984) and Milgrom and Roberts (1986), informative advertising weakens the negative effect of information frictions, and is usually associated with welfare improving. Indeed, I find that as marketing becomes cheaper, the level effect dominates the growth effect, and the overall welfare increases. However, advertising is also a taste shifter that firms use to steal customers from each other and maintain their own market shares (Dixit and Norman (1978), Becker and Murphy (1993), Benhabib and Bisin (2002), Benhabib and Bisin (2011) and Molinari and Turino (2009)). In this case, the combative marketing expenditures might result in a waste of resources. Therefore, I derive the social planner's problem and find that compared to the socially optimal allocation, firms in the decentralized economy over-invest in marketing as they ignore the negative impact of their advertisement in other firms' customer acquisition. Finally, I discuss the policy implications and find that there exists an optimal level of tax on marketing that maximizes social welfare.

Related Literature. This paper is closely related to a new literature that embeds customer acquisition and marketing into a macroeconomic framework. Relevant work including Fishman and Rob (2003), Gourio and Rudanko (2014) and Paciello et al. (2019) explore the impact of long-term customer relationship in a product market with search frictions. My paper builds on their idea of customer capital but focuses on its implication for market concentration and productivity growth. Another two closely related papers are Cavenaile and Roldan-Blanco (2021) and Cavenaile et al. (2021). These two papers also develop an endogenous growth model to analyze the implications of advertising for firm dynamics and economic growth through its interaction with R&D. The key difference between their papers and mine is the role of advertising. In their papers, advertising benefits firms by either influencing consumer tastes, while my paper emphasizes the role of advertising to mitigate informational frictions in a search environment. Although we all talk about the effects on productivity, I propose a mechanism of positive feedback loop of customer base accumulation to explain increasing market concentration. The long-term customer relationship is the key to explain the sharp rise in large firm's market share.

Rachel (2021) claims that the rise of leisure goods provided by advertising has a negative effect on welfare because less labor are allocated to R&D. This is consistent with the "growth effect" in my model. However, advertising in my paper has an additional information-based role as opposed to purely shifting customer tastes, and the resulting "level effect" explains the different welfare effect in the two papers. Greenwood et al. (2021) distinguishes traditional and digital advertising and argues that digital advertising increases welfare significantly and is disproportionately financed by better-off consumers. Perla (2019) develops a model in which consumers learn about firms through a network of connections that endogenously evolves over the life cycle. Other relevant papers include papers that use pricing to accumulate a customer base (e.g. Drozd and Nosal (2012) Foster et al. (2016), Roldan-Blanco and Gilbukh (2021)), and papers that study marketing and advertising activities in international economics (Fitzgerald et al. (2017)).

This paper also contributes a novel mechanism to the large and growing literature linking trends in concentration, productivity growth, and business dynamism using models of endogenous growth. The existing literature highlights the importance of technological change and intangible assets to explain macroeconomic trends. For example, De Ridder (2019) shows that the rise of intangible inputs such as software can jointly explain the slowdown of productivity growth and rising market power. Olmstead-Rumsey (2019) explains these trends using the declining innovativeness of smaller firms, and Aghion et al. (2019) emphasizes the importance of less technology spillover. In addition, there are also non-technological explanations including demographic changes (Hopenhayn et al. (2018) , Jones (2020), Peters and Walsh (2021), Karahan et al. (2019), Eggertsson et al. (2019), Bornstein et al. (2018)) or declining real interest rates (Kroen et al. (2021), Chatterjee and Eyigungor (2020)). Rather than emphasizing the product competition, my paper provides a novel idea of the importance of customer base to firms. Hence, marketing investment plays a key role in firms' pursuit of profits, and potentially affects competition, which further explains the enlarging gap among large and small firms and the potential threat to productivity growth.

More broadly, this paper relates to the literature that provides micro-foundations to how advertising works. The two-sided market is first proposed by Caillaud and Jullien (2003) and Rochet and Tirole (2003). These papers are built upon the idea that advertising platform serves a two-sided market between firms and consumers. The platform is a monopoly that maximizes its profit by choosing the number of ads it sells. In my paper, I model advertising as a type of investment for firms to build customer base, and do not explicitly study the relationship between ad agencies and firms. Another related strand of literature explores the role of intangibles for firm and industry dynamics (Arkolakis (2016), Dinlersoz and Yorukoglu (2012) and Perla (2017)). I contribute to this literature by exploring the interaction of advertising with R&D.

Outline. The paper proceeds as follows. Section 2 shows the motivating facts in marketing landscape and firm investment. Section 3 presents the endogenous growth model with customer acquisition and Section 4 discusses the mechanism. I structurally estimate the model in Section 5, and discuss results in Section 6. Section 7 concludes.

2 Empirical Motivation

This section describes stylized facts on marketing landscape, including the composition of advertising spending, the evolution of marketing cost and trends in firm marketing investment. I also display macroeconomic trends of increasing concentration and slowdown in productivity growth during the same period.

2.1 Changes in Marketing Landscape

The advance of digital technologies has greatly reshaped the delivery of advertising. Digital marketing, also known as online advertising, evolves quickly because of Internet technologies, with the online channels now accounting for almost half of total advertising, and strong growth in social media, search engines, mobile apps and e-commerce. Figure 1 shows the composition of various marketing/advertising platforms. Internet advertising, which was almost absent in the late 1990s, grows quickly in recent years and becomes a major marketing platform that accounts for more than 40% of total US advertising today. Advertising on print-based media and radio declines rapidly, while the share of TV advertising remains relatively stable over time.

Even traditional media is deeply influenced by the development of internet and communications technologies (ICT) capital, with the digital technologies reshaping the display of advertising and consequently affecting the cost of marketing for businesses. For example, in recent years, there has been a shift from traditional to connected TV (CTV). With CTV advertising, ad buys are not based on air times or channels as in traditional TV advertising. Instead, CTV ads are delivered one at a time based on the specific viewer watching a program. This saves the cost of ad buyers because they no longer have to guess which shows their target audiences are watching. Instead, they can build a target audience based on demographic and behavioral signals, then serve ads to specific viewers.

Goldfarb (2014)points out that the fundamental difference between traditional and digi-

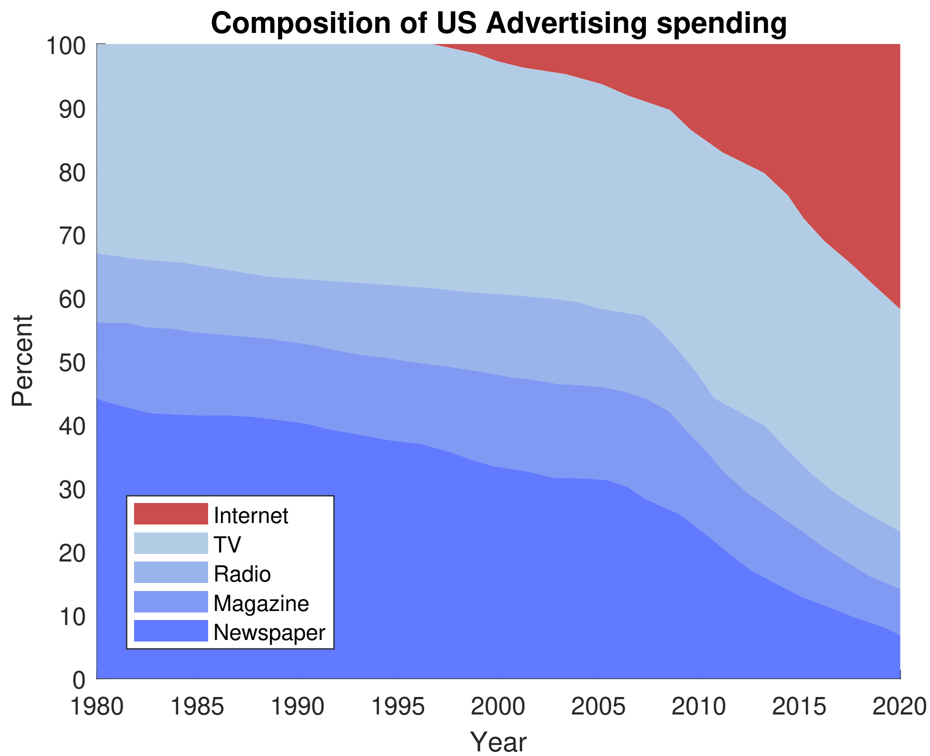


Figure 1: Composition of US advertising Spending. Data source: Zenith, McCann.

tal advertising is a substantial reduction in the cost of targeting for digital channels. With online advertising, advertisers can target the keywords customers use in search engines, demographic characteristics, and a consumer's past online behavior at relatively low cost. On the macro level, this technological change leads to a large decline of the cost of marketing/advertising over time. The left panel of Figure 2 displays the price index of information processing equipment (black curve) taken from NIPA Table 5.3.4. The price index has dropped by 2/3 from 318.5 in late 1990s to 80.4 in 2019. The red curve is the price index of Internet advertising, a main platform of advertising as aforementioned. The statistic is provided by BEA and starts from 2010. In the past 10 years, the price of Internet advertising has declined by 40%. Although the price of Internet advertising is not available for years before 2009, the downward trends between the two price indices are quite similar, with the price of ICT investment declining even slightly faster.

In order to obtain an average price index of advertising, but in the mean time not over-estimate the decline, I assume 1/3 of the TV advertising uses Internet technologies, and none of the other traditional platforms uses ICT. Next, I use the price index of ICT investment as the proxy for Internet advertising over the past 20 years. Then taking the share of different advertising platforms shown in Figure 1 as the weight, I calculate the average marketing price index, and plot it in the right panel of Figure 2 (black curve). It plummeted by 40% from 1996 to 2019.

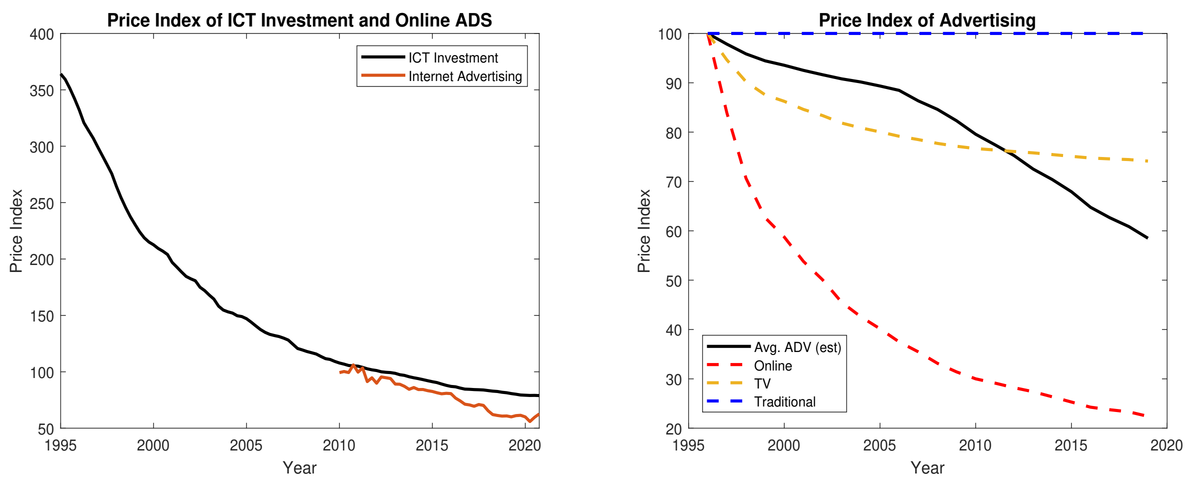


Figure 2: Price Index of Marketing input. Black: Price index of information processing equipment (Data source: NIPA). Red: Price Index of Internet advertising sales (Data source: Bureau of Economic Analysis).

2.2 Firm Marketing Investment

During the same period, the aggregate investment of marketing has increased (Figure 3). The total nominal spending of advertising of Compustat firms increased by 3 times. And taking into account the decline of advertising price, the real expenditures increased by 5 times. Digital technologies lowers the cost of advertising and induces firms to invest more in marketing and selling activities.

Although aggregate amount of marketing investment has risen, large firms increase their investment by more than the small ones. The right panel of Figure 3 shows the average share of marketing investment of the top-half and bottom-half firms in Compustat, weighted by their industry sales. This ratio increases from 68% in 1996 to 80% in 2020.

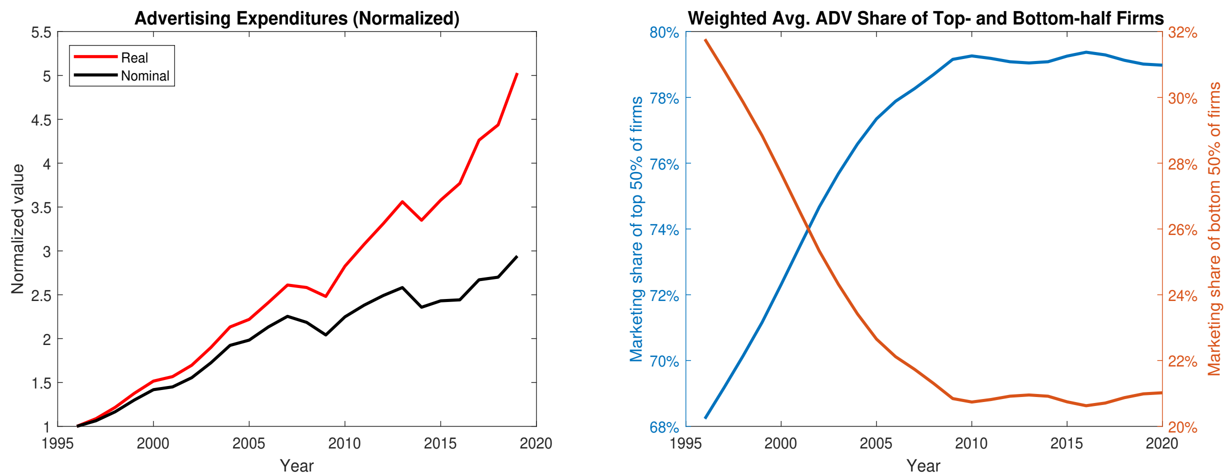


Figure 3: Evolution of Firm Advertising Investment. Left: Aggregate firm advertising expenditure. Black: nominal spending. Red: real spending by dividing the nominal spending with the average price index of advertising. Data source: Compustat variable "XAD". Right: Weighted average difference of advertising share between large and small firms within industries. Use industry sales as weights.

2.3 Market Concentration and Productivity Growth

Coinciding with changes in marketing are the dramatic changes in recent macroeconomic statistics. Figure 4 plots the evolution of average market share of industry leader for all U.S. public companies (left panel), and the total factor productivity growth rate (right panel). Using Compustat data, I show that the largest firm's sales share within 3-digit Standard Industrial Classification (SIC) industries has risen from around 24% in the 1996 to 29.2% in 2019. Annual total factor productivity growth averaged about 1.5% in the late 1990s, and slightly increased to 1.6% in the late 1990s, and has declined all the way to 0.6% in the following 20 years.

3 Model

To investigate the effect of declining marketing cost, or equivalently, increasing marketing efficiency, on firm dynamics and the macroeconomy, I develop an endogenous growth model in which the product market is frictional, and customers are a long-term asset to firms.

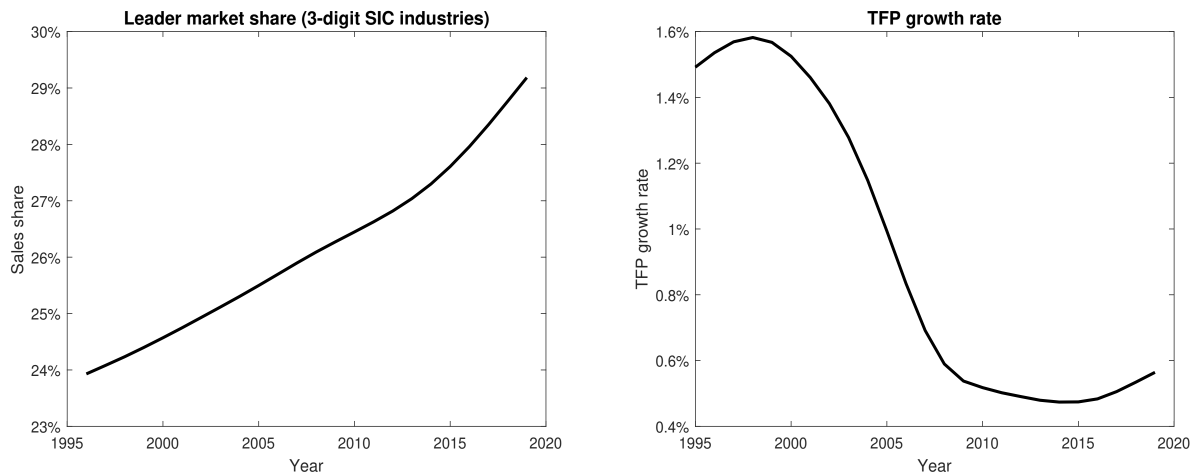
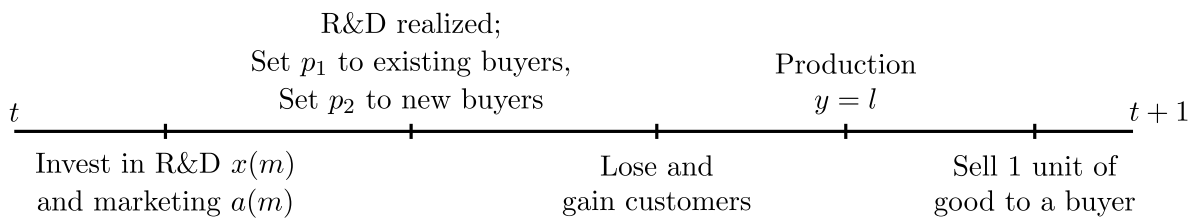


Figure 4: Total factor productivity (TFP) estimates from FRBSF Working Paper 2012-19; Average market share of the largest firm (by sale) in 3-digit SIC industries from Compustat, weighted by industry sales. The plots are smoothed using an HP filter with an annual smoothing parameter of 20 and 200, respectively.

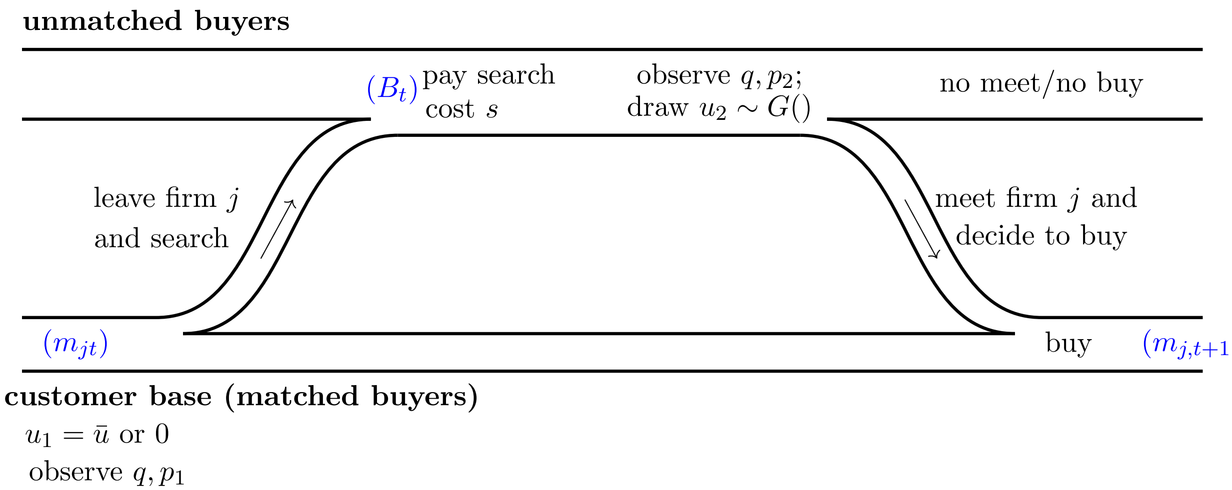
3.1 Model Setup and Timeline

Firms. Time is discrete and lasts forever. There is a measure-one continuum of firms that are risk-neutral and discount the future at rate β. Firms are heterogeneous in their customer base m, which is defined as the mass of customers who bought from that firm in the previous period. The timeline of firms is shown below. At the beginning of period t, firms invest in R&D and marketing, which are denoted by $x(m)$ and $a(m)$, respectively. Next, the outcome of R&D is realized, with a probability $x(m)$ of achieving a successful innovation. Based on quality of products, firms decide on the optimal prices that they will charge to the existing buyers, who bought from the firm last period, p_1, and the price to the new buyers, p_2. Next, customers make decisions on where to buy goods, and firms lose and gain customers. After customers are settled, firms produce using labor with a linear technology $y = l$. Finally, firms sell 1 unit of good to each buyer, and the period ends.



Assume a firm has a constant probability τ of exiting the market each period. Once a firm exits the market it loses all of its customers and has a value of zero. An exiting firm is replaced by a new entrant that starts with a customer base $m_0 = 0$.

Customers. The economy is populated by a continuum of mass N consumers, who are risk-neutral and discount the future at rate β. At the beginning of a period, some of the customers (called matched buyers) are in the customer base of the firm they bought from last period, while others are unmatched buyers. The flow of customers in each period is illustrated as follows:



Since the matched customers consumed at the firm last period, they know the product characteristics, and thus their per-quality utility u_1 on the product is a constant, $\bar{u}$ or 0.[3] After innovation has taken place, the customer observes perfectly the firm's product quality q and price p_1, and decide whether to continue their relationship with the firm.

Some will choose to stay and consume at the matched firm again, while others leave the firm and join the pool of unmatched customers. Denote B_t as the total measure of unmatched customers, which include both of those who start the period unmatched and those who are matched at the beginning but decide to leave the firm. All these people pay a search cost s for a chance to meet a firm in the search. The meeting process will be discussed

[3]The details will be discussed in the "Customer Problem" subsection later.

in detail later. Some of them successfully bump into a firm, where they observe the quality q and price p_2. They also draw their per-quality utility u_2 from a known distribution G, where $\mathbb{E}(u_2) = \bar{u}$. It is reasonable to assume u_1 is less disperse than u_2 because experienced customers know the product well and have smaller variance on utility than customers who first bump into it. Based on the information discovered upon meeting, the customer decides whether to buy the good or not. Those who buy will become part of the customer base of the newly matched firm. A customer can meet at most one firm in a period, and thus those who do not meet a firm, or who meet a firm but decide not to buy will remain unmatched and search again in the next period.

3.2 Investment in Innovation

Firms improve the quality of their current product by investing in R&D. In each period, firms innovate upon a baseline product quality q, which is the average product quality of all firms from last period $\bar{q}_{t-1} = \int q_{j,t-1} dj$. This assumption is based on the idea of "learning and forgetting" (See Benkard (2000)). Firms that used to take lead in previous innovations do not necessarily ace in the new generations of technology development. On the other hand, laggard firms may also benefit from the spillover of knowledge. Therefore, it is reasonable to assume firms innovate based on the average level of technology in the economy. Moreover, if we assume firms innovate based on their own quality q_j, the large firms even have more advantage compared to their smaller counterparts, because the large firms are usually the most productive ones. The main conclusions about the impact of concentration and productivity growth in my model will still hold, and become even stronger due to this additional benefit of being large.

Upon successful innovation, a firm increases product quality from q to $q(1 + \lambda)$, where λ is the quality increment in each round of innovation. When investing, firms choose the probability of R&D success $x \in (0, 1)$. To achieve innovation rate of x, firms employ

$$R(x) = \phi x^{\eta_x} \tag{1}$$

researchers, where $\phi > 0$ and $\eta_x > 1$. So, the total R&D investment cost is convex in the rate of innovation x. In this economy, labor is the numeraire, and therefore the detrended wage w is normalized to 1.

The decision to model R&D as a process of own-product quality improvement is consistent with the findings of Garcia-Macia et al. (2019) that growth mainly occurs through quality improvements rather than new varieties. The specification of cost function reflects the scale effect of R&D. This is built upon the innovation literature that argues ideas are non-rivalrous (Bloom et al. (2020), Jones (1995)) and that intangible inputs (R&D) can be duplicated at close to zero marginal costs (Haskel and Westlake (2017), Hsieh and Rossi-Hansberg (2019)).

3.3 Product Market and Investment in Marketing

Another essential part of firm operations involves building and maintaining customer base, because it usually costs less to keep existing customers than it does to acquire new ones. Firms that care about customer loyalty has a key measure to track: customer lifetime value (CLV). CLV is the value of a customer to a company, not just on a purchase-by-purchase basis but over the entire customer relationship.

To capture this long-term value of customer relationship, I introduce search frictions into the product market of my model, in which it takes time for an unmatched customer to find a firm. Therefore firms also invest substantially in marketing[4], which helps to increase product exposure to potential buyers. An unmatched customer does not know the existence of a product until they see an advertisement, meet a salesperson, or run into the product

[4]The marketing investment in this paper refers to the broadly defined investment to promote product exposure, including expenditures on purchasing product space, expanding distributional channels, hiring sales people, and posting advertisement. Many of these promotional activities can be done online or advanced by Internet technologies, and therefore experiences a cost reduction.

in a store of the firm. As a result, marketing investment allows firms to enhance demand at the extensive margin. The advertisement describes the product's characteristics, quality and price, which are important factors for customers to decide whether to buy or not after seeing the ads, and in this way marketing also influences demand at the intensive margin.

Marketing investment also uses labor. Assume decreasing returns in advertising: to post a units of advertisement requires

$$I(a) = \psi a^{\eta_a} \tag{2}$$

of labor, where $\psi > 0$, and $\eta_a > 1$ to have cost convexity.

Figure 5 depicts the matching process between unmatched customers and firms. Firms are indexed by j. Let $A = \int a_j dj$ denote the aggregate amount of advertising from all firms, and B the measure of potential buyers (unmatched), then the measure of total match H is given by

$$H(B, A) = \xi B^{\gamma} A^{1-\gamma}, \tag{3}$$

where $\xi > 0$ and $\gamma \in (0, 1)$. It is convenient to define the average queue length $\theta = B/A$. This allows writing the probability a customer succeeds in matching as a decreasing function of the queue length $H(1, 1/\theta)$. The more aggregate ads are there, the more likely that an unmatched customer is exposed to an ad and become aware of a product she did not know before.

At the firm level, let h_j be the measure of customers that firm j is matched with, and κ_j the share of newly matched customers visiting the firm ($\sum \kappa_j = 1$). Apparently, the more ads a firm posts, the more likely that customers see its ads and miss other firms' ads. Thus, it is reasonable to assume $\kappa_j = a_j/A$, then

$$h_j = \kappa_j H = \frac{\xi B^{\gamma}}{A^{\gamma}} \cdot a_j. \tag{4}$$

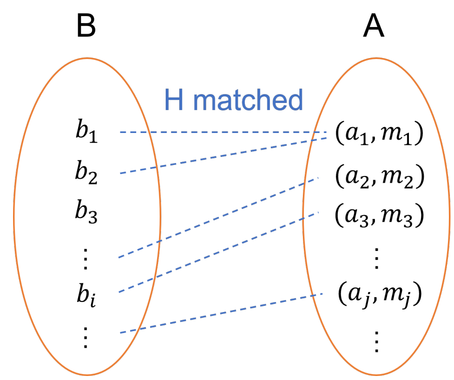


Figure 5: Illustration of the Formation of Firm-Customer Relationship.

In this specification, h_j is increasing in (i) the aggregate amount of ads A, which increases the total measure of matched customers; and (ii) the individual amount of ads posted by the firm relative to other firms.

3.4 Customer Problem

3.4.1 Matched Customers

Assume in each period, a fraction δ of the customer base leaves the matched firm for random reasons, such as moving to a different city. For the rest of matched customers, searching another firm is an outside option of continuing the customer relationship. Assume a fraction $(1 - \alpha_1)$ of customers care about quality improvement, and will leave the firm if it cannot upgrade the product because of R&D failure (Think about the "techies" who look for fancy technologies especially when they purchasing e-products). These people have $u_1 = 0$ if R&D fails. The rest α_1 share of customers are "loyal" ones who would consider continuing their customer relationship even if the firm fails to make innovations. After innovation is realized, the "techies" in R&D success and the loyal customers have per-quality uility $u_1 = \bar{u}$, and will compare the outside option (expected value of search) U_s and the value of continuing the customer relationship at the firm U_0. The latter depends on product quality q, the price charged by the current firm p_1, and the utility u_1.

Due to frictions in the product market, customer relationships become long-term in nature. Unless there is random separation (with probability δ) or R&D failure, the relationship lasts as long as $U_0 \geq U_s$. Since product appeal u_1 is the same for all the existing customers, they either all stay ($U_0 \geq U_s$) or all leave ($U_0 < U_s$). To maximize profits on existing customers, firms will charge p_1 to make them just indifferent between staying and searching, $U_0 = U_s$, extracting their entire surplus of the match. A customer's value of staying with the firm U_0 is given by

$$U_0 = \bar{u}q_j - p_1 + \beta U_0', \quad \text{where } q_j = \begin{cases} \bar{q}(1+\lambda), & \text{if R\&D succeeds} \\ \bar{q}, & \text{if R\&D fails} \end{cases} \tag{5}$$

and the price is $p_1 = \bar{u}q_j - U_0 + \beta U_0'$.

3.4.2 Unmatched Customers

Unmatched customers pay a search cost s for the chance of meeting a firm. Upon matching with the firm, she observes the quality q and price p_2. Similarly to the previous subsection, I assume a fraction $(1 - \alpha_2)$ of customers care about quality improvement, and will have $u_2 = 0$ if the firm fails in their R&D effort. The newly matched customers draw a per-quality utility u_2 from a distribution $G(\cdot)$. Firms know the distribution G, but they do not know each customer's utility u_2, and thus make a take-it-or-leave-it offer p_2 to all the customers.

If an unmatched customer meets a firm of size-m_j after searching, her utility of consuming the product will be $u_2 q_j$. She will buy the good only if $u_2 q_j \geq p_2(m_j)$. Thus, a customer's value function of meeting with a size-m firm is

$$U_s(m_j) = \int_{\frac{p_2(m)}{q_j}} \left[u_2 q_j - p_2(m_j)\right] dG(u_2) + \beta U_0' \tag{6}$$

Thus, the expected value of searching is given by

$$U_s = -s + \sum_j \left[\frac{h(m_j)}{B} U_s(m_j) \right] + \left(1 - \frac{H}{B} \right) \beta U_0' \tag{7}$$

Indifference condition requires $U_0 = U_s$.

3.5 Firm Problem

3.5.1 Value Function

Each period, firms make decisions on the R&D investment x, marketing investment a, prices to existing customers upon R&D success p_1^{rd} and failure p_1^{nrd}, and prices to new customers p_2, to maximize their value function, which reads

$$\begin{aligned} V(m) = \max_{p_1^{(n)rd}, p_2, a, x} \; & x \left\{ (p_1^{rd} - w)(1-\delta)m + (p_2 - w)y^{rd} + \tilde{\beta} V(m'^{rd}) \right\} \\ & + (1-x) \left\{ (p_1^{nrd} - w)\alpha_1 (1-\delta)m + (p_2 - w)\alpha_2 y^{nrd} + \tilde{\beta} V(m'^{nrd}) \right\} \\ & - \psi a^{\eta_a} w - \phi x^{\eta_x} w, \end{aligned} \tag{8}$$

$$s.t. \;\; p_1^{rd} = \bar{u}(1+\lambda) - \hat{U}_0, \quad p_1^{nrd} = \bar{u} - \hat{U}_0, \tag{9}$$

$$y^{rd} = \left[1 - G\left(\frac{p_2}{1+\lambda} \right) \right] \frac{\xi B^\gamma}{A^\gamma} a, \tag{10}$$

$$m'^{rd} = (1-\delta)m + y^{rd}, \tag{11}$$

$$y^{nrd} = \left[1 - G(p_2) \right] \frac{\xi B^\gamma}{A^\gamma} a, \tag{12}$$

$$m'^{nrd} = (1-\delta)\alpha_1 m + \alpha_2 y^{nrd}. \tag{13}$$

3.6 Equilibrium

To close the model, the total measure of consumers is equal to the total population N:

$$\int \left[x(m)m + (1 - x(m))\alpha_1 m \right] (1-\delta) dF(m) + B = N \tag{14}$$

Firms take the aggregate advertising level A and TFP growth rate g as given, and in equilibrium A is equal to the sum of advertising intensity of each firm, and g is the average of R&D intensity multiplied by λ:

$$\int a(m)dF(m) = A \tag{15}$$

$$\lambda \int x(m)dF(m) = g \tag{16}$$

Definition 1. A **balanced growth path equilibrium** is for every t, firm allocations of innovation and marketing $\{x(m), a(m)\}$, prices $\{p_1^{rd}, p_1^{nrd}, p_2^{rd}(m), p_2^{nrd}(m)\}$, firm value function $V(m)$, customer value functions U_0 and U_s, firm size distribution $F(m)$, aggregate measure of marketing and unmatched customers $\{A, B\}$, and the growth rate g such that

1. Firm decision rules and value function solve the problem (8);
2. Customers choose where to buy to maximize lifetime utility and the existing customers who decide to consume again are indifferent between searching and staying;
3. Customer base distribution $F(m)$ is stationary;
4. N, A, g satisfy the clearing conditions (14), (15) and (16).

4 Model Mechanism

This section analyzes the main mechanisms of the model. I show in Section 3 that firms with more customers invest more in both R&D and marketing. Based on these findings, I discuss in this section how and why the large and small firms react differently to a higher marketing efficiency, and further explain how the distinct responses lead to rising market concentration and lower productivity growth rate.

4.1 Optimal choices

Detrend the value function by dividing baseline quality q on both sides. Denote $\tilde{V}(m) = \frac{V(m;q)}{q}$, $\tilde{p}_i = \frac{p_i}{q}$, $\tilde{w} = \frac{w}{q}$, and $\tilde{\beta} = \beta(1-\tau)(1+g)$.

Firms set prices on new customers p_2 according to

$$1 - G\left(\frac{\tilde{p}_2}{1+\lambda}\right) = \left[\tilde{p}_2 - \tilde{w} + \tilde{\beta}(1-\delta)\tilde{V}'(m'^{rd})\right] G'\left(\frac{\tilde{p}_2}{1+\lambda}\right)\frac{1}{1+\lambda} \tag{17}$$

$$1 - G\left(\tilde{p}_2\right) = \left[\tilde{p}_2 - \tilde{w} + \tilde{\beta}(1-\delta)\tilde{V}'(m'^{nrd})\right] G'\left(\tilde{p}_2\right) \tag{18}$$

Increasing price $\tilde{p_2}$ increases the per unit profit by the same amount, and thus the marginal benefit is equal to the measure of new customers (recall that each customer buys at most one unit of good). The marginal cost is that it reduces the measure of new customers willing to consume, which has both a direct effect of lowering profits in the current period, and also a long-term effect through the slower accumulation of customer base.

Proposition 1. *When R&D is successful, the firm charges a higher price than when R&D fails.* $\tilde{p}_1^{rd} > \tilde{p}_1^{nrd}$.

Proof: Appendix A. Proposition 1 is intuitive because R&D success brings higher product quality, and increases customer's utility of consuming the good. Therefore, customers are willing to pay more for higher quality.

The firm's optimal innovation rate is the solution to the first order condition of the value function with respect to x,

$$\begin{aligned}\phi\eta_x x^{\eta_x-1}\tilde{w} = &\left\{(\tilde{p}_1^{rd} - \tilde{w}) - (\tilde{p}_1^{nrd} - \tilde{w})\alpha_1\right\}(1-\delta)m \\ &+ \left\{(\tilde{p}_2 - \tilde{w})\left[1 - G\left(\frac{\tilde{p}_2}{1+\lambda}\right)\right] - (\tilde{p}_2 - \tilde{w})\left[1 - G\left(\tilde{p}_2\right)\right]\alpha_2\right\}\frac{\xi B^\gamma}{A^\gamma}a \\ &+ \tilde{\beta}\tilde{V}(m'^{rd}) - \tilde{\beta}\tilde{V}(m'^{nrd})\end{aligned} \tag{19}$$

The marginal benefit of innovation is composed of three parts. The first line of equation

(19) is the increment in the profits on existing customers due to higher R&D intensity. When R&D is successful, firms charge higher unit price ($\tilde{p}_1^{rd}$ vs. $\tilde{p}_1^{nrd}$) and also sell to more customers (m vs. $\alpha_1 m$). The second line is the increment in profits on new customers. The last term is the difference in the future value of a firm between R&D success and failure. Better product quality is critical to expanding the customer base, for it not only prevents more customers from leaving but also turns more visiting customers into actual buyers, and therefore is likely to generates a higher customer base $m'^{rd} > m'^{nrd}$.

As for the choice of advertising, the first order condition with respect to advertising intensity a reads

$$\begin{aligned}\psi\eta_a a^{\eta_a-1} =& x\left[\tilde{p}_2 - \tilde{w} + \tilde{\beta}(1-\delta)\tilde{V}'(m'^{rd})\right]\left[1 - G\left(\frac{\tilde{p}_2}{1+\lambda}\right)\right]\frac{\xi B^\gamma}{A^\gamma} \\ &+ (1-x)\left[(\tilde{p}_2 - \tilde{w}) + \tilde{\beta}(1-\delta)\tilde{V}'(m'^{nrd})\right]\alpha_2\left[1 - G\left(\tilde{p}_2\right)\right]\frac{\xi B^\gamma}{A^\gamma}\end{aligned} \tag{20}$$

One additional unit of advertising helps acquire a mass of $\left[1 - G\left(\frac{\tilde{p}_2}{1+\lambda}\right)\right]\frac{\xi B^\gamma}{A^\gamma}$ customers upon R&D success and $\alpha_2\left[1 - G\left(\tilde{p}_2\right)\right]\frac{\xi B^\gamma}{A^\gamma}$ customers upon R&D failure. The new customers bring more profits in the current period and increase long-term value by enlarging the customer base.

Denote the right hand side of the value function as $H(m, x, a)$.

Proposition 2. **(Supermodularity)** *When $\frac{\partial p_2}{\partial m}$ is small, $H(m, x, a)$ is supermodular. Firms with a larger customer base m invest more in innovation x and marketing a.*

Proof: Appendix A. Supermodularity is easily checked numerically and appears to hold in all the calibrations. Intuitively, the profit on existing customers is proportional to the customer base. A successful R&D allows the firm to charge a higher price and retain more existing customers than in the case of R&D failure. Therefore, the marginal benefit of innovation is increasing in the customer base m. However, due to the scale effect of R&D, the innovation outcome can be applied to all goods at close to zero marginal cost. Therefore, a larger firm chooses higher innovation investment to take advantage of their large customer base.

As for marketing investment, the marginal benefit of marketing is to obtain $\frac{\xi B^\gamma}{A^\gamma}$ measure of new customers. Each of them generates a current profit for the firm, and increases the lifetime value of a larger customer base m'. The convexity of function V leads to larger marginal benefit of marketing for large firms. Moreover, large firm's higher likelihood of high quality also enables the firm to charge higher prices and convert more matched customers to really buying the good, than in the case of R&D failure. In this sense, innovation and marketing is complementary. This explains why large firms also invest more in marketing a.

4.2 Impact on Market Concentration

The process of customer base building falls into a positive feedback loop as follows:

large customer base	$\Rightarrow$	high R&D	$\Rightarrow$	high ADV
$\Uparrow$ $m' = (1-\delta)m + y$				$\Downarrow$ $h = \alpha_2 \frac{\xi B^\gamma}{A^\gamma} a$
more eventual purchases		$y = [1 - G(u_2^*)]h$ $\Leftarrow$		more matches

Firms with a large customer base m invest more in both R&D x and marketing a. and more marketing allows large firms to acquire a larger share of customers during the search process. This advantage of large firms is further amplified by their higher R&D, because better quality can convert more matches into purchases. The new customers add to the customer base next period, and the loop starts again from a larger customer base next period. This explains why there is distinction between large and small firm's customer base.

Then consider the effect of a shock on the cost of marketing, reflected by a reduction in ψ, which means firms now are able to produce the same amount of ads as before at a lower cost. On a balanced growth path, all firms equate their marginal cost of marketing

$MC_a = \psi \eta_a a^{\eta_a - 1}$ and the marginal value of marketing,

$$\begin{aligned} MV_a = & x \left[\tilde{p}_2 - \tilde{w} + \tilde{\beta}(1-\delta)\tilde{V}'(m'^{rd}) \right] \left[1 - G\left(\frac{\tilde{p}_2}{1+\lambda} \right) \right] \frac{\xi B^\gamma}{A^\gamma} \\ & + (1-x) \left[(\tilde{p}_2 - \tilde{w}) + \tilde{\beta}(1-\delta)\tilde{V}'(m'^{nrd}) \right] \alpha_2 \left[1 - G(\tilde{p}_2) \right] \frac{\xi B^\gamma}{A^\gamma} \end{aligned} \tag{21}$$

which is larger for firms with more customers because of their higher innovation intensity. When ψ declines, there is a sudden drop in MC_a but the marginal value MV_a does not change immediately because it is not directly affected by ψ. At this point, all firms have incentives to raise their marketing intensity a to match MC_a with MV_a. Assume ψ declines from ψ_1 to ψ_2, and firm j increases its marketing intensity from a_{1j} to a_{2j}. Then $\psi_1 \eta_a a_{1j}^{\eta_a - 1} = MV_{a,j} = \psi_2 \eta_a a_{2j}^{\eta_a - 1}$, which implies $\frac{a_{2j}}{a_{1j}} = (\frac{\psi_1}{\psi_2})^{\frac{1}{\eta_a - 1}}$. So, every firm increases its marketing intensity by the same proportion. This increases the aggregate advertising intensity $A = \int a_j dj$, and therefore enables more customers to get matched with firms in the search. However, the relative share of customers that each firm meets with remains the same as before ($a_{1j}/A_1 = a_{2j}/A_2$), because all firms increase their marketing intensity by the same proportion.

Although each firm obtains the same measure of more matched customers during search and matching, large firms turn more visiting customers into eventual buyers with their advantage in R&D. Their higher likelihood of R&D success attracts more techies to stay with them upon meeting, and the higher product quality also appeals to a wider range of customers. As a result, the variation of innovation across firm size generates different pass-through rate from potential buyers to actual new customers, and thus widens the gap in customer base between large and small firms.

As was discussed in Section 3, the innovation investment is positively related to firm size. Although all firms have incentives to increase R&D investment, the enlarging difference in customer base m will result in a larger gap between large and small firms in terms of innovation intensity x as well as prices p_2. According to equation (21), all of these factors are important determinants of the marginal value of marketing MV_a, and thus exert an indirect effect on marketing intensity a. With a larger customer base, higher innovation intensity and higher prices, large firms elevate their MV_a by more than their smaller counterparts, and this further amplifies the difference in marketing intensity between large and small firms. This unequal indirect effect on marketing reallocates more customers to large firms during the search, and creates a positive feedback loop on the gap in customer base, R&D and prices through equation (20). The spiral is further amplified through the long-term customer relationship, and eventually gives rise to superstar firms that take up a substantial market share.

4.3 Impact on Productivity Growth

The impact on aggregate productivity growth rate is more complicated because there are offsetting effects on firm innovation decisions. In response to a rise in marketing efficiency (lower ψ), each firm increases its marketing intensity which brings the firm more new customers in the search. With a larger customer base, all firms have incentives to increase their innovation intensity x. Nevertheless, due to the enlarging gap of marketing intensity across firm size as aforementioned, relatively more new customers are exposed to large firms' advertisement than in the higher-ψ economy. In other words, the largest firm's share of new customers in the match is higher while that of the smallest firm is lower than before. Moreover, there is a limited capacity of consumption because of the decreasing return of advertising in the matching technology $Y = \xi B^{\gamma} A^{1-\gamma}$. Each customer purchases at most one unit of product in a period. Even if every unmatched customer gets matched during the search, the total purchase would be N units. Under this circumstance, the uneven increase in marketing creates congestion in the product market, where the smallest firms might end up matching with fewer new customers than before. A smaller customer base discourages firms from innovation, and the resulting lower product quality further reduces the incentives of advertising and slows down the accumulation of customer base, which puts the small firms at a long-term disadvantage compared to the large ones. The lower R&D incentives by small firms negatively affects the productivity growth rate.

The analysis above indicates opposite forces on productivity growth: large firms increase their innovation intensity while small firms tend to invest less in R&D. In addition, there is also a third effect on innovation decisions through the labor market. Higher marketing efficiency facilitates matching during product search and generate more purchases. The boosted labor demand for production increases wages and also shifts more labor from R&D to production, both of which depress firm R&D investment. Due to these offsetting factors, the overall impact on productivity growth is ambiguous. In the calibrated model, the latter forces dominate and firms on average choose a lower rate of innovation.

4.4 Empirical Validations

The model predictions of enlarging gap in marketing and innovations are consistent with the observed trends in firm investment data. The left panel of Figure 6 shows the average share of the marketing investment of the top-half and bottom-half firms in Compustat, weighted by their industry sales. This ratio increases from 68% in 1996 to 80% in 2020.

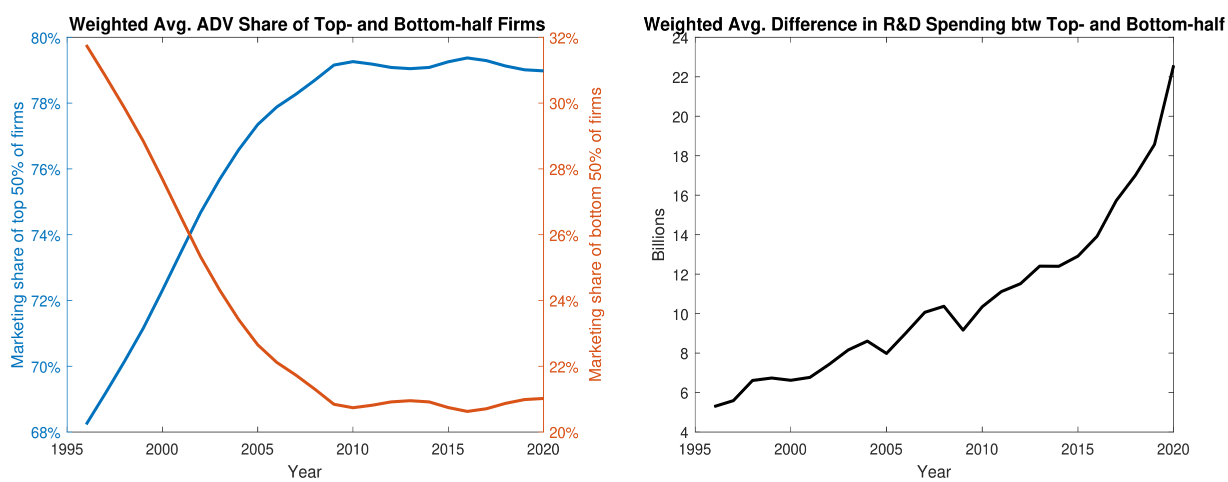


Figure 6: Weighted Average Difference (by industry sales) of Advertising Share and R&D Intensity Between Large and Small Firms

The right panel shows the difference between large and small firms' R&D expenditures is also widened in the past 20 years, validating the predictions that large firms disproportionately increase their innovation and marketing investment compared to smaller ones.

5 Quantification

In this section, I calibrate the model on the balanced growth path by matching various moments for the U.S. economy in the late 1990s (1996-1999), using data on all U.S. non-financial listed firms from Compustat as well as aggregate moments. Using this initial calibration I describe firms' pricing and innovation strategies to develop intuition about the model. I then calibrate the rise in marketing efficiency (the reduction in ψ) to fully capture the change in aggregate real marketing investment in the data.

Using the calibrated model, I conduct three exercises to infer changes to the economy between late 1990s and today. The first analysis displays how firm investments in R&D and marketing change in respond to the rise in marketing efficiency. Specifically, I show a sharp contrast between large and small firms. Second, I quantify the aggregate effects of advertising shock on innovation, economic growth and market concentration on the balanced growth path. Finally, I demonstrate the dynamics of transition from the low to high marketing equilibrium.

5.1 Calibration

The model is calibrated at an annual frequency. There are three parameters calibrated externally. I calibrate the curvature of R&D (η_x) to 2 following the literature of Acemoglu et al. (2018) and Akcigit and Kerr (2018). I calibrate the cost elasticity of advertising (η_a) to 2. This is a key parameter because it determines the concavity of the return to advertising. If marketing activities concentrate among fewer firms, the fact that $\eta_a > 1$ implies that the average effect of these investments on growth is lower. I will do a robustness check of using other values of η_a in the Appendix. The discount rate β is set to 0.95.

The statistics for firm entry rate and residents moving rate can be found directly. They are

9.6% and 11.2% respectively. The remaining seven parameters are estimated using indirect inference by matching moments from the U.S. Compustat data. I use the Genetic Algorithm to choose combinations of parameters within broad bounds on their possible values. Using the equilibrium values for innovation and entry rates, the firm-size distribution, rates of creative destruction and aggregate quantities such as the efficiency wedge, wages and output, I simulate the economy for 20,000 firms until the distribution of has converged, and simulate data for five more years to collect moments on the simulated sample. The Genetic Algorithm then updates the combinations of parameters based on a comparison of the theoretical and data moments along the following objective function:

$$\min \sum_{k=1}^{7} \frac{|\text{model}_k - \text{data}_k|}{(|\text{model}_k| + |\text{data}_k|)\, 0.5} \Omega_k \tag{22}$$

where the weights $\Omega = 2$ for TFP growth rate and leader market share, and $\Omega = 1$ for the rest of targeted moments.

Targeted Moments	Model Value	Data Value
TFP growth rate	1.52%	1.51%
Leader market share (3-digit SIC)	24.04%	24.04%
Avg. R&D/Advertising	4.03	3.85
Avg. R&D/Sales	4.79%	4.28%
Avg. customer turnover rate	14.6%	10%-20%
Avg. profit share	5.24%	5.31%
Labor share of income	0.69	0.67

Table 1: Model fit for targeted moments from calibration of 8 parameters for the late 1990s.

Table 1 display the targeted moments in model and data. The generalized model does not yield an analytical solution, and thus we cannot express the targeted moments in this form. Table 2 presents an overview of the calibrated parameters. In fact, each targeted moments is jointly affected by multiple parameters.

Then I calibrate the decline in marketing cost ψ from 1996 to 2019 to fully capture the

Parameter	Description	Method	Value
β	Discount factor	External	0.95
η_a	Cost elasticity of advertising	External	2
η_x	Cost elasticity of R&D	External	2
$\bar{u}$	Product appeal	TFP growth rate	2.5
ξ	Matching function coefficient	Leader market share	0.95
ψ	Cost of advertising	Avg. R&D/Advertising	4.98
ϕ	Cost of R&D	Avg. R&D/Sales	2.0
α	Customer loyalty	Customer turnover rate	0.4
N	Total population	Labor share of income	0.499
γ	Matching function elasticity	Avg. profit share	0.5
δ	customer depreciation rate	Share of residents moving	0.008
τ	Firm exit rate	BDS firm entry rate	0.096

Table 2: Overview of Estimated Parameters.

increase in real aggregate marketing expenditures.

As aforementioned in Section 2, digital technologies lowers the cost of advertising and induces firms to invest more in marketing and selling activities. From 1996 to 2019, the total nominal spending of advertising of Compustat firms increased by 3 times. And taking into account the decline of advertising price, the real expenditures increased by 5 times, as is shown in Figure 7. This requires efficiency parameter of advertising ψ to decrease from 4.98 to 0.0218 in the quantitative model.

5.2 Results: on the Balanced Growth Path

The effect of increasing marketing efficiency as reflected by the reduction of ψ is summarized in Table 3. It presents the variables of interest in differences from the original balanced growth path. The change in real expenditures on advertising is targeted. I compute the changes in data that are explained by a change in ψ as follows:

$$M_j\left(\boldsymbol{\theta}_{1990s}, \psi_{low}\right) - M_j\left(\boldsymbol{\theta}_{1990s}, \psi_{high}\right) \tag{23}$$

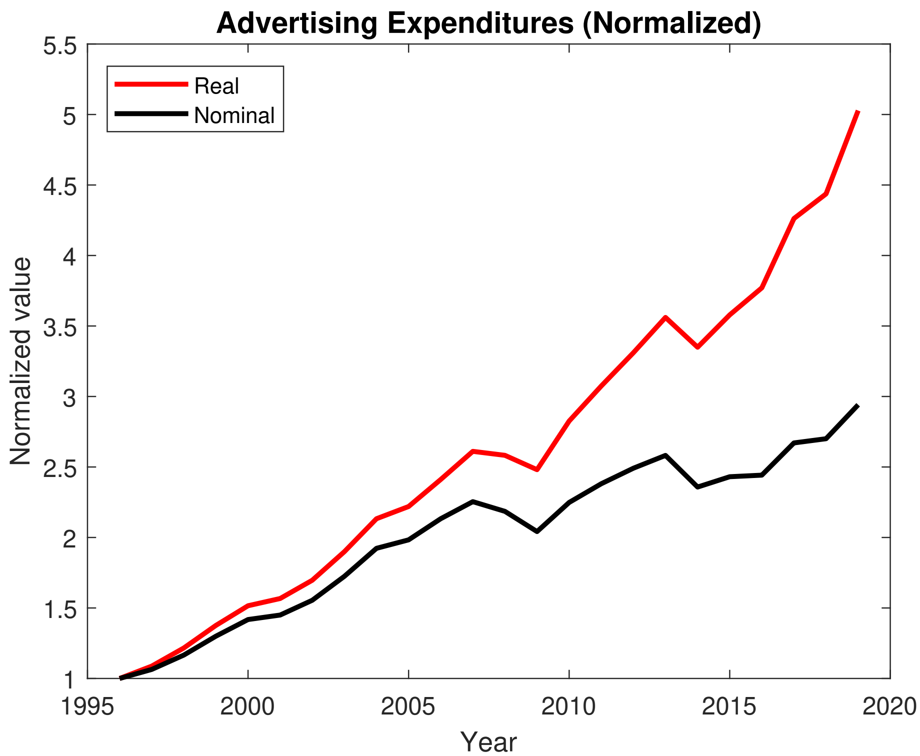


Figure 7: Evolution of Firm Aggregate Advertising Investment. Black: nominal spending. Red: real spending by dividing the nominal spending with the average price index of advertising. Data source: Compustat variable "XAD".

where M_j is moment j in the model BGP with the other parameters θ held fixed at their estimated 1990s values.

	Targeted	Δ Data	Δ Model
Expenditures on advertising	Yes	5.08	5.08
Productivity growth rate	No	-0.98 pp	-0.31 pp
Leader market share	No	5.15 pp	4.25 pp

Table 3: Balanced Growth Path Change due to Plummet of Marketing Cost

The comparison between the new and old balances growth path shows that the change in marketing cost predicts a 0.37 percentage-point drop in productivity growth, and an increase of 4.25pp for the top 1% of firms' market share[5]. As a result, the model can explain around

[5]There is around 100 firms within each SIC-3 industry, so the largest firm is approximately the top 1% of firms.

1/3 of the decline in productivity growth and about 83% of rise in aggregate concentration. It therefore seems that technological change in marketing are not responsible for most of the slowdown of growth in the US. This is reasonable because the technological progress in marketing increases production and consumption, which boosts firms' incentives to make innovations.

5.3 Results: Composition Effect in Rising Concentration

5.3.1 Distribution of Customer Base

Each period, the customer base is accumulated by adding new customers to the original customer base, adjusted for random separations. Lower ψ saves advertising cost for all firms, but more so for large firms because of their higher innovation investment and the complementarity between product quality and measure of new matches. Therefore, more new customers are reallocated to the large firms. This is graphically shown in the red components of the bar plot in Figure 8, where as ψ drops, both the top 10% and bottom 90% of firms have an increasing number of new buyers, but the former has a much higher fraction than the smaller ones.

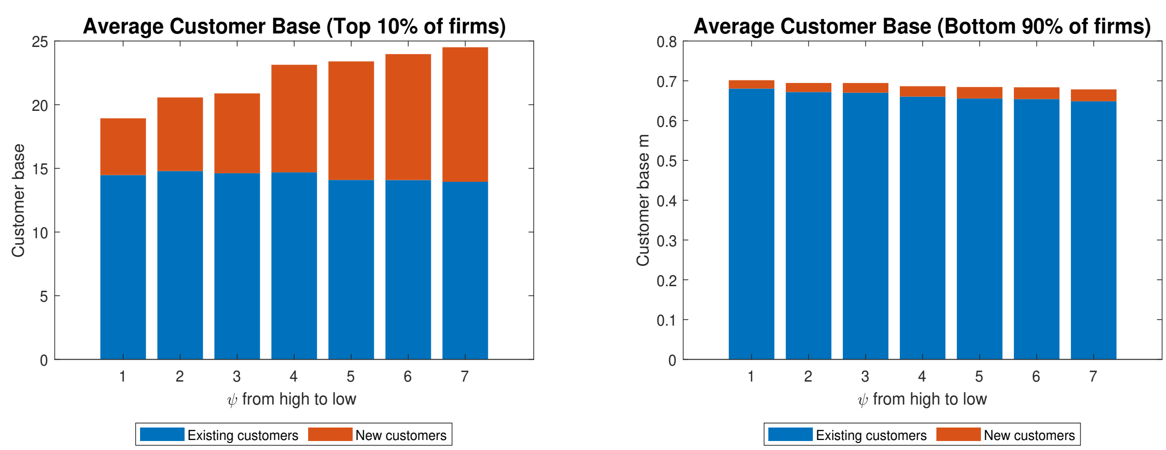


Figure 8: Changes in Customer Base of Large and Small Firms with the Decline of ψ

The blue components in Figure 8 display the measure of existing customers in customer base formation. A lower ψ has two opposite effects on top firms' existing customers. On one hand, easier matches due to higher marketing intensity increases the option value of searching and thus attract more matched customers to leave the previously matched firms. This reduces large firms' measure of existing customers. On the other hand, as ψ drops, large firms have an increasing number of new customers each period and are more likely to reach a larger customer base. The left panel of Figure 8 indicates that in the calibrated model, the former dominates the latter and the size of existing customers shrinks a little bit. This makes sense because the initial tech shock happens to advertising which directly shifts customers from staying to searching. In spite of this, the massive increase in new customers outweigh the minor drop in existing ones, so the total customer base of large firms are getting bigger with the enhanced marketing efficiency. As for the small firms, the two effects are in the same direction: more customers choose to search rather than continue the customer relationship, and fewer new customers bump into their advertisement, both of which lead to shrinking customer base.

Figure 9 demonstrate the distributions of firms' sales revenue for the top 10% of firms on initial (left panel) and new balanced growth paths (right panel)[6]. The efficiency change in marketing gives rise to a fatter-tailed distribution. The economy now is populated with more superstar firms with larger number of customers and higher revenues.

In addition to the customer base, changes in prices can affect firms' market share as well. However, a lot of studies show that the rising market share of large firms is not driven by a larger gap in prices or markups between large and small firms. Rather, it is the composition effect (reallocation of resources towards large productive firms) that explains most of the rising superstar firms. For example, Baqaee and Farhi (2020) and De Loecker et al. (2020) find that average markups have been increasing primarily due to a between-firm composition effect, whereby firms with high markups have been getting larger, and not to a within-firm

[6]It is not easy to compare the two BGP by viewing the entire distribution because the distribution is highly fat-tailed and the main difference is on the right tail. The entire distribution can be seen in Appendix A.

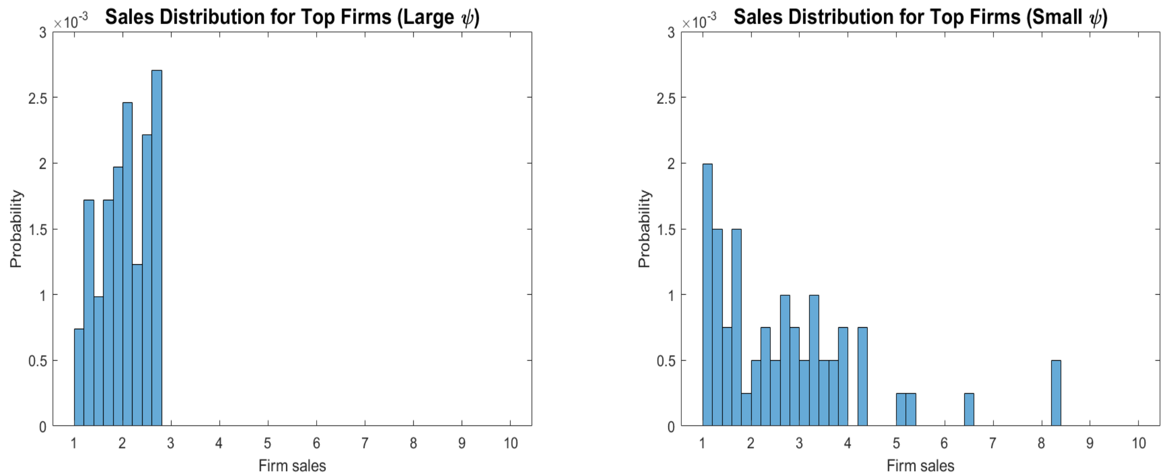


Figure 9: Sales Distribution for Initial and New Steady States.

increase in markups.

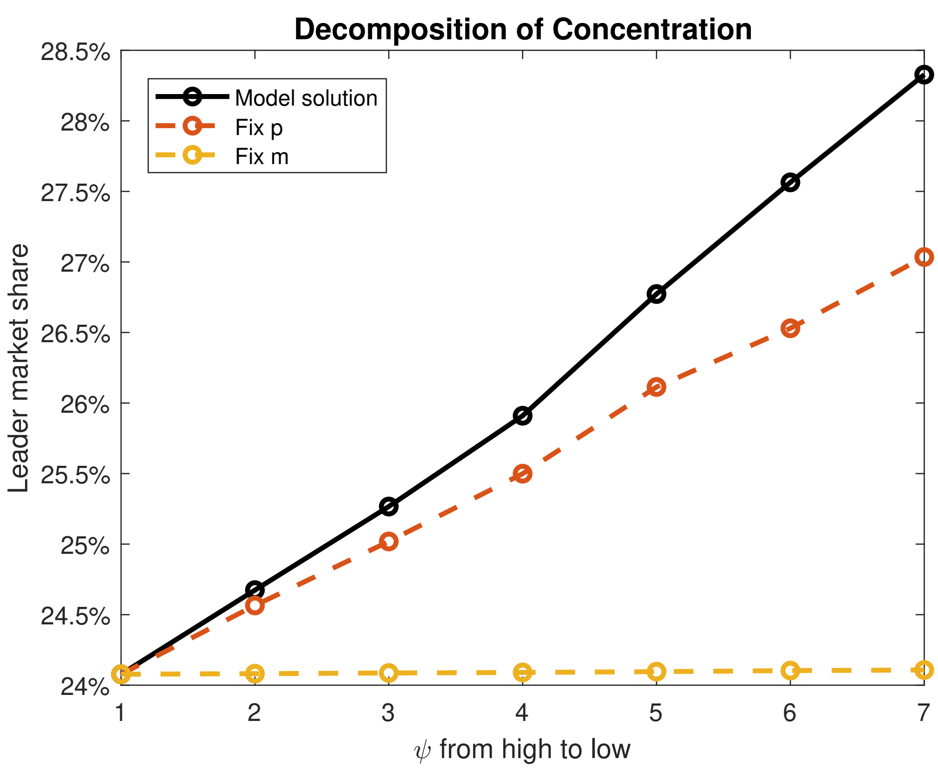


Figure 10: Decomposition of Concentration.

To understand whether the enlarging gap between large and small firms' customer base is a main driver of rising concentration, I conduct a counterfactual exercise to decompose the effect on rising leader market share into customer base (m) and prices (p_1, p_2). I study the effect for seven different values of marketing cost ψ. $\psi = 1$ corresponds to the initial balanced growth path where $\psi = 4.98$; and $\psi = 7$ corresponds to the new balanced growth path with $\psi = 0.0218$. Figure 10 shows the decomposition. The red dashed curve is obtained by fixing the prices charged to the same level as on the $\psi = 1$ balanced growth path, while allow firm customer base to change as on the new balanced growth paths. As ψ declines, the enlarging gap in customer base raises the market share of large firms. On the new balanced growth path, it explains 2.96 percentage-point rise in top 1% firms' sales share, which accounts for 70% of the total rise predicted by the model (the black curve). Next, fix the customer base as the $\psi = 1$ BGP and change prices, and the results are shown by the almost flat yellow dashed curve, which only explains a rise of 0.3 percentage-point out of the total 4.25 percentage-point. This suggests that the market concentration is mainly driven by the enlarging gap in customer base between large and small firms.

5.4 Results: Productivity Growth

Next, I study the quantitative effect of lower marketing cost on productivity growth rate. As was discussed in Section 4, the increase in marketing activities facilitates more matches, which raises the aggregate output quantity and has a positive effect on productivity growth. This is the "level effect". On the other hand, firm investment decisions on R&D have also been modified. Small firms invest less in innovations because of a lower price and smaller customer base. Large firms now obtain more customers in total. However, as marketing becomes cheaper, demand from new customers become more "elastic" because they can more easily find another firm in the economy, firms have to lower prices to retain more customers. Hence, there is ambiguous effect on their innovation. The changes in the rate of quality improvement is the "growth effect".

In Figure 11, the red curve accounts for a "level effect" of marketing: as marketing becomes cheaper (ψ decreases), firms expand demand contemporaneously, which increases the perceived utility derived from aggregate consumption. Compared to the low-marketing equi-

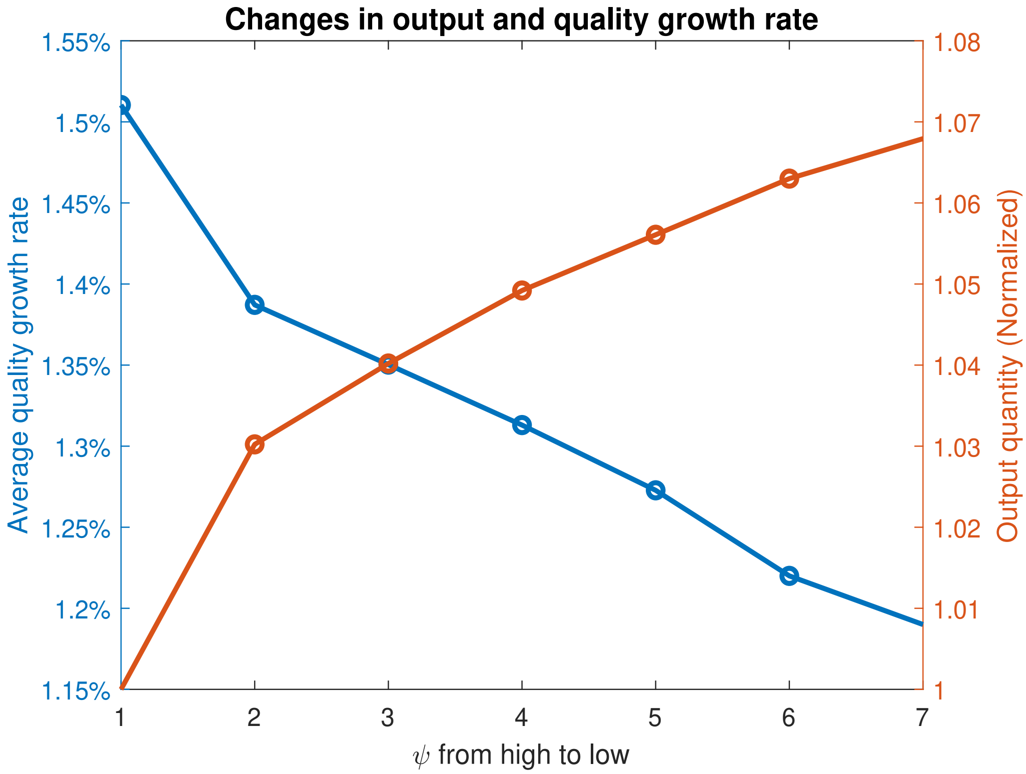


Figure 11: Level Effect and Growth Effect in Productivity Growth.

librium, output in the new economy increases by 6.79%. Additionally, marketing critically shapes firm R&D incentives and, therefore, has an indirect effect on growth (the "growth effect"). Although it is theoretically ambiguous whether the large firms will increase or decrease innovation, the calibrated model demonstrates a declining average innovation growth rate from 1.51% to 1.19%, implying the the negative effect of small firms and price drop dominates the rise of customer base in large firms.

5.5 Results: Transition Dynamics

The analysis thus far has studied the effect of a lower marketing cost along the balanced growth path. This section shows that short-term dynamics are substantially different. To quantify the transition path, I numerically solve for the path of top 1% firms' market share and productivity growth[7]. To understand the transition dynamics from the low-marketing

[7]The computational algorithm is described in Appendix B.

to high-marketing equilibrium, I consider a permanent decrease in the cost parameter ψ as documented in the previous section. I assume the transition takes 120 years, with the marketing efficiency $1/\psi$ rising at a steady pace.

Over the transition path, the market leader share (left panel of Figure 16) increases rapidly at the beginning and gradually grows to the new balanced growth path. As for aggregate productivity growth (right panel of Figure 16), there is an initial surge in the growth rate for 5 years, and then it gradually declines to a growth rate lower than the initial value as it reaches the new balanced growth path. This "first-rise-then-fall" pattern is consistent with the trend in the right panel of Figure 4, which shows the TFP growth rate experienced a temporary boom during the late 1990s and then declined all the way down to today.

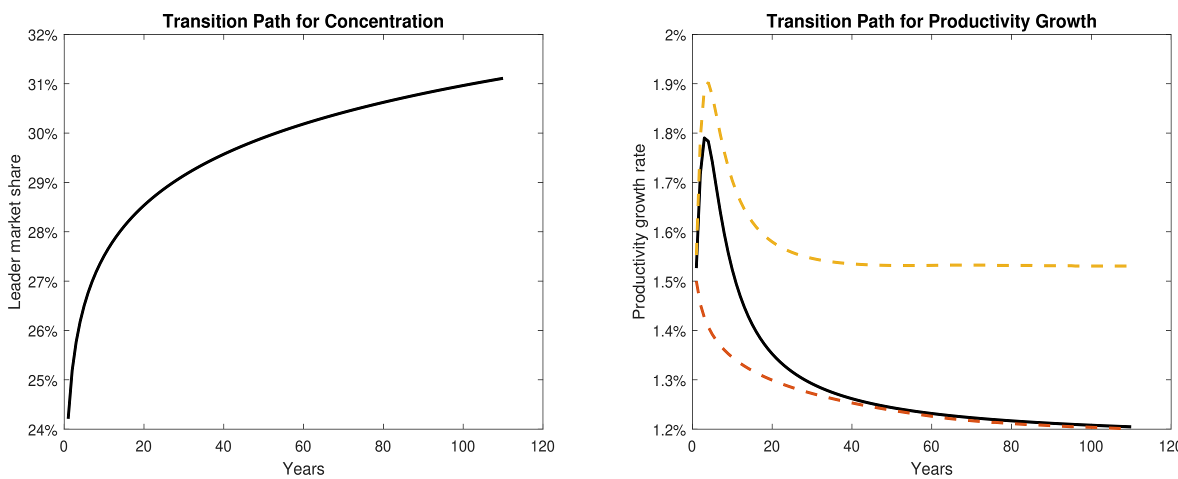


Figure 12: Transition Dynamics for Concentration and Productivity Growth.

To understand the mechanism behind the hump shape immediately after the marketing shock, I decompose the change in productivity growth into the level effect and the growth effect. In the right panel of Figure 16, the yellow dashed curve is obtained by keeping fixed the average quality growth rate g to its initial level on the high-ψ balanced growth path, and allowing the output quality to change as it is on the transition path. The productivity growth increases by 37 basis point in the first few years right after marketing becomes cheaper. Then the positive effect on growth gradually fades out as the economy converges to the new balanced growth path. Next, I keep the output quantity fixed and let R&D investment change on the transition path and find that the average innovation growth rate monotonically declines from 1.51% to 1.19%. The long-run TFP growth on the new balanced growth path is the same as the innovation growth rate as the output will eventually become stable over time. This implies that the increase in overall TFP growth at the beginning is all driven by the increase in output quantity due to more matches in the product market.

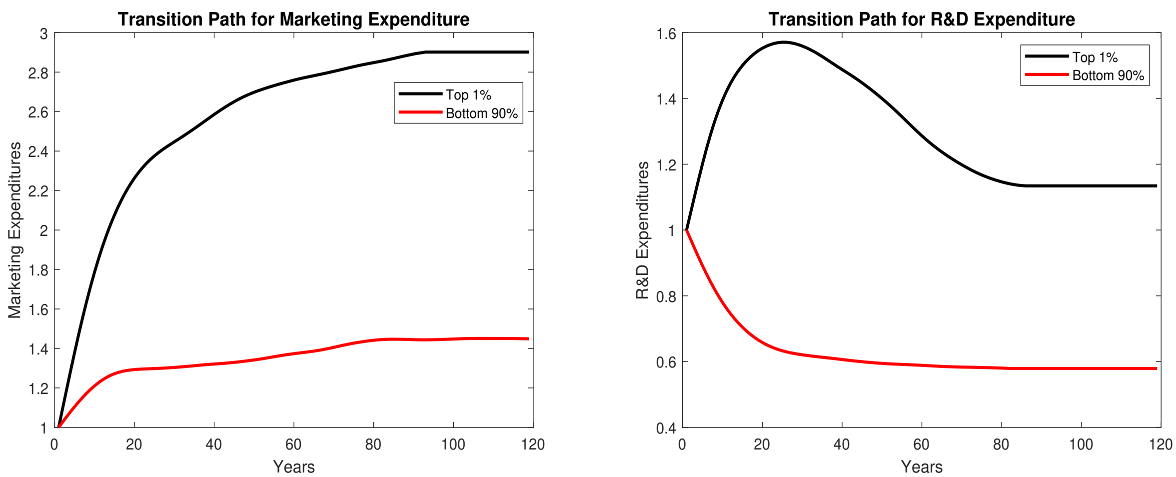


Figure 13: Transition Dynamics for Marketing and R&D Expenditures.

Figure 13 demonstrates the transitional dynamics of marketing and R&D expenditures for large and small firms, respectively. The left panel indicates that large firms increase their marketing expenditures by much more than small firms, which is consistent with the trends in data. The right panel shows that large firms increase innovation investment in the first 20 years or so, because of their large customer base, but the will gradually downsize their R&D investment as they have to offer lower prices to retain customers. As for small firms, there is always an adverse effect, leading to less innovation efforts.

6 Welfare Analysis

6.1 Welfare Effect of Marketing

In Section 5, I pointed out that there are two offsetting effect of marketing: (i) overcomes search frictions and allow more customers to buy products they like; (ii) discourages firms from innovation and thus harm the growth in the calibrated model. In this section, I study the consequences of lower marketing cost on welfare. There has been a long-lasting debate in economics about the welfare implications of advertising.

In this paper, following the classic literature of Nelson (1974), Butters (1977), Grossman and Shapiro (1984) and Milgrom and Roberts (1986), I focus on the informative role of marketing/advertising, where advertisement helps remove information frictions by providing relevant information about product characteristics and quality, or simply about the existence of the product. Informative advertising weakens the negative effect of information frictions and is usually associated with welfare improving.

On the other hand, advertising is also a taste shifter that firms use to steal customers from each other and maintain their own market shares (See e.g. Dixit and Norman (1978), Becker and Murphy (1993), Benhabib and Bisin (2002), Benhabib and Bisin (2011) and Molinari and Turino (2009)). In this case, marketing expenditures are a pure combative tool for a limited number of customers and thus might result in a waste of resources. Moreover, in my model, the decline of marketing cost also affects firm's R&D decisions, which further create another side effect from the perspective of productivity growth rate. Given both the pros and cons, it is important to quantify the welfare implications of the declining cost of marketing.

In this economy, labor is the numeraire and customers have a quasi-linear utility function. The welfare effect of the decline of marketing cost is given by the change in the discounted sum of consumption utility. The total welfare in the economy can be written as

$$\mathbb{E}\sum_{t=0}^{\infty}[\beta(1-\tau)(1+g)]^t \int \left\{ x_{jt}\left[\bar{u}(1+\lambda)m_{jt} + \int_{u_{2j}^{*}{}^{rd}} u_2(1+\lambda)dG(u_2)\frac{\xi B_t^{\gamma}}{A_t^{\gamma}}a_{jt}\right] + (1-x_{jt})\left[\alpha_1\bar{u}m_{jt} + \alpha_2\int_{u_{2j}^{*}{}^{nrd}} u_2 dG(u_2)\frac{\xi B_t^{\gamma}}{A_t^{\gamma}}a_{jt}\right]\right\} dj - sB_t \quad (24)$$

where u_{2j}^{*} stands for the threshold of utility above which the matched customers will buy. As ψ decreases, the trade-off is between a decline of long-run growth rate g and a rise in the output driven by more matched new customers a.

Figure 14 shows that the level effect dominates the growth effect, and the overall welfare increases as marketing cost drops. This is reasonable considering that the shock is a technological progress that overcomes information frictions in the market. Despite the negative side effects on growth and business stealing, the first-order effect of facilitating matches dominates and raises total welfare.

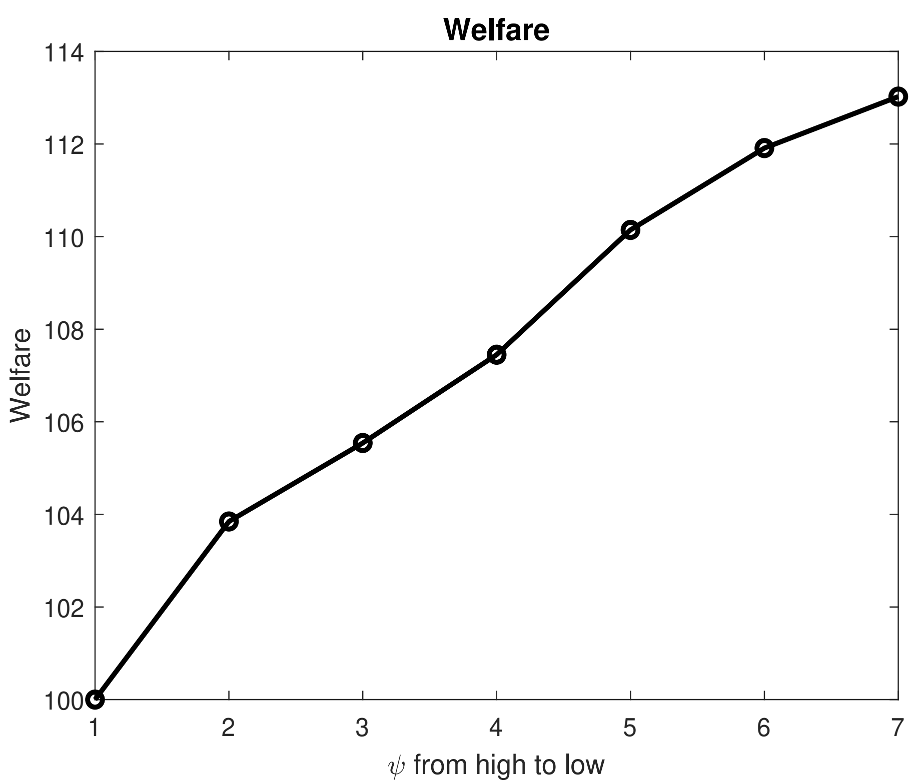


Figure 14: Change in Welfare with the Decline in ψ.

6.2 Constrained Efficiency

Despite the increase in welfare, trade-off between the level effect and the growth effect is still present. Is the heavy spending in advertising socially efficient? Does the decentralized equilibrium optimally allocate resources between innovation and marketing? To answer these questions, I derive the social planner's problem.

The goal of the social planner is to maximize lifetime utility of all customers subject to the search frictions and technological constraints of the economy. The planner generally does not set prices to customers, but to maintain the market structure where existing and new customers co-exist, I assume there are imaginary prices that make the existing customers indifferent between searching and continuing the relationship. This implies that there is still a threshold on product utility $u_2^{*(n)rd}$ for the new buyers[8]. With this restriction in the market, the social planner chooses the optimal marketing $a(m)$, innovation $x(m)$ and the thresholds $u_2^{*(n)rd}$ to solve the following maximization problem:

$$\max_{a_{jt}, x_{jt}, u_{2j}^{*(n)rd}} \mathbb{E} \sum_{t=0}^{\infty} [\beta(1-\tau)(1+g)]^t \int \left\{ x_{jt} \left[\bar{u}(1+\lambda)(1-\delta)m_{jt} + \int_{u_{2j}^{*rd}} u_2(1+\lambda)dG(u_2) \frac{\xi B_t^\gamma}{A_t^\gamma} a_{jt} \right] \right.$$
$$\left. +(1-x_{jt}) \left[\alpha_1 \bar{u}(1-\delta)m_{jt} + \alpha_2 \int_{u_{2j}^{*nrd}} u_2 dG(u_2) \frac{\xi B_t^\gamma}{A_t^\gamma} a_{jt} \right] \right\} dj - sB_t \tag{25}$$

$$s.t. \quad \int a_t(m_{jt})dj = A_t \tag{26}$$

$$B_t + \int m_{jt} dj = N \tag{27}$$

$$\int \left\{ \phi x_{jt}^{\eta_x} + \psi a_{jt}^{\eta_a} + x_{jt} m_{j,t+1}^{rd} + (1-x_{jt}) m_{j,t+1}^{nrd} \right\} dj = N \tag{28}$$

$$m_{j,t+1}^{rd} = (1-\delta)m_{jt} + y_{jt}^{rd}, \;\; y_{jt}^{rd} = \left[1 - G\left(u_2^{*rd}\right)\right] \frac{\xi B_t^\gamma}{A_t^\gamma} a_{jt} \tag{29}$$

$$m_{j,t+1}^{nrd} = (1-\delta)\alpha_1 m_{jt} + \alpha_2 y_{jt}^{nrd}, \;\; y_{jt}^{nrd} = \left[1 - G\left(u_2^{*nrd}\right)\right] \frac{\xi B_t^\gamma}{A_t^\gamma} a_{jt} \tag{30}$$

[8] Absent this assumption, the planner would always want the matched customers to buy because their $u_2 \geq 0$ and there is not price p_2.

Proposition 3. *The constrained efficient allocation specifies lower marketing investment and higher innovation investment compared to the decentralized equilibrium:* $a_j^{sp} < a_j$, $x_j^{sp} > x_j$.

Proof: Appendix A. Intuitively, marketing creates a negative "congestion externality", where one firm's marketing reduces all other firms' chance of meeting customers $\frac{\xi B^\gamma}{(A_{-j}+a_j)^\gamma}$, and therefore in the decentralized economy firms over-invest in advertising.

6.3 Policy Implications

The results so far raise some questions in terms of policy implications, especially regarding advertising. In this section, I study whether a tax on marketing or a subsidy on firm R&D activities could be welfare-improving. In particular, I focus on linear taxes and subsidies. The revenues from taxes are rebated back to the consumers, and subsidies are financed through lump-sum taxes.

With the quasi-linear utility function, the welfare function with marketing tax is

$$\mathbb{E}\sum_{t=0}^{\infty}[\beta(1-\tau)(1+g)]^t \int \left\{ x_{jt}\left[\bar{u}(1+\lambda)m_{jt} + \int_{u_{2j}^{*}{}^{rd}} u_2(1+\lambda)dG(u_2)\frac{\xi B_t^\gamma}{A_t^\gamma}a_{jt}\right] \right.$$
$$\left. +(1-x_{jt})\left[\alpha_1\bar{u}m_{jt} + \alpha_2\int_{u_{2j}^{*}{}^{nrd}} u_2 dG(u_2)\frac{\xi B_t^\gamma}{A_t^\gamma}a_{jt}\right]\right\} dj + T_t - sB_t \tag{31}$$

where the proceeds of marketing tax is rebated to consumers. Assume the linear tax rate on marketing is ε, then

$$T_t = \varepsilon \int \psi a_j^{\eta_a} dj \tag{32}$$

There are several forces associated with taxation of advertising that go in opposite directions regarding welfare. I find that there exists an optimal level of tax on marketing that maximizes welfare equal to 24% (see Figure 15). This tax is associated to a 0.24% increase in growth and a 0.54% reduction in quantity of output, and an overall increase in welfare of

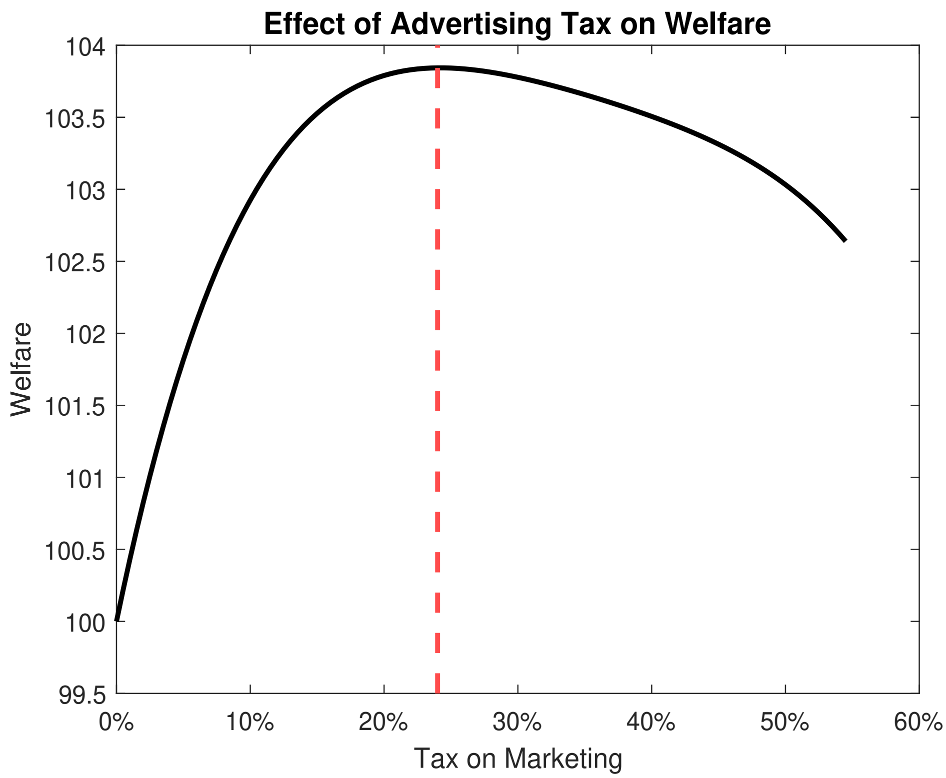


Figure 15: Effect of Tax on Marketing on Welfare.

3.8%.

7 Conclusion

This paper documents a large decline in the cost of marketing and advertising due to the development of digital technologies and the rise of online marketing platforms. The average price index of advertising has plummeted by 40% in the past 20 years. In response to this technological change, firms invest more in marketing and advertising, with the large firms increasing their investment by much more than their smaller counterparts.

To understand the macroeconomic consequences of lower marketing cost, I develop an endogenous growth model in which the product market is frictional, and customers are a valuable long-term asset to firms. Firms invest in marketing and advertising to increase the visibility of their products to customers, and invest in R&D to increase product quality.

Large firms invest more in both R&D and advertising due to the complementarity between these two types of investments.

As marketing becomes cheaper, it is less costly for all firms to make advertisement and match with new customers. However, since large firms invest more in R&D and generate better product quality, they convert a higher proportion of customers to buyers, and gain a higher return of advertising investment. Therefore, more new customers are reallocated to the large firms. This widens the gap in customer base between large and small firms, which further affects future R&D and marketing investment, giving rise to increasing concentration and the rise of superstar firms.

Rising concentration caused by cheaper marketing does not necessarily do harm to aggregate productivity. Instead, it generates a positive level effect due to more efficient matching in the product market, and a negative growth effect mainly due to lower innovation incentives of small firms. These offsetting factors make effects on productivity growth ambiguous.

The quantitative analysis implies that the declining marketing cost is a critical driving force of rising marketing concentration: it explains 83% of the rise in the largest firm's market share within 3-digit SIC industries. Moreover, consistent with other classical studies, rising market concentration is mainly driven by the enlarging gap in customer base across firm size rather than large firms charging higher markups.

Nevertheless, lower marketing cost generates mixed effects on productivity growth. On average, it only accounts for 1/3 of the decline in TFP growth rate. The initial surge of productivity growth at the beginning of transition path is also consistent with the temporary boom in TFP growth rate observed in data. This contributes to the heated discussion of whether rising concentration depresses long-run economic performance by squeezing out competitors. The findings in my paper suggest that increasing concentration caused by lower marketing cost is not responsible for the majority of productivity growth slowdown. It could be driven by other competing factors, such as the fact that ideas are getting harder to find or an increasing effort for firms to protect their intellectual properties. Further work should investigate these potential mechanism that account for the productivity growth slowdown.

References

Acemoglu, D., U. Akcigit, H. Alp, N. Bloom, and W. Kerr (2018): "Innovation, reallocation, and growth," *American Economic Review*, 108, 3450–91.

Aghion, P., A. Bergeaud, T. Boppart, P. J. Klenow, and H. Li (2019): "A theory of falling growth and rising rents," Tech. rep., National Bureau of Economic Research.

Akcigit, U. and W. R. Kerr (2018): "Growth through heterogeneous innovations," *Journal of Political Economy*, 126, 1374–1443.

Argente, D., D. Fitzgerald, S. Moreira, and A. Priolo (2021): "How do entrants build market share? The role of demand frictions," Tech. rep., Mimeo, Penn State University.

Arkolakis, C. (2016): "A unified theory of firm selection and growth," *The Quarterly Journal of Economics*, 131, 89–155.

Baqaee, D. R. and E. Farhi (2020): "Productivity and misallocation in general equilibrium," *The Quarterly Journal of Economics*, 135, 105–163.

Becker, G. S. and K. M. Murphy (1993): "A simple theory of advertising as a good or bad," *The Quarterly Journal of Economics*, 108, 941–964.

Benhabib, J. and A. Bisin (2002): "Advertising, mass consumption and Capitalism," *manuscript, Department of Economics NYU.*

——— (2011): "Social construction of preferences: advertising," in *Handbook of Social Economics*, Elsevier, vol. 1, 201–220.

Benkard, C. L. (2000): "Learning and forgetting: The dynamics of aircraft production," *American Economic Review*, 90, 1034–1054.

Bloom, N., C. I. Jones, J. Van Reenen, and M. Webb (2020): "Are ideas getting harder to find?" *American Economic Review*, 110, 1104–44.

Bornstein, G. et al. (2018): "Entry and profits in an aging economy: The role of consumer inertia," .

Butters, G. R. (1977): "Equilibrium distributions of sales and advertising prices," *The*

Review of Economic Studies, 44, 465–491.

Caillaud, B. and B. Jullien (2003): "Chicken & egg: Competition among intermediation service providers," *RAND journal of Economics*, 309–328.

Cavenaile, L., M. A. Celik, P. Roldan-Blanco, and X. Tian (2021): "Style over Substance? Advertising, Innovation, and Endogenous Market Structure," *Advertising, Innovation, and Endogenous Market Structure (March 13, 2021)*.

Cavenaile, L. and P. Roldan-Blanco (2021): "Advertising, innovation, and economic growth," *American Economic Journal: Macroeconomics*, 13, 251–303.

Chatterjee, S. and B. Eyigungor (2020): "Policy inertia, election uncertainty, and incumbency disadvantage of political parties," *The Review of Economic Studies*, 87, 2600–2638.

De Loecker, J., J. Eeckhout, and G. Unger (2020): "The rise of market power and the macroeconomic implications," *The Quarterly Journal of Economics*, 135, 561–644.

De Ridder, M. (2019): "Market power and innovation in the intangible economy," .

Dinlersoz, E. M. and M. Yorukoglu (2012): "Information and industry dynamics," *American Economic Review*, 102, 884–913.

Dixit, A. and V. Norman (1978): "Advertising and welfare," *The Bell Journal of Economics*, 1–17.

Drozd, L. A. and J. B. Nosal (2012): "Understanding international prices: Customers as capital," *American Economic Review*, 102, 364–95.

Eggertsson, G. B., N. R. Mehrotra, and J. A. Robbins (2019): "A model of secular stagnation: Theory and quantitative evaluation," *American Economic Journal: Macroeconomics*, 11, 1–48.

Fishman, A. and R. Rob (2003): "Consumer inertia, firm growth and industry dynamics," *Journal of Economic theory*, 109, 24–38.

Fitzgerald, D., S. Haller, and Y. Yedid-Levi (2017): "Appendix for How Exporters Grow," Tech. rep., Federal Reserve Bank of Minneapolis.

Foster, L., C. Grim, J. Haltiwanger, and Z. Wolf (2016): "Firm-level dispersion in productivity: is the devil in the details?" *American Economic Review*, 106, 95–98.

Garcia-Macia, D., C.-T. Hsieh, and P. J. Klenow (2019): "How destructive is innovation?" *Econometrica*, 87, 1507–1541.

Goldfarb, A. (2014): "What is different about online advertising?" *Review of Industrial Organization*, 44, 115–129.

Gourio, F. and L. Rudanko (2014): "Customer capital," *Review of Economic Studies*, 81, 1102–1136.

Greenwood, J., Y. Ma, and M. Yorukoglu (2021): "'You Will:'A Macroeconomic Analysis of Digital Advertising," Tech. rep., National Bureau of Economic Research.

Grossman, G. M. and C. Shapiro (1984): "Informative advertising with differentiated products," *The Review of Economic Studies*, 51, 63–81.

Haskel, J. and S. Westlake (2017): "Capitalism without capital," in *Capitalism without Capital*, Princeton University Press.

Hopenhayn, H., J. Neira, and R. Singhania (2018): "The rise and fall of labor force growth: Implications for firm demographics and aggregate trends," *NBER Working Paper*, 1–28.

Hsieh, C.-T. and E. Rossi-Hansberg (2019): "The industrial revolution in services," Tech. rep., National Bureau of Economic Research.

Jones, C. I. (1995): "R & D-based models of economic growth," *Journal of political Economy*, 103, 759–784.

——— (2020): "The end of economic growth? Unintended consequences of a declining population," Tech. rep., National Bureau of Economic Research.

Karahan, F., B. Pugsley, and A. Şahin (2019): "Demographic origins of the startup deficit," Tech. rep., National Bureau of Economic Research.

Kroen, T., E. Liu, A. R. Mian, and A. Sufi (2021): "Falling rates and rising superstars," Tech. rep., National Bureau of Economic Research.

Milgrom, P. and J. Roberts (1986): "Price and advertising signals of product quality," *Journal of political economy*, 94, 796–821.

Molinari, B. and F. Turino (2009): "Advertising and business cycle fluctuations," *Instituto Valenciano de Investigaciones Económicas Working Paper No. AD*, 9.

Nelson, P. (1974): "Advertising as information," *Journal of political economy*, 82, 729–754.

Olmstead-Rumsey, J. (2019): "Market concentration and the productivity slowdown," .

Paciello, L., A. Pozzi, and N. Trachter (2019): "Price dynamics with customer markets," *International Economic Review*, 60, 413–446.

Perla, J. (2017): "Product awareness, industry life cycles, and aggregate profits," *Mimeograph, University of British Columbia.*

——— (2019): "A model of product awareness and industry life cycles," *Unpublished Paper, University of British Columbia.*

Peters, M. and C. Walsh (2021): "Population growth and firm dynamics," Tech. rep., National Bureau of Economic Research.

Rachel, L. (2021): "Leisure-enhancing technological change," *Job market paper, London School of Economics.*

Rochet, J.-C. and J. Tirole (2003): "Platform competition in two-sided markets," *Journal of the european economic association*, 1, 990–1029.

Roldan-Blanco, P. and S. Gilbukh (2021): "Firm dynamics and pricing under customer capital accumulation," *Journal of Monetary Economics*, 118, 99–119.

Thomas, J. S. (2001): "A methodology for linking customer acquisition to customer retention," *Journal of marketing research*, 38, 262–268.

A Mathematical Proofs

Each period, firms make decisions on the R&D investment x, marketing investment a, as well as the prices upon R&D success p_i^{rd} ($i = 1$ for existing customers, $i = 2$ for new), and prices upon R&D failure p_i^{nrd}, to maximize their value function, which reads

$$
\begin{aligned}
V(m; q) = \max_{p_i^{rd}, p_i^{nrd}, a, x} \; & x \left\{ (p_1^{rd} - w)m + (p_2^{rd} - w)y^{rd} + \beta(1-\tau)V(m'^{rd}; (1+g)q) \right\} \\
& + (1-x) \left\{ (p_1^{nrd} - w)\alpha_1 m + (p_2^{nrd} - w)\alpha_2 y^{nrd} + \beta(1-\tau)V(m'^{nrd}; (1+g)q) \right\} \\
& - \psi a^{\eta_a} w - \phi x^{\eta_x} w \qquad (33)
\end{aligned}
$$

$$
\begin{aligned}
s.t. \;\; & p_1^{rd} = \bar{u}(1+\lambda) - U_0 + \beta U_0' && (34) \\
& p_1^{nrd} = \bar{u} - U_0 + \beta U_0' && (35) \\
& y^{rd} = \left[1 - G\left(\frac{p_2^{rd}}{(1+\lambda)q} \right) \right] \frac{\xi B^\gamma}{A^\gamma} a && (36) \\
& m'^{rd} = (1-\delta)m + y^{rd} && (37) \\
& y^{nrd} = \left[1 - G\left(\frac{p_2^{nrd}}{q} \right) \right] \frac{\xi B^\gamma}{A^\gamma} a && (38) \\
& m'^{nrd} = (1-\delta)\alpha_1 m + \alpha_2 y^{nrd} && (39)
\end{aligned}
$$

There are two types of customers. Existing customer's value of continuing relationship with the firm

$$U_0 = \bar{u} q_j - p_1 + \beta U_0' \qquad (40)$$

The value function of an unmatched customer to meet with a size-m firm is

$$U_s(m_j) = \int_{\frac{p_2(m)}{q_j}} \left[u_2 q_j - p_2(m_j) \right] dG(u_2) + \beta U_0' \qquad (41)$$

Thus, the expected value of searching is given by

$$U_s = -s + \sum_j \left[\frac{h(m_j)}{B} U_s(m_j) \right] + \left(1 - \frac{Y}{B}\right) \beta U_0' \tag{42}$$

Indifference condition requires $U_0 = U_s$.

A.1 Proof of Proposition 1. $p_1^{rd} > p_1^{nrd}$

Firms charge p_1^{rd} (when they succeed in R&D) and p_1^{nrd} (when they fail in R&D) to the existing customers:

$$p_1^{rd} = \bar{u}(1 + \lambda) - \hat{U}_0 \tag{43}$$

$$p_1^{nrd} = \bar{u} - \hat{U}_0 \tag{44}$$

Apparently, $p_1^{rd} > p_1^{nrd}$ because $\lambda > 0$.

A.2 Proof of Proposition 2. (Supermodularity) Firms with a large customer base m invest in more innovation x and more marketing a.

Assume value function $V(m)$ is convex.

The value function can be written as form:

$$V(m) = \max_{x,a,p_2} R(m, x, a, p_2) + \tilde{\beta}\mathbb{E}V(g(m, x, a, p_2)) \tag{45}$$

where $R(m, x, a, p_2)$ is the profit function once prices are maximized out:

$$\begin{aligned} R(m, x, a, p_2) =& x \left[\left(p_1^{rd} - w \right) (1 - \delta) \, m + (p_2 - w) \, y^{rd} \right] \\ & + (1 - x) \left[\left(p_1^{nrd} - w \right) \alpha_1 \, (1 - \delta) \, m + (p_2 - w) \, \alpha_2 y^{nrd} \right] \\ & - \psi a^{\eta_a} w - \phi x^{\eta_x} w \end{aligned} \tag{46}$$

and the FOC of p_2 reads

$$\begin{aligned} & x \left[1 - G \left(\frac{p_2}{1 + \lambda} \right) \right] + (1 - x) \alpha_2 \left[1 - G \left(p_2 \right) \right] \\ =& x \left[p_2 - w + \tilde{\beta} V' \left(m'^{rd} \right) \right] \frac{G' \left(\frac{p_2}{1+\lambda} \right)}{1 + \lambda} + (1 - x) \alpha_2 \left[p_2 - w + \tilde{\beta} V' \left(m'^{nrd} \right) \right] G'(p_2) \end{aligned} \tag{47}$$

$m' = g(m, x, a, p_2)$ is the law of motion for customer base.

$$m'^{rd} = g^{rd}(m, x, a, p_2) = (1 - \delta) \, m + y^{rd} \quad \text{w.p. } x \tag{48}$$

$$m'^{nrd} = g^{nrd}(m, x, a, p_2) = \alpha_1 \, (1 - \delta) \, m + \alpha_2 y^{nrd} \quad \text{w.p. } (1 - x) \tag{49}$$

A.2.1 Prove the supermodularity of $R + \tilde{\beta} \mathbb{E} V(g)$

Denote

$$H(m, x, a, p_2) = R(m, x, a, p_2) + \tilde{\beta} \left[x V(g^{rd}(m, x, a, p_2)) + (1 - x) V(g^{nrd}(m, x, a, p_2)) \right] \tag{50}$$

1. $\frac{\partial^2 H}{\partial m \partial a}$

$$\frac{\partial H}{\partial m} = \frac{\partial R}{\partial m} + \tilde{\beta} \left[x V'(m'^{rd}) \frac{\partial g^{rd}}{\partial m} + (1 - x) V'(m'^{nrd}) \frac{\partial g^{nrd}}{\partial m} \right] \tag{51}$$

where

$$\frac{\partial R}{\partial m} = \left[x\left(p_1^{rd} - w\right) + (1-x)\left(p_1^{nrd} - w\right)\alpha_1\right](1-\delta) \tag{52}$$

and

$$\frac{\partial g^{rd}}{\partial m} = 1-\delta, \quad \frac{\partial g^{nrd}}{\partial m} = (1-\delta)\alpha_1 \tag{53}$$

Second derivative are as follows:

$$\begin{aligned} \frac{\partial^2 H}{\partial m \partial a} =& 0 + \tilde{\beta}\left\{xV''(m'^{rd})\frac{\partial g^{rd}}{\partial a} + (1-x)\alpha_1 V''(m'^{nrd})\frac{\partial g^{nrd}}{\partial a}\right\}(1-\delta) & (54)\\ =& \tilde{\beta}\left\{xV''(m'^{rd})\left[1 - G\left(\frac{p_2}{1+\lambda}\right)\right] + (1-x)\alpha_1 V''(m'^{nrd})\left[1 - G\left(p_2\right)\right]\right\}(1-\delta) & (55)\end{aligned}$$

By convexity of V, $\frac{\partial^2 H}{\partial m \partial a} > 0$.

2. $\frac{\partial^2 H}{\partial m \partial x}$

$$\frac{\partial^2 H}{\partial m \partial x} = \left\{\left(p_1^{rd} - w\right) - \left(p_1^{nrd} - w\right)\alpha_1 + \tilde{\beta}\left[V'(m'^{rd}) - \alpha_1 V'(m'^{nrd})\right]\right\}(1-\delta) \tag{56}$$

It can be shown that

$$m'^{rd} = (1-\delta)m + \left[1 - G\left(\frac{p_2}{1+\lambda}\right)\right]\frac{\xi B^\gamma}{A^\gamma}a > \alpha_1(1-\delta)m + \alpha_2\left[1 - G\left(p_2\right)\right]\frac{\xi B^\gamma}{A^\gamma}a = m'^{nrd} \tag{57}$$

Since V is convex and $p_1^{rd} > p_1^{nrd}$, $\frac{\partial^2 H}{\partial m \partial x} > 0$.

3. $\frac{\partial^2 H}{\partial x \partial a}$

$$\frac{\partial H}{\partial x} = \frac{\partial R}{\partial x} + \tilde{\beta} \left\{ V(g^{rd}) - V(g^{nrd}) \right\} \tag{58}$$

and the second derivative is

$$\frac{\partial^2 H}{\partial x \partial a} = \frac{\partial^2 R}{\partial x \partial a} + \tilde{\beta} \left[V'(m'^{rd}) \frac{\partial g^{rd}}{\partial a} - V'(m'^{nrd}) \frac{\partial g^{nrd}}{\partial a} \right]. \tag{59}$$

Note that

$$\begin{aligned} \frac{\partial R}{\partial x} &= \left[\left(p_1^{rd} - w \right) (1-\delta) m + (p_2 - w) \, y^{rd} \right] \\ &\quad - \left[\left(p_1^{nrd} - w \right) \alpha_1 (1-\delta) m + (p_2 - w) \, \alpha_2 y^{nrd} \right] - \phi \eta_x x^{\eta_x - 1} w \end{aligned} \tag{60}$$

So,

$$\frac{\partial^2 R}{\partial x \partial a} = \left\{ (p_2 - w) \left[1 - G\left(\frac{p_2}{1+\lambda} \right) \right] - (p_2 - w) \, \alpha_2 \left[1 - G\left(p_2 \right) \right] \right\} \frac{\xi B^\gamma}{A^\gamma}, \tag{61}$$

which is obviously positive because $\alpha_2 < 1$ and $\lambda > 0$.

Also,

$$\frac{\partial g^{rd}}{\partial a} = \left[1 - G\left(\frac{p_2}{1+\lambda} \right) \right] \frac{\xi B^\gamma}{A^\gamma} > \alpha_2 \left[1 - G\left(p_2 \right) \right] \frac{\xi B^\gamma}{A^\gamma} = \frac{\partial g^{nrd}}{\partial a} \tag{62}$$

As shown before, $V'(m'^{rd}) > V'(m'^{nrd})$. So, the second term of (59) is also positive. We have shown $\frac{\partial^2 H}{\partial x \partial a} > 0$.

4. $\frac{\partial^2 H}{\partial m \partial p_2}$.

Apparently,

$$\frac{\partial^2 R}{\partial m \partial p_2} = 0, \quad \frac{\partial^2 g}{\partial m \partial p_2} = 0 \tag{63}$$

Thus, the second derivative is 0.

5. $\frac{\partial^2 H}{\partial a \partial p_2}$.

$$\frac{\partial H}{\partial a} = \frac{\partial R}{\partial a} + \tilde{\beta}\left\{xV'(m'^{rd})\frac{\partial g^{rd}}{\partial a} + (1-x)V'(m'^{nrd})\frac{\partial g^{nrd}}{\partial a}\right\} \tag{64}$$

$$\begin{aligned} =& x\left(p_2 - w\right)\left[1 - G\left(\frac{p_2}{1+\lambda}\right)\right]\frac{\xi B^\gamma}{A^\gamma} + (1-x)\left(p_2 - w\right)\alpha_2\left[1 - G\left(p_2\right)\right]\frac{\xi B^\gamma}{A^\gamma} \\ &- \psi \eta_a a^{\eta_a - 1} w + \tilde{\beta}\left\{xV'(m'^{rd})\frac{\partial g^{rd}}{\partial a} + (1-x)V'(m'^{nrd})\frac{\partial g^{nrd}}{\partial a}\right\} \end{aligned} \tag{65}$$

and the second derivative is

$$\begin{aligned} \frac{\partial^2 H}{\partial a \partial p_2} =& x\left\{1 - G\left(\frac{p_2}{1+\lambda}\right) - \left[p_2 - w + \tilde{\beta}V'(m'^{rd})\right]\frac{G'\left(\frac{p_2}{1+\lambda}\right)}{(1+\lambda)}\right\}\frac{\xi B^\gamma}{A^\gamma} \\ &+ (1-x)\alpha_2\left\{1 - G\left(p_2\right) - \left[p_2 - w + \tilde{\beta}V'(m'^{nrd})\right]G'\left(p_2\right)\right\}\frac{\xi B^\gamma}{A^\gamma} \\ &- \tilde{\beta}\left\{xV''(m'^{rd})\frac{G'\left(\frac{p_2}{1+\lambda}\right)}{(1+\lambda)} + (1-x)V''(m'^{nrd})G'\left(p_2\right)\right\} \end{aligned} \tag{66}$$

Denote

$$Y_1 \equiv 1 - G\left(\frac{p_2}{1+\lambda}\right) - \left[p_2 - w + \tilde{\beta}V'(m'^{rd})\right]\frac{G'\left(\frac{p_2}{1+\lambda}\right)}{(1+\lambda)} \tag{67}$$

Y_1 is the marginal value of increasing p_2 when R&D is successful.

Denote

$$Y_2 \equiv \alpha_2\left\{1 - G\left(p_2\right) - \left[p_2 - w + \tilde{\beta}V'(m'^{nrd})\right]G'\left(p_2\right)\right\} \tag{68}$$

Y_2 is the marginal value of increasing p_2 when R&D fails.

The FOC of p_2 (equation (47)) can be rewritten as

$$xY_1 + (1-x)Y_2 = 0 \tag{69}$$

which is exactly the first two lines of equation (66). The third line of (66) is obviously negative, so $\frac{\partial^2 H}{\partial a \partial p_2} < 0$.

6. $\frac{\partial^2 H}{\partial x \partial p_2}$.

$$\begin{aligned}
\frac{\partial H}{\partial x} =& \frac{\partial R}{\partial x} + \tilde{\beta}\left\{V(g^{rd}) - V(g^{nrd})\right\} && (70)\\
=& \left[\left(p_1^{rd} - w\right)(1-\delta)m + (p_2 - w)\, y^{rd}\right]\\
&- \left[\left(p_1^{nrd} - w\right)\alpha_1(1-\delta)m + (p_2 - w)\,\alpha_2 y^{nrd}\right] - \phi\eta_x x^{\eta_x - 1} w\\
&+ \tilde{\beta}\left\{V(g^{rd}) - V(g^{nrd})\right\} && (71)
\end{aligned}$$

and the second derivative is

$$\begin{aligned}
\frac{\partial^2 H}{\partial x \partial p_2} =& \left\{1 - G\left(\frac{p_2}{1+\lambda}\right) - \left[p_2 - w + \tilde{\beta}V'(m'^{rd})\right]\frac{G'\left(\frac{p_2}{1+\lambda}\right)}{(1+\lambda)}\right\}\frac{\xi B^\gamma}{A^\gamma}a\\
&- \alpha_2\left\{1 - G(p_2) - \left[p_2 - w + \tilde{\beta}V'(m'^{nrd})\right]G'(p_2)\right\}\frac{\xi B^\gamma}{A^\gamma}a && (72)\\
=& \left[Y_1 - Y_2\right]\frac{\xi B^\gamma}{A^\gamma}a && (73)
\end{aligned}$$

Intuitively, $Y_1 < Y_2$ requires the MV of p_2 upon R&D success is less than the MV of p_2 in R&D failure.

If $Y_2 > 0$, then by FOC of p_2, $Y_1 < 0$, and $\frac{\partial^2 H}{\partial x \partial p_2} < 0$.

If $Y_2 < 0$, assume $\alpha_2 < \frac{1}{1+\lambda}$, which is a reasonable assumption because the quality increment

λ is usually very small in data. Then,

$$Y_1 - Y_2 < \frac{\lambda}{1+\lambda} - \tilde{\beta}\left[V'(m'^{rd}) - V'(m'^{nrd})\right] \tag{74}$$

When λ is very small or when G is narrowly distributed, the RHS< 0, then $Y_1 < Y_2$, which contradicts $Y_2 < 0$. So, $Y_2 > 0$ and $Y_1 < 0$.
When λ is not very small, the relationship between Y_1 and Y_2 depends on the distribution of G and the functional form of V.

Denote $\hat{p} = -p_2$, then $\frac{\partial^2 H}{\partial z \partial \hat{p}} \geq 0$ $(z = m, x, a)$. Hence, we have shown that when λ is quite small, $H(m, x, a, \hat{p})$ is supermodular.

A.2.2 Prove convexity of $V(m)$

According to the envelope condition,

$$V'(m) = \frac{\partial H(m, x, a, \hat{p})}{\partial m} \tag{75}$$

$$= \left\{x(\bar{u}(1+\lambda) - \hat{U}_0 - w) + (1-x)\alpha_1(\bar{u} - \hat{U}_0 - w)\right\}(1-\delta)$$
$$+ x\tilde{\beta}V'\left(m'^{rd}\right)\frac{\partial g^{rd}}{\partial m} + (1-x)\tilde{\beta}V'\left(m'^{nrd}\right)\frac{\partial g^{nrd}}{\partial m} \tag{76}$$

where

$$\frac{\partial g^{rd}}{\partial m} = 1 - \delta, \quad \frac{\partial g^{nrd}}{\partial m} = (1-\delta)\alpha_1 \tag{77}$$

Thus,

$$V'(m) = \left\{x\left[\bar{u}(1+\lambda) - \hat{U}_0 - w\right] + (1-x)\alpha_1\left[\bar{u} - \hat{U}_0 - w\right] + x\tilde{\beta}V'\left(m'^{rd}\right) + (1-x)\tilde{\beta}V'\left(m'^{nrd}\right)\right\}(1-\delta) \tag{78}$$

Next, I will show the derivative of the RHS with m ($\frac{d}{dm}\left(\frac{\partial H}{\partial m}\right)$) is positive.

$$\frac{d}{dm}\left(\frac{\partial H}{\partial m}\right) \tag{79}$$

$$=\left\{\left[\bar{u}\lambda+(1-\alpha_1)(\bar{u}-\hat{U}_0-w)\right]x'(m)+\tilde{\beta}\left[x'(m)V'\left(m'^{rd}\right)+xV''\left(m'^{rd}\right)\frac{dg^{rd}}{dm}\right]\right.$$
$$\left.+\tilde{\beta}\left[-x'(m)V'\left(m'^{nrd}\right)+(1-x)V''\left(m'^{nrd}\right)\frac{dg^{nrd}}{dm}\right]\right\}(1-\delta) \tag{80}$$

$$=\left\{\left[\bar{u}\lambda+(1-\alpha_1)(p_1^{nrd}-w)\right]x'(m)+\tilde{\beta}x'(m)\left[V'\left(m'^{rd}\right)-V'\left(m'^{nrd}\right)\right]\right.$$
$$\left.+\tilde{\beta}\left[xV''\left(m'^{rd}\right)\frac{dg^{rd}}{dm}+(1-x)V''\left(m'^{nrd}\right)\frac{dg^{nrd}}{dm}\right]\right\}(1-\delta) \tag{81}$$

where (note that $\hat{p}=-p_2$)

$$\frac{dg^{rd}}{dm}=\frac{\partial g^{rd}}{\partial m}+\frac{\partial g^{rd}}{\partial a}a'(m)+\frac{\partial g^{rd}}{\partial \hat{p}}\hat{p}'(m) \tag{82}$$

$$=1-\delta+\left[1-G\left(\frac{p_2}{1+\lambda}\right)\right]\frac{\xi B^\gamma}{A^\gamma}a'(m)+\frac{1}{2\bar{u}(1+\lambda)}\hat{p}'(m)\frac{\xi B^\gamma}{A^\gamma} \tag{83}$$

$$\frac{dg^{nrd}}{dm}=(1-\delta)\alpha_1+\left[1-G\left(p_2\right)\right]\alpha_2\frac{\xi B^\gamma}{A^\gamma}a'(m)+\frac{1}{2\bar{u}}\hat{p}'(m)\frac{\xi B^\gamma}{A^\gamma} \tag{84}$$

We have shown supermodularity of $H(m,x,a,\hat{p})$, so, $x'(m)>0$, $a'(m)>0$ and $\hat{p}'(m)>0$. Thus, both $\frac{dg^{rd}}{dm}>0$ and $\frac{dg^{nrd}}{dm}>0$. The third line of (A.2.2) is positive.

Since $p_1^{nrd}-w>0$, the first term of the first line of (A.2.2) is positive.

In terms of the second term of the first line, $g^{rd}>g^{nrd}$ has been shown before. Combined with the convexity of V, $V'\left(m'^{rd}\right)>V'\left(m'^{nrd}\right)$.

Hence, $\frac{d}{dm}\left(\frac{\partial H}{\partial m}\right)>0$. The derivative of the LHS $\frac{d}{dm}V'(m)>0$, which gives $V''(m)>0$. V is convex. -x'(m)V'$\left(m'^{nrd}\right)+(1-x)V''\left(m'^{nrd}\right)\frac{dg^{nrd}}{dm}(1-\delta)$
$=\left\{\left[\bar{u}\lambda+(1-\alpha_1)(p_1^{nrd}-w)\right]x'(m)+\tilde{\beta}x'(m)\left[V'\left(m'^{rd}\right)-V'\left(m'^{nrd}\right)\right]\right.$
$\left.+\tilde{\beta}\left[xV''\left(m'^{rd}\right)\frac{dg^{rd}}{dm}+(1-x)V''\left(m'^{nrd}\right)\frac{dg^{nrd}}{dm}\right]\right\}(1-\delta)$

where (note that $\hat{p} = -p_2$)

$$\frac{dg^{rd}}{dm} = \frac{\partial g^{rd}}{\partial m} + \frac{\partial g^{rd}}{\partial a} a'(m) + \frac{\partial g^{rd}}{\partial \hat{p}} \hat{p}'(m) \tag{85}$$

$$= 1 - \delta + \left[1 - G\left(\frac{p_2}{1+\lambda} \right) \right] \frac{\xi B^\gamma}{A^\gamma} a'(m) + \frac{1}{2\bar{u}(1+\lambda)} \hat{p}'(m) \frac{\xi B^\gamma}{A^\gamma} \tag{86}$$

$$\frac{dg^{nrd}}{dm} = (1-\delta)\alpha_1 + [1 - G(p_2)]\, \alpha_2 \frac{\xi B^\gamma}{A^\gamma} a'(m) + \frac{1}{2\bar{u}} \hat{p}'(m) \frac{\xi B^\gamma}{A^\gamma} \tag{87}$$

We have shown supermodularity of $H(m, x, a, \hat{p})$, so, $x'(m) > 0$, $a'(m) > 0$ and $\hat{p}'(m) > 0$. Thus, both $\frac{dg^{rd}}{dm} > 0$ and $\frac{dg^{nrd}}{dm} > 0$. The third line of (A.2.2) is positive.

Since $p_1^{nrd} - w > 0$, the first term of the first line of (A.2.2) is positive.

In terms of the second term of the first line, $g^{rd} > g^{nrd}$ has been shown before. Combined with the convexity of V, $V'\left(m'^{rd}\right) > V'\left(m'^{nrd}\right)$.

Hence, $\frac{d}{dm}\left(\frac{\partial H}{\partial m}\right) > 0$. The derivative of the LHS $\frac{d}{dm} V'(m) > 0$, which gives $V''(m) > 0$. V is convex.

A.3 Proof of Proposition 3.

The planner maximizes

$$\mathbb{E} \sum_{t=0}^{\infty} [\beta(1-\tau)(1+g)]^t \int \left\{ x_{jt} \left[\bar{u}(1+\lambda)(1-\delta) m_{jt} + \int_{u_{2j}^{*\,rd}} u_2 (1+\lambda) dG(u_2) \frac{\xi B_t^\gamma}{A_t^\gamma} a_{jt} \right] \right.$$
$$\left. + (1 - x_{jt}) \left[\alpha_1 \bar{u}(1-\delta) m_{jt} + \alpha_2 \int_{u_{2j}^{*\,nrd}} u_2 dG(u_2) \frac{\xi B_t^\gamma}{A_t^\gamma} a_{jt} \right] \right\} dj - sB_t \tag{88}$$

$$s.t. \quad \int a_t(m_{jt}) dj = A_t \tag{89}$$

$$B_t + \int m_{jt} dj = N \tag{90}$$

$$m_{j,t+1}^{rd} = (1-\delta) m_{jt} + y_{jt}^{rd}, \;\; y_{jt}^{rd} = \left[1 - G\left(u_2^{*rd}\right) \right] \frac{\xi B_t^\gamma}{A_t^\gamma} a_{jt} \tag{91}$$

$$m_{j,t+1}^{nrd} = (1-\delta)\alpha_1 m_{jt} + \alpha_2 y_{jt}^{nrd}, \;\; y_{jt}^{nrd} = \left[1 - G\left(u_2^{*nrd}\right) \right] \frac{\xi B_t^\gamma}{A_t^\gamma} a_{jt} \tag{92}$$

The planner generally does not set prices to customers, but to maintain the market structure where existing and new customers co-exist, I assume there are imaginary prices that make the existing customers indifferent between searching and continuing the relationship. This implies that there is still a threshold on product utility $u_2^{*(n)rd}$ for the new buyers.

The planner's objective function

$$\mathbb{E}\sum_{t=0}^{\infty}[\beta(1-\tau)(1+g)]^t \int \Bigg\{x_{jt}\left[\bar{u}(1+\lambda)(1-\delta)m_{jt} + \int_{u_{2j}^{*}{}^{rd}} u_2(1+\lambda)dG(u_2)\frac{\xi B_t^{\gamma}}{A_t^{\gamma}}a_{jt}\right]$$
$$+(1-x_{jt})\left[\alpha_1\bar{u}(1-\delta)m_{jt} + \alpha_2\int_{u_{2j}^{*}{}^{nrd}} u_2 dG(u_2)\frac{\xi B_t^{\gamma}}{A_t^{\gamma}}a_{jt}\right]\Bigg\} dj - sB_t \tag{93}$$

$$=\mathbb{E}\sum_{t=0}^{\infty}[\beta(1-\tau)(1+g)]^t \int \{x_{jt}\left[\bar{u}(1+\lambda)-(1-\beta(1+g))U_0 - w\right](1-\delta)m_{jt}$$
$$+x_{jt}\left(p_{2j}^{rd}-w\right)\left[1-G(u_2^{*rd})\right]\frac{\xi B_t^{\gamma}}{A_t^{\gamma}}a_{jt}$$
$$+(1-x_{jt})\left[\bar{u}-(1-\beta(1+g))U_0 - w\right]\alpha_1(1-\delta)m_{jt}$$
$$+(1-x_{jt})\left(p_{2j}^{nrd}-w\right)\left[1-G(u_2^{*nrd})\right]\frac{\xi B_t^{\gamma}}{A_t^{\gamma}}a_{jt}$$
$$+\,x_{jt}\left[(1-\beta(1+g))U_0 + w\right](1-\delta)m_{jt}$$
$$+x_{jt}\left\{\int_{u_2^{*rd}} u_2(1+\lambda)dG(u_2) - \left(p_{2j}^{rd}-w\right)\left[1-G(u_2^{*rd})\right]\right\}\frac{\xi B_t^{\gamma}}{A_t^{\gamma}}a_{jt}$$
$$+(1-x_{jt})\left[(1-\beta(1+g))U_0 + w\right]\alpha_1(1-\delta)m_{jt}$$
$$+(1-x_{jt})\alpha_2\left\{\int_{u_2^{*nrd}} u_2 dG(u_2) - \left(p_{2j}^{nrd}-w\right)\left[1-G(u_2^{*nrd})\right]\right\}\frac{\xi B_t^{\gamma}}{A_t^{\gamma}}a_{jt}\Bigg\} dj - sB_t \tag{94}$$

Denote

$$p_1^{rd} = \bar{u}(1+\lambda) - (1-\beta(1+g))U_0 \tag{95}$$
$$p_1^{nrd} = \bar{u} - (1-\beta(1+g))U_0 \tag{96}$$

Then the first four rows of equation (94) has the same functional form as the firm value function in decentralized equilibrium, except lack the cost functions of innovation and marketing.

Next, recall the value functions of customers,

$$U_s(m_j) = \int_{u_2^{*rd}} \left[u_2(1+\lambda)\mathbb{I}(rd) - p_2(m_j)\right] dG(u_2) + \beta U_0' \tag{97}$$

$$U_0 = -s + \int_j \left[\frac{h(m_j)}{B} U_s(m_j)\right] dj + \left(1 - \frac{Y}{B}\right) \beta U_0' \tag{98}$$

Multiply B on both sides of equation 98,

$$sB_t = \frac{\xi B_t^\gamma}{A_t^\gamma} \int_j a_{jt} \int_{u_2^{*rd}} \left[u_2(1+\lambda)\mathbb{I}(rd) - p_2(m_j)\right] dG(u_2) dj - \left[1 - \beta(1+g)\right] U_0 B_t \tag{99}$$

The total measure of labor used in the economy is L_t, which satisfies

$$\int \left\{\phi x_{jt}^{\eta_x} + \psi a_{jt}^{\eta_a} + x_{jt} m_{j,t+1}^{rd} + (1 - x_{jt}) m_{j,t+1}^{nrd}\right\} dj = L_t \tag{100}$$

Plug equations (99) and (100) back into (94). The objective function becomes

$$\begin{aligned}
&\mathbb{E}\sum_{t=0}^{\infty}[\beta(1-\tau)(1+g)]^t \int \{x_{jt}\left[\bar{u}(1+\lambda) - (1-\beta(1+g))U_0 - w\right](1-\delta)m_{jt} \\
&+x_{jt}\left(p_{2j}^{rd} - w\right)\left[1 - G(u_2^{*rd})\right]\frac{\xi B_t^\gamma}{A_t^\gamma} a_{jt} \\
&+(1-x_{jt})\left[\bar{u} - (1-\beta(1+g))U_0 - w\right]\alpha_1(1-\delta)m_{jt} \\
&+(1-x_{jt})\left(p_{2j}^{nrd} - w\right)\left[1 - G(u_2^{*nrd})\right]\frac{\xi B_t^\gamma}{A_t^\gamma} a_{jt} \\
&-\psi a_{jt}^{\eta_a} w - \phi x_{jt}^{\eta_x} w\} dj + L_t + \left[1 - \beta(1+g)\right] U_0 N
\end{aligned} \tag{101}$$

It is equivalent to the sum of all firms' value function in the decentralized equilibrium. However, when individual firms make decision on a_{jt} and x_{jt}, other firm's value does not enter their objective function, and thus they ignore the negative effect of increasing aggregate A_t on other firms' probability of meeting. Think of the decision on a_{jt}. The decentralized firms maximize

$$D_1 a_j / A^\gamma - \psi a_j^{\eta_a} \tag{102}$$

while the planner maximizes

$$D_1 a_j / (a_j + A_{-j})^\gamma - \psi a_j^{\eta_a} + \int [D_1 a_i / (a_j + A_{-j})^\gamma - \psi a_i^{\eta_a}]\, di \tag{103}$$

where D_1 is a complicated function independent of a_j. Obviously, $a_j^{sp} < a_j$ due to the effects of a_j on the denominator of all other firm's matching function.

B Computational Algorithm

Computational approach to solving the stationary equilibrium.

1. Guess the values of g, B, A, U_0. ($Y = \xi B^{\gamma} A^{1-\gamma}$)

2. Guess the functional form of value function $V(m)$ for the case of R&D success and failure, respectively.

3. Calculate firm's optimal choices of $p_2^{rd}(m), p_2^{nrd}(m)$ for R&D success and failure, using equations (17) and (18), and firm's innovation decision $x(m)$ and advertising decision $a(m)$.

4. Plug the results from Step 4 into equations (8) and update the guesses of value functions $V(m)$ until they converge.

5. Perform firm simulation to obtain firm size distribution $F(m)$.

6. Plug the results in Step 4 and 5 and the simulated $F(m)$ into the closing model conditions, and update the guesses of g, B, A, U_0 until the model converges.

To calculate comparative statics, there is one more outer loop on the value of ψ.

To calculate transition path, there is one more outer loop of guessing the value functions $V(m)$ over the transition path.

C Firm Size Distribution

Firm size distribution

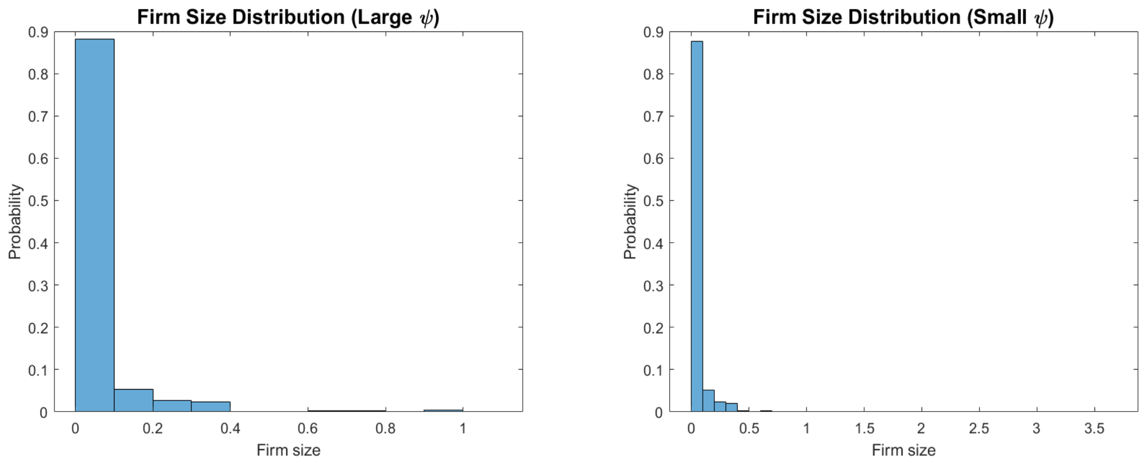


Figure 16: Entire Firm Size Distribution on the Initial and New BGP.

D Comparison across Firm Size

D.1 Advertising

As was analyzed in Section 4, there is both a direct effect and indirect effect on firm's advertising investment. Although all firms have incentives to raise marketing intensity at first, the uneven pass-through rate amplifies the gap between large and small firms in customer base, innovation and marketing. Small firms thus get a smaller fraction of customers in the matches. Due to the limited consumption capacity, the measure of customers exposed to small firms could even shrink as ψ decreases.

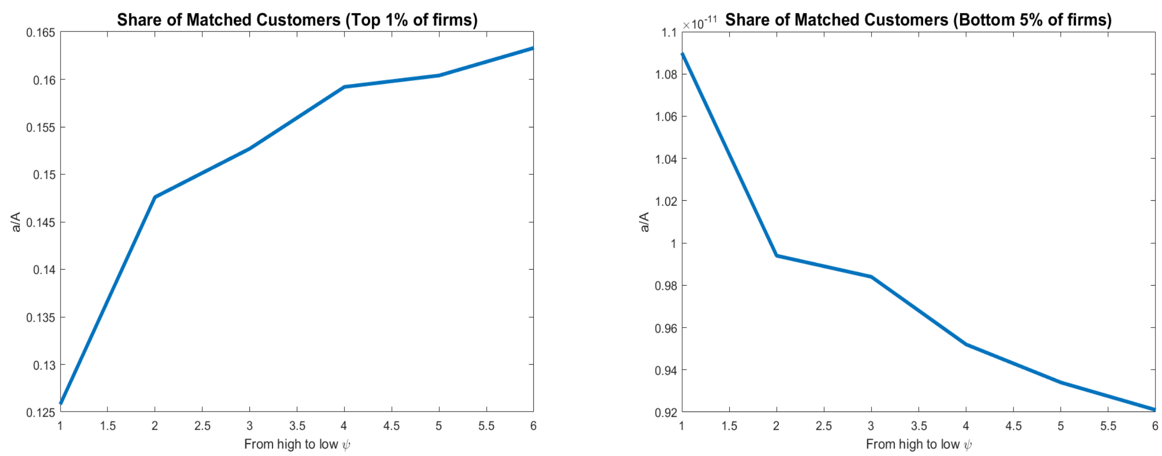


Figure 17: Changes in the Share of Matched Customers for Large and Small Firms

Figure 17 shows in the calibrated model, how the share of matched customers change with ψ for the top 1% of firms and the bottom 5% of firms. At the initial level of marketing efficiency ($\psi = 4.98$), the largest firms attract 12.5% of the searching customers. Then as efficiency rises (ψ decreases), the share surges to 16.5%. On the other hand, the already tiny fraction of visitors for small firms has deteriorated from 1.1E-11 to 9E-12. This means the small firms are having less exposure to the searching customers via advertising when marketing efficiency increases, because their advertisement become shaded in the outburst of marketing by larger competitors.

D.2 Innovation

There is also a sharp contrast between large and small firms in terms of the impact of a lower ψ on firm R&D investment. As equation 19 indicates, the marginal benefit of innovation can be divided into three parts: profits on existing customers, profits on new customers and the increase in future value.

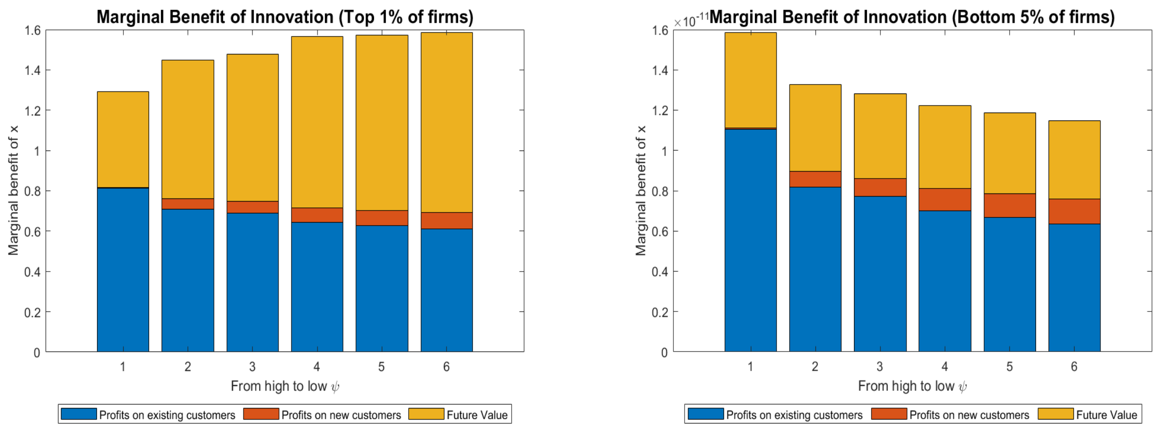


Figure 18: Changes in Marginal Benefit of Innovation for Large and Small Firms

With higher marketing efficiency, searching becomes more attractive and fewer customers choose to be loyal to their original firm. This reduces firms' incentives of extracting surplus from loyal customers by improving product quality. On the other hand, the increasing number of new customers matched through searching boost firm innovation incentives. Regardless of the firm size, an additional unit of innovation brings less profit on existing customers and more on new customers. The third important component in innovation is the difference in future value between a successful and failed innovation. As I have discussed in Section 4, the advantage of more advertising from large firms is weaved into the period-by-period accumulation of customer capital. To take advantage of the richer future profits, large firms have incentives to make more innovations. In contrast, the shrink of customer base for small firms makes them less willing to invest in innovations.