Inequality, Business Cycles, and Growth: A Unified Approach to Stabilization Policies*

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Abstract

This paper studies the efficacy of discretionary stabilization policies, when household heterogeneity, business cycles and long-run growth interact. Consistent with empirical evidence, we develop a unified heterogeneous agent New Keynesian growth framework in which: (i) growth arises from innovative investment, (ii) a demand externality shapes economic activity, and (iii) household heterogeneity acts on those channels through the joint distribution of marginal propensities to consume (MPCs) and marginal propensities to invest (MPIs). First, we analytically show that an income redistribution channel from high-MPI "entrepreneurs" to high-MPC households increases technology growth if the stabilizer is sufficiently persistent. Second, while the aggregate demand externality pushes toward progressive stabilizers, the endogenous growth channel pushes toward regressive stabilizers. When policymakers do not balance among both extremes, a short- versus long-run stabilization tradeoff arises. We quantitatively investigate this tension based on temporary unemployment insurance extensions during the U.S. Great Recession. The government maximizes the short-run output multiplier at a value of 1.2 by financing the policy with progressive income taxes. This is, however, at the cost of a long-run output loss with a multiplier of -0.2. Overall, our analysis provides a rationale to finance stabilizers by shifting the tax incidence on middle-class households.

Keywords: Inequality, Business Cycles, Growth, Stabilization, Redistribution Policies.
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1 Introduction

Economic downturns are accompanied by a set of recurrent empirical facts. At the micro level, low-income households face higher unemployment risk (Storesletten et al., 2004) and earnings losses (Guvenen et al., 2017; Heathcote et al., 2020), resulting in higher income inequality (Coibion et al., 2017; Ampudia et al., 2018). At the macro level, aggregate consumption (Patterson, 2022) and innovative investment (Barlevy, 2007) – such as research and development (R&D) expenditures – strongly contract and may cast long shadows on economic growth.[1] To stabilize aggregate fluctuations and reduce individual hardship, a set of discretionary monetary, fiscal, and social insurance policies are frequently used.[2] This type of discretionary "stabilizers" share a common feature; they redistribute income across households who experience not only a heterogeneous exposure to economic downturns, but also differ in their consumption and investment behavior. How does this redistribution channel affect the efficacy of discretionary stabilization policies? Are there circumstances under which such policies stabilize short-run fluctuations by expanding aggregate demand but harm, at the same time, the long-run recovery from economic downturns by lowering innovative investment and technology growth?

To make progress on these questions, this paper puts forward a theory on discretionary macroeconomic stabilization policies, in which household heterogeneity jointly interacts with business cycle fluctuations and long-run technology growth. Consistent with empirical evidence, we develop a unified *heterogeneous agent New Keynesian growth* framework (hereafter HANK-GS) that synthesizes three main features. First, technology growth and potential output are determined endogenously as the result of innovative investment. Second, the presence of nominal rigidities generates a tight link between aggregate demand and economic activity, i.e., a temporary fall in aggregate demand reduces firm profits, innovative investment, and consequently the long-run level of output. Third, household heterogeneity drives aggregate demand and innovative investment through the joint distribution of marginal propensities to consume (MPCs), marginal propensities to invest (MPIs), and the household exposure to aggregate income.

Through the lens of this framework, we show that the efficacy of stabilization policies fundamentally changes relative to the conventional wisdom that either neglects household heterogeneity, nominal rigidities, or long-run effects on technology growth. While nominal rigitidies provide through the aggregate demand channel a rationale for progressive stabilizers, the endogenous growth channel provides a force toward regressive stabilizers. When policymakers do not bal-

[1] For a multitude of economic downturns, output does not revert to its prerecession trend level such that transitory economic downturns permanently depress the long-run level of economic activity (Cerra and Saxena, 2008; Blanchard et al., 2015; Fatás and Summers, 2018). The Great Recession, for instance, left a permanent output loss of 15% in the US.

[2] During the Great Recession, for instance, monetary authorities of many countries massively lowered nominal interest rates, and even hit negative territories for a prolonged time. The U.S. government implemented a large fiscal stimulus on, among others, unemployment insurance, supplemental nutrition assistance programs, and Medicaid.

ance among both extremes, a discretionary policy engenders a short- versus long-run stabilization tradeoff, i.e., it expands output but harms technology growth and, hence, slows down the long-run recovery from recessions. We illustrate this tension by analyzing, as our leading example, one of the biggest stabilization policies that was implemented during the Great Recession: temporary extensions in the unemployment insurance (UI) duration.[3] A policymaker maximizes the short-run output multiplier at a value of 1.2 after one year by financing the UI extensions through progressive income taxes, which is, however, at the cost of a long-run output loss with a multiplier of -0.2. Overall, our analysis provides a rationale to finance discretionary stabilizers by additional debt issuance and/or by shifting the additional tax incidence on middle-class households. We develop our results in two steps. First, we expose the tradeoffs based on a tractable setup. Second, we quantify aggregate effects based on a rich quantitative setup of the US economy.

The backbone of the theoretical framework builds upon a tractable version of the heterogeneous agent New Keynesian economy (HANK) that we merge with endogenous growth (GS). A continuum of households consume, supply labor, and face idiosyncratic shocks that generate a precautionary self-insurance motive. Due to limited asset market participation, only a fraction of households are unconstrained and participate in financial markets to save and self-insure themselves against income risk. Those households are also the owners of innovative firms and execute managerial control over investment decisions. The remaining households are constrained hand-to-mouth with high MPCs, who consume their disposable income. Nominal wage contracts generate an aggregate demand externality which drives fluctuations in economic activity. On the firm side, our framework relies on an endogenous model of vertical innovation. Firms improve the quality of their goods through innovative investment decisions that are closely linked to the cyclicality of profits. As reward, they are compensated by monopoly rents which, in turn, increase stockholders' dividend income and, thus, inequality. Finally, the government redistributes income across both household types, while a monetary authority sets the nominal interest rate.

The stationary balanced growth path equilibrium of this economy admits a four-equation representation that comprises: (i) an IS equation that captures aggregate demand; (ii) an endogenous growth equation that embodies firms' innovation decisions; (iii) a New Keynesian wage Phillips curve; and (iv) a Taylor-type monetary policy rule. The key novelty of this representation lies in the unified treatment of the short- and long-run propagation of stabilizers, while preserving comparability with existing models studying inequality, business cycles and growth in isolation.

Our first main theoretical result concerns the *income redistribution channel* of discretionary stabi-

[3] In the US, the duration of unemployment benefits was extended in every major recession since 1958 that was characterized by high unemployment. During the Great Recession, unemployment benefit extensions reached up to 99 weeks. As such, unemployment assistance amounted to roughly 25% of the total fiscal stimulus according to the CBO. Such episodes are often accompanied by a rise in the progressivity of income taxation, i.e., income tax progressivity approximately increased by 5–10% in the US during the Great Recession (Bayer et al., 2020).

lization policies, leaving aside potential financing distortions. We show that the effects of temporary variations in income inequality on technology growth depend on three statistics: the income exposure of high-MPI households, that of high-MPC households, and the persistence of the stabilization policy. On the one hand, if a stabilization policy redistributes income away from high-MPI households, expected consumption growth of investors decreases, which raises investment costs in terms of foregone consumption and reduces innovative investment (i.e., a *cost of funds effect*). On the other hand, if a stabilization policy redistributes income toward high-MPC households, aggregate demand overproportionally expands such that firm profits and innovative investment increase (i.e., a *market size effect*). When both forces move in opposite directions, the persistence of the discretionary policy determines the sign of the overall effect. A higher persistence increases the weight on the market size effect as aggregate demand expands for a prolonged amount of time, whereas it reduces the weight on the cost of funds effect as expected investor consumption growth fluctuates less. While a countercyclical income redistribution channel generically leads to greater output stabilization, it, thus, reduces (resp. increases) innovative investment under a low (resp. high) persistent policy. If the stabilization policy is persistent enough, there arises a complementarity between cyclical inequality, innovative investment and output, suggesting long-run scars from inequality. A stabilizer that reduces inequality increases innovative investment, which, in turn, expands aggregate demand and output such that income inequality falls even more.

We summarize these forces in an insightful diagram, which is close in spirit to a *sufficient statistics* approach. It decomposes the effects of the income redistribution channel on the short- and long-run level of output into the cost of funds and the market size effect. This diagram is a powerful device to organize substantial parts of the heterogeneous agent literature in terms of their inequality-investment-efficiency relationship. The previous case in which countercyclical income inequality redistributes income from high-MPC to high-MPI households, for instance, falls into a particular region of the diagram. Despite the parsimony of our analytical framework, we argue that richer quantitative models admit a similar representation with model-specific sign and weights on the cost of fund and market size effects. These insights carry over to a larger class of models with other forms of investment, such as physical capital or skill accumulation.

Our second main theoretical result concerns the *financing channel* of discretionary stabilization policies. We highlight the role of progressive income taxes that may generate additional investment distortions. Depending on the strength of the short-run stabilization in aggregate demand, the efficacy of stabilization policies can be classified into three regimes, in which: (i) both output and technology growth increase; (ii) output increases but technology growth decreases; and, (iii) both output and technology growth decrease. These regimes arise as a heterogeneous tax incidence moves the aggregate demand and the innovative investment channel into opposite directions. Absent both channels, progressive income redistribution reduces inequality but is irrelevant for output, which is purely supply determined. In the presence of an aggregate demand externality, progressive income redistribution expands output, as it redistributes income to high-MPC households. This is in stark contrast to an environment with endogenous growth. Without aggregate demand externality, progressive income redistribution is contractionary for short-run output and technology growth, as it distorts innovation decisions of high-MPI households beyond the cost of funds effect. As a result, if the aggregate demand expansion is sufficiently strong (resp. weak), then output and innovative investment increase (resp. contract). Importantly, for moderate aggregate demand expansions, output increases in the short-run while innovative investment falls. As a result, the policy engenders a long-run output loss after a certain time horizon.

To assess the overall efficacy of a discretionary stabilizer based on the income redistribution and the financing channel, it is, thus, important to put a realistic quantitative structure on aggregate demand, innovative investment, and the policy itself. We therefore build a full-blown incomplete markets HANK-GS framework that extends the analytical setup along three dimensions. First, we incorporate multiple layers of household heterogeneity through discount factors, labor productivity, unemployment risk, and entrepreneurial talent. Households that suffer from bad labor market outcomes end up at the bottom of the wealth distribution and exhibit, on average, a high MPC. Households with entrepreneurial talent exert costly effort to accumulate innovations. Within this group, the most successful entrepreneurs self-select at the top of the income distribution, which generates a positive cross-sectional correlation between MPIs and income. Second, households face idiosyncratic earnings risk on the intensive margin through hours worked and on the extensive margin through the employment status. Both adjustment margins are correlated with worker's ability, which generates countercyclical income inequality. Third, the government partially insures households against income risk through a rich set of policies comprising safety-net programs, unemployment insurance benefits, and progressive income taxes.

We use household panel data from the Current Population Survey (CPS) to discipline the distributional burden of economic downturns and their demand repercussions. We pin down nonuniform adjustments in hours worked and unemployment risk across different wage bins such that the model replicates well the joint distribution of MPCs and earnings exposures. Moreover, we target the elasticity of innovative investment to corporate income taxes as estimated by Akcigit et al. (2022). They provide extensive empirical evidence on the responsiveness of innovative activities. The model generates, thus, a reasonable average MPI regarding innovative investment. Moreover, the presence of a small fraction of entrepreneurs who experience high returns to innovative investment brings about the empirically observed concentration of income and wealth.

To quantitatively highlight the pure income redistribution channel, we consider an unanticipated monetary policy tightening, which substantially reduces innovative investment. The reduction in technology growth at impact is, perhaps surprisingly, around 50% smaller in comparison to a representative agent economy. This is the case because the cost-of-fund effect is less pronounced in heterogeneous agent economies, i.e., entrepreneurs are relatively richer and, thus, less sensitive to income shocks. We also find that the persistence of monetary policy has non-linear effects on innovative investment. Countercyclical income inequality leads to a sizeable fall in short- and long-run output above a quarterly persistence value of 0.7. As a result, there are long-run scars from cyclical inequality but, at the same time, long-run gains from household heterogeneity *per se*.

To jointly study the income redistribution and the financing channel, we evaluate the temporary extension in the UI benefit duration. The efficacy of this policy in stabilizing the US economy heavily depends on its financing. Debt financing and lump-sum taxes both generate large short-run and moderate long-run output gains. Under both financing instruments, the aggregate demand expansion is large and increases innovative investment. In contrast, government spending cuts crowd out consumption and lead to mild short- and long-run effects. Progressive income taxation stabilizes short-run output most effectively, which is at the cost of a large long-run loss. Importantly, the ordering of these financing instruments is reversed when abstracting from either nominal rigidities or endogenous growth. Progressive income taxation becomes the most powerful instrument to stabilize output absent endogenous growth, while it turns out to be the least effective instrument absent nominal rigidities. Finally, we zoom into the progressive income tax results and take a detailed look at the effects of a heterogeneous, non-monotonous tax incidence. When gradually shifting the tax incidence from low-income to middle-income households, short-run output and technology growth increase. Instead, when high-incomes are relatively more taxed, short-run output still expands but technology growth falls. Our results, thus, provide a rationale for higher taxes on middle-class households to stabilize economic downturns. The UI extension engenders in this case both sizable short- and long-run output gains.

Related literature This paper is part of a vast literature studying the interaction between household heterogeneity and macroeconomic outcomes. As our main contribution, we provide a novel perspective on the effectiveness of discretionary stabilizer when household heterogeneity, business cycles and long-run growth interact. We relate foremost to four strands of the literature.

First, our paper relates to the analytical and quantitative HANK literature. On the analytical side, we relate to Bilbiie (2008, 2020, 2021), Debortoli and Galí (2018), Acharya and Dogra (2020), Broer et al. (2020), Ravn and Sterk (2020), and Cantore and Freund (2021). We propose an alternative tractable HANK representation based on a general earnings incidence specification in an environment with limited asset markets participation and sticky wages, and thus conceptually follow Werning (2015).[4] On the quantitative side, we relate to McKay et al. (2016), Kaplan et al.

[4] We follow a recent echo advocating the use of sticky wages over sticky prices (Nekarda and Ramey, 2020), applied in quantitative HANK models among others by Hagedorn et al. (2019a), Hagedorn et al. (2019b), Auclert et al. (2021), or Dávila and Schaab (2022) and in tractable ones by Colciago (2011), Ascari et al. (2017), and Walsh (2017).

(2018), Auclert et al. (2018) Bayer et al. (2019), Auclert (2019), Auclert and Rognlie (2020), and Bayer et al. (2020). We contribute to this literature by synthesizing HANK with a vertical innovation model of endogenous growth. As the latter acts through an investment channel, our paper shares similarities with Galí et al. (2007) and Bilbiie et al. (2022) highlighting the propagation of a consumption-investment accelerator on consumption (Samuelson, 1939). Relative to them, we provide a decomposition of the effects of cyclical inequality on investment (Luetticke, 2021; Kekre and Lenel, 2022) and link it to the persistence of redistributionary stabilizers.

Second, our paper relates to an older, but rapidly growing, literature that jointly studies business cycles and endogenous growth (cf. Romer (1990), Grossman and Helpman (1991), Aghion and Howitt (1992)). Stadler (1990), Fatás (2000), and Comin and Gertler (2006) are early contributions, while Benigno and Fornaro (2018), Moran and Queralto (2018), Anzoategui et al. (2019), and Garga and Singh (2021) advanced those models toward a Keynesian growth economy.[5] We enrich this strand by inequality and a heterogeneous income exposure during economic downturns that affects aggregate outcomes through the joint distribution of MPCs and MPIs.

Third, the HANK-GS framework is empirically consistent with a growing literature that argues how transitory economic downturns permanently depress the level of economic activity, i.e., generate *hysteresis* or *scarring*. Fatás (2000) and Blanchard et al. (2015) show that business cycles cast long shadows, while Cerra and Saxena (2008) find that output drops are particularly persistent in the aftermath of financial and political crises. Fatás and Summers (2018) and Cerra et al. (2021) provide a detailed overview on this strand. Recently, structural vector autoregression models have been used to identify the long-lasting effects of transitory stabilization policies. Moran and Queralto (2018), Jordá et al. (2020) and Garga and Singh (2021) show that loose monetary policy has a positive and long-lived effect on output, R&D expenditures and total factor productivity. Antolin-Diaz and Surico (2022) and Ilzetzki (2022) document such effects for government spending increases, while Cloyne et al. (2022) obtains similar patterns for corporate tax cuts.[6]

Fourth, by analyzing the efficacy of fiscal stabilization policies in heterogeneous agent incomplete markets frameworks with nominal rigidities, our paper is closely related to McKay and Reis (2016), McKay and Reis (2021), Kekre (2022) and Ferriere and Navarro (2022). While the former two papers highlight the contribution of the *ex-ante* level of automatic stabilizers to the US business cycle, the latter two focus on the effects of discretionary stabilizers. Kekre (2022) quantitatively shows that temporary UI extensions raised output and lowered unemployment during the

[5] Bianchi et al. (2019), Queralto (2020), Fornaro and Wolf (2020, 2021), Cozzi et al. (2021), Licandro and Vinci (2021) and Queralto (2022) are further important contributions in this strand.

[6] Related, Furlanetto et al. (2021) identify demand shocks that have permanent effects on output and show that they are quantitatively relevant in driving the US business cycle and disproportionally affecting the least productive workers. Similarly, Maffei-Faccioli (2021) documents that demand-side factors explain large parts of the slow-down in productivity in the US over the past two decades. Bertolotti et al. (2022) suggest that complementary demand and supply factors led to a downward quality adjustment in durable-goods purchases during the Great Recession. Ignaszak and Sedláček (2021) confirm those findings based on an endogenous growth model calibrated to US census firm data.

Great Recession, when they are financed by taxing employed households. Ferriere and Navarro (2022) empirically and quantitatively show that U.S. fiscal multipliers are larger when they are financed by a higher degree of income tax progressivity. Instead, our paper points toward an ambivalent role of progressive stabilization policies: while they improve the short-run recovery from recessions, they are harmful for the long-run recovery by worsening innovation incentives. Our analysis is, thus, consistent with inconclusive empirical evidence on the effects of tax cuts on economic growth (Stokey and Rebelo, 1995; Jaimovich and Rebelo, 2017; Jones, 2022).

Roadmap In section 2, we construct an analytical heterogeneous agent New Keynesian growth economy in order to highlight the main theoretical tradeoffs and insights. In section 3, we set out the quantitative environment and discuss the model calibration. Section 4 inspects properties of the quantitative model. Section 5.1 studies the transmission of monetary policy, while section 5.2 analyzes UI generosity as macroeconomic stabilization policy. Section 6 concludes. We refer analytical derivations, proofs, empirical analyses and computational details to the Appendix.

2 An Analytical HANK Growth Economy

This section analytically characterizes the short- and long-run propagation of macroeconomic stabilization policies. For this purpose, we develop a "limited heterogeneity" setup to shed light on the key mechanisms that are present in rich quantitative models, as the one in section 3. In this economy, technology growth arises from innovative investment; aggregate demand shapes economic activity due to nominal rigidities; household heterogeneity impacts aggregate consumption and investment decisions; and the welfare state insures against individual hardship.

2.1 Environment

Time is discrete. The economy consists of infinitely-lived households, final and intermediary good firms, a government and a central bank.

2.1.1 Households There is a unit mass $i \in [0,1]$ of households who discount future periods at $\beta \in (0,1)$. Their instantaneous utility depends on consumption $C_{i,t}$ and hours worked $L_{i,t}$, i.e., $\mathcal{U}(C_{i,t}, L_{i,t}) = \ln C_{i,t} - \nu \frac{(L_{i,t})^{1+\varphi}}{1+\varphi}$, where $\nu > 0$ is a labor disutility shifter and φ^{-1} the Frisch elasticity.

There are two household groups. *Saver* households S participate in asset markets. Their portfolio is composed of liquid bond holdings $b^S_{i,t+1}$ with gross return $R_t = 1 + i_t$, and a share of illiquid stocks $\omega^S_{i,t+1} \in [0,1]$ of intermediary good firms that are priced at q_t. Instead, *hand-to-mouth* households H cannot participate in stock markets and only use liquid bond holdings $b^H_{i,t+1}$ to insure against income risk. Households face a sequence of idiosyncratic shocks that induce them to switch across saver and hand-to-mouth states. Saver households stay in state S with probability $s \equiv \mathbb{P}(S_{t+1}|S_t)$, while hand-to-mouth households remain in state H with probability $h \equiv \mathbb{P}(H_{t+1}|H_t)$, such that $1-s \equiv \mathbb{P}(H_{t+1}|S_t)$ and $1-h \equiv \mathbb{P}(S_{t+1}|H_t)$. We restrict our focus to stationary equilibria, where the mass of hand-to-mouth households is $\lambda = \frac{1-s}{2-h-s}$.

States S and H can be viewed as separate islands. At the beginning of period t, households on the same island pool resources and face an aggregate shock before they make equivalent consumption and saving decisions. At the end of period t, they observe their future $t+1$ state and settle on the corresponding island while transferring only liquid bonds. B^S_{t+1} (resp. B^H_{t+1}) denotes *total* real bond holdings of all households on island S (resp. H) at the beginning of period $t+1$, after they have moved across islands. On the contrary, b^S_{t+1} (resp. b^H_{t+1}) denotes *per capita* real bond holdings at the end of period t before households change states. The laws of motion are

$$B^S_{t+1} = (1-\lambda)s b^S_{t+1} + \lambda(1-h) b^H_{t+1}\,, \qquad B^H_{t+1} = (1-\lambda)(1-s) b^S_{t+1} + \lambda h b^H_{t+1}\,. \tag{1}$$

Saver households work L^S_t hours, specified below, at a real wage $\frac{W_t}{P_t}$, where P_t is the aggregate price level. They also receive a real after-tax dividend income D_t. The value of their program is

$$V^S(B^S_t, \omega^S_t) = \max_{\{C^S_t,\, b^S_{t+1},\, \omega^S_{t+1}\}} \mathcal{U}(C^S_t, L^S_t) + \beta \mathbb{E}_t \left[V^S(B^S_{t+1}, \omega^S_{t+1}) + \frac{\lambda}{1-\lambda} V^H(B^H_{t+1}) \right]$$
$$s.t. \qquad C^S_t + b^S_{t+1} + q_t \frac{\omega^S_{t+1}}{1-\lambda} = \frac{W_t}{P_t} L^S_t + \frac{R_{t-1}}{\pi_t}\frac{B^S_t}{1-\lambda} + (q_t + D_t)\frac{\omega^S_t}{1-\lambda}\,, \quad b^S_{t+1} \geq 0\,, \quad \text{and} \quad (1)\,,$$

where $\pi_t \equiv P_t/P_{t-1}$ is the gross inflation rate.[7] Similarly, hand-to mouth households work L^H_t hours and receive real transfers T^H_t from the government. The value of their program is

$$V^H(B^H_t) = \max_{\{C^H_t,\, b^H_{t+1}\}} \mathcal{U}(C^H_t, L^H_t) + \beta \mathbb{E}_t \left[V^H(B^H_{t+1}) + \frac{1-\lambda}{\lambda} V^S(B^S_{t+1}, \omega^S_{t+1}) \right]$$
$$s.t. \qquad C^H_t + b^H_{t+1} = \frac{W_t}{P_t} L^H_t + \frac{R_{t-1}}{\pi_t}\frac{B^H_t}{\lambda} + \frac{T^H_t}{\lambda}\,, \quad b^H_{t+1} \geq 0\,, \quad \text{and} \quad (1)\,.$$

We now specify the insurance scheme across agents that we select and solve for in equilibrium.

Assumption 1 (INSURANCE). *(A1.a) there is perfect insurance among households being in the same state, but not across states; (A1.b) stocks are illiquid and cannot be transferred across states; (A1.c) liquid bond holdings are weakly positive and the positivity constraint of hand-to-mouth households binds in each period; and (A1.d) no bonds are traded in equilibrium.*

$(A1.a)$-$(A1.d)$ jointly apply and ensure a tractable representation of the bond market equilibrium. Under $(A1.a)$-$(A1.b)$, households living on the same island make equivalent consumption and saving decisions. $(A1.c)$ guarantees that H households are strictly hand-to-mouth, while

[7] The next period hand-to-mouth value function is scaled by $\frac{\lambda}{1-\lambda}$ as the state variable is written in terms of total island bond holdings. As such, the relative transition probability $\frac{\lambda}{1-\lambda}$ maps into individual probabilities $(s, 1-s)$.

$(A1.d)$ imposes a zero liquidity limit (Krusell et al., 2011).

2.1.2 Labor Market Nominal rigidity arises from sticky wages. As such, hours worked are determined by labor demand in equilibrium. Sticky wages generate empirically consistent *procyclical* dividends, which are crucial for innovation decisions in an endogenous growth environment.[8] Subsequently, "hat"-variables refer to percentage deviations from the steady-state.

Assumption 2 (EARNINGS INCIDENCE). *Hours worked are uniformly distributed across household types in the steady state, i.e., $L^H = L^S = L$, but they may fluctuate dis-proportionally off the steady-state, i.e., $\hat{L}_t^H = \mu \hat{L}_t$, where $\mu \in [0, \overline{\mu})$ and $\lambda^{-1} < \overline{\mu}$.*

The parameter μ summarizes the distributional burden of adjustments in aggregate hours worked across the income distribution.[9] The upper bound $\overline{\mu}$ ensures that a tighter monetary policy restrains aggregate demand (Bilbiie, 2008; Bilbiie and Straub, 2013).

The incidence modelling allows to flexibly cover a number of cases. From Assumption 2 it follows that $\hat{L}_t^S = \frac{1-\lambda\mu}{1-\lambda}\hat{L}_t$. Suppose now that aggregate hours worked increase. First, if $\mu = 1$, hours worked are split proportionally off the steady state. Second, if $\mu = 0$, hand-to-mouth households permanently work the amount of steady state hours and the labor adjustment is completely borne by saver households. In contrast, if $\mu = \lambda^{-1}$, the entire labor adjustment is borne by hand-to-mouth households and our setup mimics the capitalist-worker dynamics of Broer et al. (2020, 2021a). Third, if $\mu \in (0, 1)$, hours worked of hand-to mouth households increase underproportionally, whereas hours worked of savers increase overproportionally. On the interval $\mu \in (1, \lambda^{-1})$, hand-to-mouth households overproportionally increase hours worked, while savers increase hours worked underproportionally. Finally, $\mu \in (\lambda^{-1}, \overline{\mu})$ implies that hours worked of hand-to-mouth households increase overproportionally, whereas hours worked of savers fall.[10]

Wage Rigidity Aggregate labor L_t is composed of a continuum $l \in [0, 1]$ of differentiated labor inputs $L_t(l)$ that are bundled according to a CES aggregator with elasticity $\epsilon_w > 1$. Each union specifies a nominal wage $W_t(l)$. The demand for labor input l is $L_t(l) = (W_t(l)/W_t)^{-\epsilon_w} L_t^d$, where

[8]The use of sticky prices would not only generate countercyclical markups and profits but also cause a negative wealth effect which increases hours worked (Bilbiie, 2008, 2020). The distribution of profits thus substantially affects aggregate labor dynamics in sticky price HANK models.

[9]Assumption 2 is a two state analog to the general $\gamma(i, L_t)$ incidence functions used in Werning (2015), Auclert and Rognlie (2020), Alves et al. (2020) and Patterson (2022). Auclert and Rognlie (2020), for instance, write labor income as $\frac{W_t}{P_t} \cdot L_t \cdot \gamma(i, L_t)$. Applied to our setting, it follows that $\mu = 1 + \epsilon_{\gamma,L}^H$, where $\epsilon_{\gamma,L}^H$ is the elasticity of the incidence function with respect to total labor L_t in state H. Normalization implies $\epsilon_{\gamma,L}^S = -\frac{\lambda}{1-\lambda}\epsilon_{\gamma,L}^H$.

[10]Empirically, the earnings incidence elasticity μ maps into extensive margin exposure through unemployment risk (Storesletten et al., 2004; Guvenen et al., 2014; Kramer, 2022), as well as intensive margin exposure through *worker betas* (Guvenen et al., 2017). Those papers point out that the bottom of the income distribution overreacts to aggregate shocks, pushing toward $\mu > 1$. Moreover, Coglianese et al. (2022) provide a detailed overview of a growing number of papers documenting rising income inequality conditional on contractionary monetary policy shocks (Heathcote et al., 2010; Coibion et al., 2017; Ampudia et al., 2018; Holm et al., 2021).

W_t denotes an average wage index, and L_t^d aggregate labor demand. Each union is run by a manager who sets wages to maximize the utility of a hypothetical average household that is composed out of a hand-to-mouth and saver member with $L_t(l) = \lambda L_t(l)^H + (1-\lambda)L_t^S(l)$. Wage setting is subject to quadratic adjustment costs (Rotemberg, 1982). The manager of union l solves

$$\max_{\{W_t(l)\}} \mathcal{U}(C_t(l), L_t(l)) \quad s.t. \quad C_t(l) = \frac{W_t(l)}{P_t} L_t(l) + D_t(l) + T_t^H - \frac{\theta}{2}\left(\frac{W_t(l)}{W_{t-1}} - g^A\right)^2 Y_t \,, \tag{2}$$

where costs are symmetric around the gross technology growth rate g^A, proportional to production Y_t and scaled by $\theta > 0$. In equilibrium, all unions set the same wage, i.e., $W_t = W_t(l)$.

2.1.3 Production A final good Y_t^G is produced competitively by using a continuum $j \in [0,1]$ of intermediary goods $X_{j,t}$ of available quality $A_{j,t}$ with technology

$$Y_t^G = (Z_t L_t)^{1-\alpha} \int_j A_{j,t}^{1-\alpha} X_{j,t}^{\alpha} dj \,, \tag{3}$$

where $\alpha \in (0,1)$ is the share of intermediary goods. Moreover, $Z_t = Ze^{\hat{z}_t}$ denotes an aggregate technology shock with $Z > 0$ and $\hat{z}_t$ follows an AR(1) process with persistence ρ_z. The final good is sold at price P_t, while intermediary goods are bought at price $P_{j,t}$. The final good firm maximizes profits by choosing the demand for labor L_t^d and intermediary goods $X_{j,t}^d$.

Intermediary Goods Intermediary goods are produced by identical firms. Each firm behaves as a monopolist and sets a price $P_{j,t}$ to maximize profits. Additionally, it invests an amount $I_{j,t}$ into technology-enhancing activities, such as $R\&D$ expenditures, to improve the future quality $A_{j,t+1}$.

Price Setting. To produce $X_{j,t}$, intermediary good firms transform one unit of the final good into one unit of the respective intermediary good. They maximize nominal profits, i.e.,

$$\max_{\{P_{j,t}\}} \Theta_{j,t}^n \equiv (P_{j,t} - P_t) X_{j,t}^d \quad s.t. \quad \frac{P_{j,t}}{P_t} = \alpha (Z_t L_t^d)^{1-\alpha} A_{j,t}^{1-\alpha} (X_{j,t}^d)^{\alpha-1} \,, \tag{4}$$

where the constraint specifies the demand for a particular intermediary input. The solution to this problem is characterized by $\alpha P_{j,t} = P_t$, where $\alpha^{-1} > 1$ is the gross markup over marginal costs. Substituting the price setting relation into the maximization constraint yields $X_{j,t} = \alpha^{\frac{2}{1-\alpha}} A_{j,t} Z_t L_t$ such that final good production is $Y_t^G = \alpha^{\frac{2\alpha}{1-\alpha}} A_t Z_t L_t$, where $A_t \equiv \int_0^1 A_{j,t} dj$ denotes the average quality index. Real profits, in turn, are $\Theta_{j,t} = \Theta_\alpha A_{j,t} Z_t L_t$ with $\Theta_\alpha = \alpha^{-1}(1-\alpha)\alpha^{\frac{2}{1-\alpha}}$ and, thus, proportional to quality $A_{j,t}$, labor L_t, and exogenous technology Z_t.

Innovative Investment. Intermediary firms use the final good to invest an amount $I_{j,t}$ into innovative activities by maximizing the discounted sum of real after tax profits. To allow social investment returns to exceed private returns, we assume that an incumbent firm dies with probability $\delta \in (0,1)$ and is replaced by a new entrant that inherits the previous technology stock. As stocks are illiquid assets, the discount factor of intermediary good producers is adjusted by the survival probability $1-\delta$ and the probability s to stay on island S.[11] Given $A_{j,0} > 0$, their objective is

$$\max_{\{A_{j,t+1}\}} \mathbb{E}_0 \left[\sum_{t=0}^{\infty} \frac{(\beta s(1-\delta))^t}{C_t^S} \left((1-\tau_t^D)\Theta_{j,t} - s(1-\delta) I_{j,t} \right) \right] , \quad s.t. \quad A_{j,t+1} = A_{j,t} + \psi I_{j,t} , \tag{5}$$

where $\psi \geq 0$ is the efficacy of innovative investment on quality growth. The law of motion implies constant returns to scale in the production of new quality, i.e., a higher quality stock requires higher investment to maintain a certain quality growth rate. Finally, $\tau_t^D \in \left[\underline{\tau}^D, \overline{\tau}^D \right]$ denotes a distortive profit tax. As steady-state earnings are equalized across households under Assumption 2, τ_t^D reflects the progressivity of income taxation.

2.1.4 Fiscal and Monetary Policy The government partially insures against income risk by redistributing income through lump-sum transfers to hand-to-mouth agents. Its budget balances

$$T_t^H = \tau_t^D \Theta_t , \tag{6}$$

where $\Theta_t \equiv \int_j \Theta_{j,t} dj$ are aggregate intermediary goods profits. Moreover, monetary policy sets the nominal interest rate i_t according to a Taylor rule, which reacts to gross wage inflation $\pi_t^w \equiv W_t / W_{t-1}$, and an AR(1) shock ε_{mt} with persistence ρ_m, i.e.,

$$i_t = r + \phi_\pi \ln \left(\frac{\pi_t^w}{\pi^w} \right) + \varepsilon_{mt} , \quad \text{with} \quad \phi_\pi > 0 . \tag{7}$$

2.1.5 Market Clearing Aggregate labor and consumption are denoted by $L_t = \lambda L_t^H + (1-\lambda) L_t^S$ and $C_t = \lambda C_t^H + (1-\lambda) C_t^S$, respectively. Similarly, aggregate investment is $I_t \equiv \int_j I_{j,t} dj$, and $X_t \equiv \int_j X_{j,t} dj$ is the total amount of intermediate production. Real dividends are the sum of final and intermediary good profits, i.e., $D_t = Y_t^G - \frac{W_t}{P_t} L_t - \alpha^{-1} X_t + (1-\tau_t^D) \left(\alpha^{-1} - 1 \right) X_t - I_t$. Stock market clearing implies $\omega_t^S = \omega_{t+1}^S = 1$ in all periods. Aggregate quality growth follows from the law of motion by $g_{t+1}^A \equiv \frac{A_{t+1}}{A_t} = 1 + \psi \frac{I_t}{A_t}$. Finally, by using the household budget constraints and the labor union problem, the aggregate resource constraint is written as

$$Y_t = C_t + I_t + \frac{\theta}{2} \left(\pi_t^w - g^A \right)^2 Y_t , \tag{8}$$

where we define $Y_t \equiv Y_t^G - X_t = Y_\alpha A_t Z_t L_t$ as net gross domestic product, with $Y_\alpha = (1-\alpha^2)\alpha^{\frac{2\alpha}{1-\alpha}}$.

[11] These assumptions may reflect the prevalence of competitors or temporary rents from the exclusive use of patents. The presence of s reflects additional under-investment due to the income risk of savers, e.g., their entrepreneurial talent.

2.2 Balanced Growth Path Equilibrium

As a solution concept to the HANK-GS economy, we focus on a competitive stationary balanced growth path equilibrium. It is defined as the sequence of trending variables and prices $\{C_t, C_t^S, C_t^H, W_t, D_t, T_t^H, Y_t, Y_t^G, X_t, I_t\}$ that are normalized by the level of endogenous technology A_t. Subsequently, we denote stationary variables in small letters and impose without loss of generality $Z = 1$ and $\pi = 1$. To solve for the *approximate* stationary equilibrium, we log-linearize the economy around its non-stochastic steady-state. Subsequently, we restrict the analysis to a balanced growth path with positive household consumption and technology growth.

Proposition 1 (BALANCED GROWTH PATH (BGP))**.** *If $L > L^*$, the economy admits a unique steady-state with strictly positive individual consumption $(c^H, c^S) \in \mathbb{R}_+^2$ and technology growth $g^A > 1$ on $\tau^D \in \left(\underline{\tau}^D, \tau^D_{g^A}\right)$, where $\tau^D_{g^A} < \overline{\tau}^D$. The associated labor threshold and tax bounds are given by*

$$L^* \equiv \frac{\beta^{-1} - s(1-\delta)}{\psi\alpha^{-1}(\alpha+\lambda)\Theta_\alpha}\,, \qquad \underline{\tau}^D \equiv -\frac{\lambda}{\alpha}\,, \quad \textit{and} \quad \overline{\tau}^D \equiv 1 + \frac{1-\lambda}{\alpha(1-\beta)} + \frac{1-\beta s(1-\delta)}{\psi(1-\beta)\Theta_\alpha L}\,.$$

Proposition 1 states that the BGP features strictly positive technology growth if hours worked are sufficiently large, i.e., labor disutility ν is below a certain value ν^*. In this case, intermediate good firms face a sufficiently large market size that raises profits and innovative investment. Steady-state hours worked are spelled out as an implicit function in Appendix B. The tax bounds $\underline{\tau}^D$ and $\overline{\tau}^D$ ensure strictly positive hand-to-mouth and saver consumption, while $\tau^D_{g^A}$ guarantees strictly positive technology growth.

BGP Inequality The steady-state may feature consumption and income inequality across household types, which we measure by the ratio of S over H consumption and income, i.e., $\Gamma \equiv \frac{c^S}{c^H}$ and $\Gamma_y \equiv \frac{y^S}{y^H}$, respectively. If the consumption share in total income, s_c, is lower than unity, income inequality is larger than consumption inequality as $\Gamma_y = \frac{\Gamma}{s_c} + \frac{\lambda}{1-\lambda}\frac{1-s_c}{s_c}$.

2.3 Cyclical Income Inequality

The short- and long-run effects of stabilization policies propagate through cyclical variations in income inequality. Lemma 1 provides a measure of cyclical hand-to-mouth income.

Lemma 1 (INCOME CYCLICALITY)**.** *Given the share of hand-to-mouth agents λ, the inverse price markup α and the degree of profit redistribution τ^D, hand-to-mouth income fluctuates with ($\hat{y}_t$, $\hat{z}_t$, $\hat{\tau}_t^D$) according to*

$$\hat{y}_t^H = \chi\hat{y}_t + (1-\chi)\hat{z}_t + \chi_\tau\hat{\tau}_t^D\,, \quad \textit{with} \quad \chi \equiv 1 + (\mu-1)\frac{\lambda}{\lambda+\alpha\tau^D}\,, \quad \chi_\tau \equiv \frac{\alpha\tau^D}{\lambda+\alpha\tau^D}\,,$$

where χ serves as a sufficient statistic, which is strictly increasing in the earnings incidence elasticity μ and positive on the support of $\mu \in [0, \overline{\mu})$ if $\tau^D > 0$. For $\tau^D > \underline{\tau}^D$, we obtain $\chi \leq 1$ (> 1) iff $\mu \leq 1$ (> 1).

To interpret the cyclical behavior of hand-to-mouth income (or consumption) and isolate the pure HANK channel from the endogenous growth forces, we first abstract from aggregate technology or tax shocks, as well as from quality-enhancing investment, i.e., $\hat{z}_t = \hat{\tau}_t^D = 0$ and $\psi = 0$. If, for instance, $\mu > 1$ holds, χ decreases in α and τ^D, while it increases in λ if $\tau^D > 0$ (resp. decreases if $\tau^D < 0$). To understand these comparative statics, consider hand-to-mouth consumption, which is the sum of earnings $w_t L_t^H$ and transfers t_t^H. Under Assumption 2, earnings fluctuate with aggregate income according to $\mu \hat{y}_t$. As intermediary firm profits vary proportionally and procyclically with aggregate income, transfers fluctuate with aggregate income by $\hat{y}_t$. Thus, hand-to-mouth consumption fluctuates as a weighted sum of earnings and transfers. Under Lemma 1, the sign and degree of cyclical income inequality are captured by χ, such that

$$\hat{y}_t^S - \hat{y}_t^H = \frac{1}{1-\lambda}\frac{y}{y^S}\left((1-\chi)\hat{L}_t - \chi_\tau \hat{\tau}_t^D\right), \quad \text{with} \quad \frac{\partial(\hat{y}_t^S - \hat{y}_t^H)}{\partial \hat{y}_t} = \frac{1-\chi}{1-\lambda}\frac{y}{y^S}.$$

Conditional on technology and tax shocks, one recovers three cases: (i) if $\chi = 1$ $(\mu = 1)$ earned income inequality is acyclical, (ii) if $\chi > 1$ $(\mu > 1)$ earned income inequality is countercyclical, and (iii) if $\chi < 1$ $(\mu < 1)$ earned income inequality is procyclical. Higher profit taxes raise the sensitivity of hand-to-mouth income to aggregate income, rather than idiosyncratic earnings, and thus lower the degree of countercyclical income inequality.[12]

An interesting property of the earnings incidence elasticity μ lies in its tight relation to aggregate demand fluctuations. In Appendix OA2.3, we show that the earnings incidence is identified by $\mu = 1 + \frac{cov(mpc_{ie}, v_{ie})}{\beta\lambda}$, where $cov\,(mpc_{ie}, v_{ie})$ denotes the covariance between individual MPCs out of earnings and the elasticity of earnings with respect to aggregate earnings. As such, if $cov\,(mpc_{ie}, v_{ie}) > 0$, aggregate demand reacts overproportionally to recessionary shocks, and stabilization policies for aggregate demand become, *ceteris paribus*, more desirable.

2.4 A General Four-Equation Representation

To analyze the efficacy of stabilization policies, we now derive a general four-equation representation of the HANK-GS economy. It builds upon (i) an aggregate demand (IS) equation, (ii) an endogenous growth (EG) equation, (iii) a sticky wage Phillips curve, and (iv) a Taylor rule.[13]

2.4.1 Aggregate IS Equation

[12] *Comparison:* the sensitivity of hand-to-mouth consumption w.r.t. aggregate income is $\chi^{\text{Bilbiie}} = 1 + \varphi\left(1 - \tau^D/\lambda\right)$ in the sticky price tractable HANK model of Bilbiie (2020). Both statistics coincide if $\mu = 1 + \left(1 + \alpha\tau^D/\lambda\right)\left(\chi^{\text{Bilbiie}} - 1\right)$. Under exogenous growth, the aggregate Euler equations are isomorphic under this parameter specification such that one recovers equivalent individual dynamics, with the sole distinction that profits are procyclical.

[13] Subsequently, we state the four-equation representation without exogenous technology shocks and defer a complete treatment to Appendix B.3.

Proposition 2 (IS EQUATION)**.** *The aggregate IS equation is given by*

$$\hat{y}_t = \underbrace{\zeta_f \mathbb{E}_t\left[\hat{y}_{t+1}\right]}_{\text{① transitory income}} - \underbrace{\zeta_r\left(\hat{i}_t - \mathbb{E}_t\left[\hat{\pi}_{t+1}\right]\right)}_{\text{② real interest rate}} + \underbrace{\zeta_g \hat{g}^A_{t+1} - \zeta_{g'}\mathbb{E}_t\left[\hat{g}^A_{t+2}\right]}_{\text{③ permanent income}} + \underbrace{\zeta_\tau \hat{\tau}^D_t - \zeta_{\tau'}\mathbb{E}_t\left[\hat{\tau}^D_{t+1}\right]}_{\text{④ redistributive tax}}, \quad \text{(E.1)}$$

where $\zeta_f = 1 + (1-\tilde{s})\frac{s_c\chi - 1}{1-\lambda\chi\frac{y^H}{y}}$, $\zeta_r = \frac{(1-\lambda)\Gamma s_c}{(1-\lambda)\Gamma + \lambda - \lambda s_c \chi}$, $\zeta_g = \frac{(1-\lambda)\frac{c^S}{y} + \frac{g^A}{\psi y}}{1-\lambda\chi\frac{y^H}{y}}$, *and* $\zeta_{g'} = \tilde{s}\frac{g^A}{\psi y}\frac{1}{1-\lambda\chi\frac{y^H}{y}}$ *with* $\tilde{s} = s/(s+(1-s)\Gamma)$ *and* $1-\tilde{s}$ *denoting the inequality-weighted transition probability measure of risk. The remaining coefficients* ζ_τ*, and* $\zeta_{\tau'}$ *are provided in Appendix B.3.*

The first two components are standard. The forward-looking *transitory income channel* ① captures the sensitivity of today's income with respect to future income. Innovative investment reduces this sensitivity as $s_c < 1$, while a higher steady-state hand-to-mouth income share y^H/y increases this sensitivity as the average MPC level rises. Whenever the transition probability $\tilde{s} < 1$, countercyclical income inequality ($\chi > 1$) is no longer sufficient for compounding of the IS equation (i.e., $\zeta_f > 1$). The *real interest rate channel* ② reflects the elasticity of today's output with respect to real interest rate changes. Similar to the parameter ζ_f, innovative investment reduces direct aggregate demand effects in response to real interest rate changes as $s_c < 1$. However, a higher degree of countercyclical inequality χ raises the real interest rate elasticity ζ_r.

The remaining components concern direct effects from endogenous growth and fiscal policies. The *permanent income channel* ③ captures the sensitivity of today's output to changes in contemporaneous and future permanent income. In a rational expectations equilibrium with persistence $\rho < 1$, higher permanent income has a positive indirect effect on current aggregate demand as $\zeta_g - \rho\zeta_{g'} > 0$. Notice that this channel is stronger the higher the steady-state growth rate g^A is. Finally, the *redistributive tax channel* ④ embeds the effects of transitory profit tax shocks on current output. As profit taxes redistribute income toward high-MPC households, this channel boosts aggregate demand, i.e., $\zeta_\tau - \rho\zeta_{\tau'} > 0$.

We now state properties of the IS equation regarding the first two channels.

Corollary 1 (IS EQUATION PROPERTIES)**.** *There exist threshold values* $(\underline{\mu}, \overline{\mu})$ *with* $1 < \underline{\mu} < \overline{\mu}$ *such that (a) the elasticity of aggregate demand w.r.t. real interest rates* ζ_r *is strictly positive iff* $\mu < \overline{\mu}$*; (b) the IS equation features compounding* ($\zeta_f > 1$) *iff* $\mu > \underline{\mu}$*, and discounting* ($\zeta_f < 1$) *iff* $\mu < \underline{\mu}$*; and (c)* ζ_f *is U-shaped in* τ^D *on* $\left[\underline{\tau}^D, \tau^D_{disc}\right)$ *if* $s_c = 1$ *and* $1 < \mu < 1 + (1+\alpha)/\lambda$*. Appendix B.3 specifies* $(\underline{\mu}, \overline{\mu}, \tau^D_{disc})$*.*

The first two statements augment the standard IS properties of HANK models by steady-state consumption inequality and investment. Statement (a) provides an upper bound $\overline{\mu}$ such that a real interest rate tightening restrains aggregate demand. Moreover, statement (b) shows that hand-to-mouth consumption needs to be more countercyclical compared to HANK to establish *compounding* of the IS equation, as the investment channel lowers direct aggregate demand effects.

While we elaborate on the aggregate short- and long-run effects of progressive redistribution in Section 2.7, we briefly discuss in Statement (c) how such redistribution affects the transitory income channel ① as an *automatic stabilizer*. Under $\mu > 1$, the sign of the marginal effect of higher profit taxes on the compounding coefficient ζ_f is generally ambiguous. First, greater redistribution decreases steady-state consumption inequality and, thus, the inequality-weighted risk probability measure $1 - \tilde{s}$. Second, it reduces the degree of countercyclical income inequality χ and stabilizes demand. On the other hand, it increases the income share of hand-to-mouth agents, and, thus, the average economy-wide MPC. For low (resp. high) initial tax levels, the first two channels dominate (resp. are dominated) and a marginal increase in redistribution weakens (resp. strengthens) the sensitivity of current demand to future income.

2.4.2 Endogenous Growth (EG) Equation

Proposition 3 (EG EQUATION)**.** *The endogenous growth equation is*

$$\hat{g}^A_{t+1} = \frac{1}{1+\mathcal{E}_g}\Big(\underbrace{\mathcal{E}_y \hat{y}_t + (\mathscr{M} - \mathcal{E}_y)\mathbb{E}_t\left[\hat{y}_{t+1}\right]}_{⑤\ \textit{transitory income}} + \underbrace{\mathcal{E}_g \mathbb{E}_t\left[\hat{g}^A_{t+2}\right]}_{⑥\ \textit{perm. income}} - \underbrace{\mathcal{E}_\tau \hat{\tau}^D_t + (\mathcal{E}_\tau - \mathscr{M}_\tau)\mathbb{E}_t\left[\hat{\tau}^D_{t+1}\right]}_{⑦\ \textit{redistributive tax}}\Big), \quad \text{(E.2)}$$

where $\mathcal{E}_y = \frac{(1-\lambda)\Gamma+\lambda-\lambda s_c \chi}{(1-\lambda)\Gamma s_c}$, $\mathcal{E}_g = \frac{(1-\lambda)\Gamma+\lambda}{(1-\lambda)\Gamma}\frac{1-s_c}{s_c}\frac{g^A}{g^A-1}$, *and* $\mathcal{E}_\tau = \frac{\lambda\chi_\tau}{(1-\lambda)\Gamma}$ *denote the partial equilibrium elasticities of current saver consumption w.r.t. aggregate income, endogenous technology growth, and redistributive taxes, respectively.* $\mathscr{M} = \frac{g^A - \beta s(1-\delta)}{g^A}$ *denotes the partial equilibrium elasticity of endogenous technology growth w.r.t. future aggregate demand and* $\mathscr{M}_\tau = \frac{\beta\tau^D\psi\Theta_\alpha L}{g^A}$ *with respect to taxes.*

The components of the EG equation are tied to a *cost of funds effect* and a *market size effect*.

Cost of Funds Effect It describes that current technology growth depends inversely on expected consumption growth of savers and arises through the stochastic discount factor shaping intermediary innovative firms' decisions. If current consumption is high, the current marginal utility of consumption and investment costs in terms of forgone consumption equivalents are low. If an expansionary shock redistributes away from savers, for example, through a higher cyclical inequality χ, the sensitivity of saver consumption with respect to aggregate income decreases. This lowers, *ceteris paribus*, expected savers' consumption growth; they smooth their consumption by paying themselves dividends rather than investing. Thus, countercyclical income inequality stabilizes technology growth, i.e., it lowers (resp. increases) investment in booms (resp. recessions).

Market Size Effect It describes that current technology growth depends on future aggregate demand. Higher aggregate demand allows firms to sell more goods, increases the discounted sum of profits and raises innovative investment. If a persistent expansionary shock redistributes away from savers toward hand-to-mouth households, aggregate demand expands overproportionally and endogenous technology growth increases *ceteris paribus*. In this case, countercyclical income inequality amplifies technology growth.

We now discuss how the components ⑤–⑦ shape the cost of fund and market size effect in partial equilibrium. The *transitory income channel* ⑤ affects the cost of fund effect through $\mathcal{E}_y$ and the market size effect through $\mathscr{M}$. The strength of the latter effect increases, *ceteris paribus*, with steady-state growth g^A. A persistent increase in aggregate demand increases current technology growth. The *permanent income channel* ⑥ acts on the cost of fund effect through $\mathcal{E}_g$. By lowering future expected consumption of savers, it raises expected consumption growth and, thus, innovative investment. This effect is greater the larger the steady-state investment share $1 - s_c$ is and the lower steady-state growth g^A is. Finally, the *redistributive tax channel* ⑦ acts on two key margins. First, $\mathcal{E}_\tau$ reflects the cost of fund effect by lowering expected savers' consumption growth. Second, $\mathscr{M}_\tau$ lowers technology growth by distorting innovation decisions. Both forces decrease technology growth in equilibrium.

2.4.3 Phillips Curve and Taylor Rule There are two further equations to be specified, the Phillips curve and the Taylor rule. Price and wage inflation are linked through $\hat{\pi}_t = \hat{\pi}_t^w - g_t^A$ in our economy. Using the unions' wage setting, we state nominal rigidities in terms of price inflation.

Proposition 4 (PHILLIPS CURVE). *The static price Phillips curve is given by*

$$\hat{\pi}_t = \kappa_y \hat{y}_t - \kappa_g \hat{g}_{t+1}^A - \hat{g}_t^A \ , \tag{E.3}$$

where $\kappa_y = \kappa \frac{1+\varphi s_c}{s_c}$*, and* $\kappa_g = \kappa \frac{1-s_c}{s_c} \frac{g^A}{g^A - 1}$ *are positive, and* $\kappa > 0$ *denotes the common slope of the wage Phillips curve, provided in Appendix B.3.*

An expansion in aggregate demand increases labor demand, inducing unions to set higher wages, which translates into higher marginal costs and price inflation. As $s_c < 1$, this channel is strengthened relative to a model without endogenous growth, because a part of the increase in output is spent on innovative investment. Second, higher technology growth expands the production frontier, and reduces marginal costs and, hence, price inflation. If this channel is strong, the slope of the Phillips curve flattens, or even reverses, such that prices do not respond much to demand shocks. The model is closed by specifying nominal interest rates, i.e.,

$$\hat{i}_t = \phi_\pi \hat{\pi}_t^w + \epsilon_{mt} \ . \tag{E.4}$$

2.4.4 HANK-GS as Unified Framework Our representation unifies a number of frameworks. The model corresponds to (i) a two agent (TA) Keynesian growth economy with permanent heterogeneity if $s = 1$, and to a representative agent (RA) Keynesian growth economy (Benigno and

Fornaro, 2018) if also $\lambda = 0, \chi = \Gamma = 1$; (ii) it collapses to an alternative tractable HANK model (Bilbiie, 2021) if $\psi = 0$ and $g^A = 1$ in all periods, to a TANK model (Bilbiie, 2008) if additionally $s = 1$; and (iii) to a baseline RANK model (Galí, 2015) if $\psi = \lambda = 0, s = \chi = 1$. Absent nominal rigidities, one recovers real model variants: (iv) a flexible price HA growth economy (Cozzi, 2018) if $\theta = 0$; (v) a RA growth economy (Aghion and Howitt, 1992) if $\theta = \lambda = 0, s = \chi = \Gamma = 1$; (vi) a tractable Aiyagari (1994) model if $\theta = \psi = 0$; and (vii) a standard RBC if $\theta = \psi = \lambda = 0, s = \chi = \Gamma = 1$. As such, the model is well-suited to study short- and long-run effects of stabilization policies, while preserving comparability to existing frameworks.

2.5 Dissecting the Role of Income Inequality

We now use the four-equation representation to analyze how income inequality arising from stabilization policies, such as monetary or fiscal policy, shape aggregate output fluctuations in the short- and long-run. We begin by studying the effects of inequality *per se*. Therefore, we compare the HA economy with its RA benchmark to capture the difference between realized long-run output from its unshocked potential counterpart that can be purely attributed to heterogeneity. We then dissect the role of cyclical inequality and highlight the key forces through which it acts.

2.5.1 Measuring Long-Run Effects of Inequality Throughout the paper, we use two distinct measures to evaluate the long-run effects of household heterogeneity on technology growth.

Definition 1 (SCARS FROM INEQUALITY). *Consider a contractionary policy of persistence $\rho \in (0,1)$:*

(a) Scars from inequality are permanent output losses that can be attributed to inequality per se, i.e.,

$$\mathscr{L}(\infty) \equiv \lim_{T\to\infty} \left((\ln Y_{t+T} - \ln Y^P_{t+T}) - (\ln Y^{RA}_{t+T} - \ln Y^{RA,P}_{t+T}) \right) = \frac{\mathcal{M}^{HA}_g - \mathcal{M}^{RA}_g}{1-\rho} ,$$

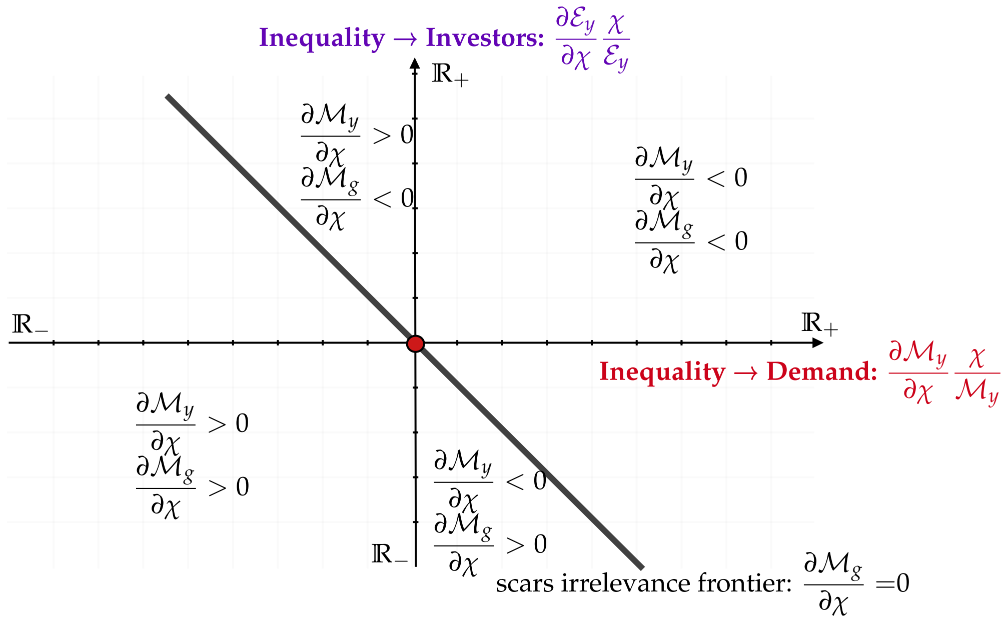
where Y_{t+T} denotes realized output T periods after the shock, Y^P_{t+T} potential output in the unshocked economy, Y^{RA}_{t+T} and $Y^{RA,P}_{t+T}$ their RA counterparts. Scars from inequality, $\mathscr{L}(\infty)$, depend on the HA technology impact multiplier, $\mathcal{M}^{HA}_g \equiv \mathcal{M}_g|_{\lambda,\chi,\tilde{s},\Gamma}$, and the RA multiplier $\mathcal{M}^{RA}_g \equiv \mathcal{M}_g|_{\lambda=0,\chi=1,\tilde{s}=1,\Gamma=1}$.

(b) Scars from cyclical variations in inequality are $\frac{\partial \mathscr{L}(\infty)}{\partial \chi} = \frac{1}{1-\rho}\frac{\partial \mathcal{M}^{HA}_g}{\partial \chi}$.

2.5.2 Understanding the Role of Income Inequality In Proposition 5, we unpack the effects of cyclical inequality on technology growth using the endogenous growth equation. To do so, we implicitly use the IS equation to specify aggregate demand.

Proposition 5 (INEQUALITY-EFFICIENCY RELATION). *Let us define the impact multiplier of output by $\mathcal{M}_y \equiv \frac{\partial \hat{y}_t}{\partial \epsilon_t}$ and the one for technology growth by $\mathcal{M}_g \equiv \frac{\partial \hat{g}^A_{t+1}}{\partial \epsilon_t}$, respectively. Using Proposition 3, the*

Figure 1. The inequality-efficiency-diagram under HANK-GS for a contractionary shock ($\mathcal{M}_y < 0$).



marginal effect of cyclical income inequality on technology growth is

$$\frac{\partial \mathcal{M}_g}{\partial \chi} = \frac{1}{1 + \mathcal{E}_g(1-\rho)} \left((1-\rho)\mathcal{M}_y \frac{\partial \mathcal{E}_y}{\partial \chi} + \left(\mathcal{E}_y(1-\rho) + \rho \mathscr{M}\right) \frac{\partial \mathcal{M}_y}{\partial \chi} - \frac{\partial \Xi_{\tau,m}}{\partial \chi} \right), \tag{9}$$

where $\Xi_{\tau,m} = 0$ *for a monetary policy shock and* $\Xi_{\tau,m} = (1-\rho)\mathcal{E}_\tau + \mathscr{M}_\tau$ *for a tax shock, respectively.*

Proposition 5 shows that the effect of higher cyclical income inequality on endogenous technology growth depends on three forces: (i) how it affects the elasticity of investor consumption with respect to aggregate income $\frac{\partial \mathcal{E}_y}{\partial \chi}$ (cost of funds effect); (ii) how it affects aggregate demand $\frac{\partial \mathcal{M}_y}{\partial \chi}$ (market size effect); and (iii) how it affects the elasticity of saver consumption with respect to tax shocks $\frac{\partial \Xi_{\tau,r}}{\partial \chi}$. The strength and sign of the forces (i)-(iii) are model-specific. When they move in opposite directions, the sign of the overall effect of cyclical inequality on endogenous technology growth is ambiguous and depends on the weighting coefficient in front of each particular force.[14]

Based on equation (9), Figure 1 illustrates the aggregate effects of cyclical income inequality

[14] Equation (9) implies that there are an infinite number of combinations of (i)-(iii) that are consistent with a certain marginal effect of cyclical inequality on technology growth. This can be formalized through the lens of isoquants in case of a contractionary policy (i.e., $\mathcal{M}_y < 0$). The *scars isoquant* at level $\overline{g}$ is analytically given by

$$\frac{\partial \mathcal{E}_y}{\partial \chi} \frac{\chi}{\mathcal{E}_y} = \frac{\chi}{(1-\rho)\mathcal{E}_y \mathcal{M}_y} \left(\left[1 + \mathcal{E}_g(1-\rho)\right] \overline{g} + \frac{\partial \Xi_{z,\tau,m}}{\partial \chi} \right) - \left(1 + \frac{\rho}{1-\rho} \frac{\mathscr{M}}{\mathcal{E}_y}\right) \frac{\partial \mathcal{M}_y}{\partial \chi} \frac{\chi}{\mathcal{M}_y},$$

and properly defined for $|\rho| < 1$, $\mathcal{E}_y \neq 0$, and $\mathcal{M}_y \neq 0$. A special case ensues for $\overline{g} = 0$, for which the *Scars Irrelevance Frontier* of cyclical inequality (SIF) separates the scars amplification region, i.e., $\frac{\partial \mathcal{M}_g}{\partial \chi} < 0$, from the scars stabilization region, i.e., $\frac{\partial \mathcal{M}_g}{\partial \chi} > 0$. The SIF strictly decreases in the elasticity of aggregate demand to cyclical inequality, rotates clockwise for a higher persistence, and shifts upward for a lower level of $\overline{g}$ and downward for tax shocks.

from a contractionary monetary policy shock (i.e., $\frac{\partial \Xi_{\tau,m}}{\partial \chi} = 0$). There are four regions, which are delimited by the sign of short- ($\mathcal{M}_y$) and long-run ($\mathcal{M}_g$) dynamics following an increase in cyclical inequality. In the top-right dashed region, a higher degree of countercyclical inequality amplifies short- and long-run fluctuations. This is the case either because cyclical inequality jointly redistributes away from investors and spenders, or because demand amplification outweighs the increase in saver's innovation incentives through the cyclicality of her consumption level. In the bottom left cross-shaded region, a higher degree of countercyclical inequality stabilizes the short- and long-run propagation of contractionary policies either because they jointly redistribute toward investors and spenders, or because the stabilization in demand outweighs the amplification through higher investment. In the two blue-shaded regions, the short- and long-run propagation move in opposite directions; in the bottom right region, output is amplified and technology growth stabilized; and in the top-left region, output is stabilized and technology growth amplified.

Despite its simplicity, the analytical decomposition derived above summarizes four key statistics to characterize the short- and long-run propagation of cyclical inequality arising from, for example, a redistributive policy: (i) the elasticity of saver's consumption exposure to aggregate income with respect to cyclical inequality, $\frac{\partial \mathcal{E}_y}{\partial \chi} \frac{\chi}{\mathcal{E}_y}$; (ii) the elasticity of aggregate demand with respect to cyclical inequality, $\frac{\partial \mathcal{M}_y}{\partial \chi} \frac{\chi}{\mathcal{M}_y}$; (iii) the persistence of the shock, ρ; and (iv) a combination of a cost of funds and a behavioral tax effect resulting from redistributive shocks, $\frac{\partial \Xi_{\tau,m}}{\partial \chi}$. These statistics locate models, such as the one studied in section 3, in a particular region of Figure 1.

Discussion While we have derived investment decisions in an endogenous growth environment, the relevant tradeoffs of the inequality-efficiency-diagram in Figure 1 apply to a larger class of heterogeneous agent models with *other* forms of investment as well, such as physical capital or skill accumulation. In the baseline two agent NK model with physical capital accumulation (Galí et al., 2007; Bilbiie et al., 2022), the investment response is very much insensitive to the degree of countercyclical inequality. By interpreting the investment response through $\mathcal{M}_g$, this model is located in the vicinity of the scars irrelevance frontier in the bottom right region, i.e., amplifying output but not investment. Kekre and Lenel (2022) argue that the investment response is amplified if a monetary policy shock redistributes income to households with high propensities to take risk. Neglecting MPC heterogeneity, the model is located above the scars irrelevance frontier in the top right quadrant. Luetticke (2021) studies the transmission of monetary policy in a HANK model with heterogeneity in MPCs and MPIs. He documents that aggregate consumption is amplified, while investment and output are stabilized relative to a RA economy. Such a model is located below the scars irrelevance frontier in the left two quadrants. Similarly, David and Zeke (2022) derive investment wedges based on a model with limited asset markets participation and price markup shocks and thus locate below the scars irrelevance frontier. Finally, Heathcote et al. (2020) study a framework in which workers invest and accumulate labor skills, while being overproportionally harmed by recessions. Such a model locates in the top right quadrant.

2.6 Redistributive Monetary Policy, Business Cycles, and Growth

We now study short- and long-run effects of monetary shocks and highlight the role of persistence.

Proposition 6 (IMPACT MULTIPLIERS). *Output, technology and inflation multipliers to a monetary policy shock of persistence $\rho_m \in (0,1)$ are recursively determined by*

$$\text{(output)} \qquad \frac{\partial \hat{y}_t}{\partial \epsilon_{mt}} \equiv \mathcal{M}_y = -\left[\zeta_r^{-1}(1-\rho_m\zeta_f) + (\kappa_y - \Omega\kappa_g)(\phi_\pi - \rho_m) - (1-\rho_m\tilde{s})\,\mathcal{E}_g\Omega\right]^{-1} ,$$

$$\text{(technology)} \qquad \frac{\partial \hat{g}^A_{t+1}}{\partial \epsilon_{mt}} \equiv \mathcal{M}_g = \Omega\mathcal{M}_y ,$$

$$\text{(inflation)} \qquad \frac{\partial \hat{\pi}_t}{\partial \epsilon_{mt}} \equiv \mathcal{M}_\pi = \left(\kappa_y - \Omega\kappa_g\right)\mathcal{M}_y ,$$

where the technology-output elasticity $\Omega \in \mathbb{R}_+$ is defined as

$$\Omega \equiv \omega\mathscr{F} + (1-\omega)\mathscr{M} , \quad \text{with} \quad \omega \equiv (1-\rho_m)\frac{1+\mathcal{E}_g}{1+(1-\rho_m)\mathcal{E}_g} \in [0,1] , \quad \text{and} \quad \mathscr{F} \equiv \frac{\mathcal{E}_y}{1+\mathcal{E}_g} .$$

Consider the standard case in which a monetary tightening reduces output, i.e., $\mathcal{M}_y < 0$. The second equation of Proposition 6 states that a transitory output drop translates into a permanent technology loss depending on the magnitude of the elasticity Ω, which is a weighted sum between the contemporaneous cost of funds effect $\mathscr{F}$ and the market size effect $\mathscr{M}$. The weight on the former, ω, strictly decreases in the persistence of the monetary policy shock. If monetary policy is a one-time surprise ($\rho_m = 0$), only the cost of funds effect matters. On the contrary, if the monetary policy shock is permanent ($\rho_m = 1$), only the market size effect matters. Moreover, the sign of the impact multiplier of inflation, i.e., $\kappa_y - \Omega\kappa_g$, is, a priori, ambiguous.[15]

To isolate the interplay between inequality, aggregate demand and technology growth, we first decompose the short-run output impact multiplier.

Corollary 2 (OUTPUT DECOMPOSITION). *For a given level of steady-state consumption inequality Γ,*

[15] In Appendix OA1.1, we show that the sign of the long-run AS slope together with the shock persistence pins down the sign of $\mathcal{M}_\pi$. In a standard upward sloping AS regime, $\text{sgn}(\mathcal{M}_\pi) = \text{sgn}(\mathcal{M}_y)$ applies. In this case, cyclical income inequality amplifies the inflation response under HANK-GS relative to HANK if $|\varepsilon_{\Omega,\chi}| > |\varepsilon_{\mathcal{M}_y,\chi}|$, i.e., the elasticity of the technology-output elasticity w.r.t. cyclical income inequality is larger than that of the output multiplier.

the short-run output response to a monetary policy shock can be decomposed into

$$-\left(\mathcal{M}_y\right)^{-1} = \underbrace{(1-\rho_m)+\tilde{\kappa}_y(\phi_\pi-\rho_m)}_{\text{① RANK multiplier}} + \underbrace{(1-\rho_m)(\tilde{\mathcal{E}}_y-1)}_{\text{② TANK interest rate elasticity}} + \underbrace{\rho_m(1-\tilde{s})(\tilde{\mathcal{E}}_y-\chi)}_{\text{③ HANK cyclical inequality}}$$
$$+\underbrace{\left(\kappa(\phi_\pi-\rho_m)+(1-\rho_m\tilde{s})\frac{(1-\lambda)\Gamma+\lambda}{(1-\lambda)\Gamma}\right)\frac{1-s_c}{s_c}}_{\text{④ reduction direct effects}} - \underbrace{(1-\rho_m)\tilde{s}\mathcal{E}_g\Omega}_{\text{⑤ TANK permanent income}}$$
$$-\underbrace{(1-\tilde{s})\mathcal{E}_g\Omega}_{\text{⑥ HANK permanent income}} - \underbrace{\Omega^{RA}\kappa_g(\phi_\pi-\rho_m)}_{\text{⑦ NKPC RANK-GS slope}} + \underbrace{(\Omega^{RA}-\Omega)\kappa_g(\phi_\pi-\rho_m)}_{\text{⑧ NKPC HANK-GS slope change}},$$

where $\tilde{\kappa}_y$ is the NKPC slope under RANK, $1/\tilde{\mathcal{E}}_y$ the demand elasticity w.r.t. real interest rates absent growth, and Ω^{RA} the endogenous technology elasticity w.r.t. aggregate income changes in an RA economy.

Corollary 2 shows how the interaction of heterogeneity and endogenous growth adds on the RANK multiplier ① through the following: ② a change in the real interest rate elasticity arising from cyclical inequality due to permanent household heterogeneity between saver and hand-to-mouth households (TANK); ③ cyclical income inequality interacting with idiosyncratic risk (HANK); and ④ a reduction in direct demand effects arising from a steeper wage Phillips curve and a lower sensitivity of aggregate demand with respect to future output and real interest rates due to the investment channel ($s_c < 1$). The first two channels amplify the output multiplier under a higher degree of countercyclical inequality ($\chi > 1$), whereas the last channel reduces the output multiplier. Two additional permanent income demand channels increase the output multiplier: ⑤ relies on permanent heterogeneity (TANK), while ⑥ arises from idiosyncratic risk and vanishes if heterogeneity is permanent. Those channels are stronger when the elasticity of saver consumption to aggregate income $\mathcal{E}_g$ or the elasticity of technology growth to aggregate income Ω increase. Finally, ⑦ and ⑧ state that the Phillips curve slope flattens under HANK-GS if $\Omega^{RA} < \Omega$ and, hence, leads to further amplification in this case. In Corollary 7 in Appendix OA1.1, we further show that the HANK-GS output multiplier is amplified relative to HANK if the technology-output elasticity is sufficiently large, i.e., $\Omega \geq \underline{\Omega}$. This is the case as the reduction of direct effects needs to be outweighed by the flattening of the wage Phillips curve and additional aggregate demand effects arising from permanent income changes.

We now characterize the role of cyclical income inequality χ. As Ω decreases in χ, it is *a priori* ambiguous whether higher cyclical inequality amplifies or stabilizes the output response.

Proposition 7 (Cyclical Inequality and Output Fluctuations). *If $\phi_\pi \leq \kappa_g^{-1}$, countercyclical income inequality amplifies output, i.e., $\frac{\partial \mathcal{M}_y}{\partial \chi} < 0$, $\forall\ \rho_m \in (0,1)$. If $\phi_\pi > \kappa_g^{-1}$, countercyclical income inequality stabilizes output, i.e., $\frac{\partial \mathcal{M}_y}{\partial \chi} > 0$, on $\rho_m \in (0, \rho_m^{SR})$, and amplifies output, i.e., $\frac{\partial \mathcal{M}_y}{\partial \chi} < 0$, on $\rho_m \in (\rho_m^{SR}, 1)$. The persistence threshold ρ_m^{SR} is specified in Appendix B.*

The first statement of Proposition 7 states that cyclical income inequality amplifies the output response for any admissible shock persistence if monetary policy reacts moderately to wage inflation, i.e., $\phi_\pi \leq \kappa_g^{-1}$. On the contrary, if the feedback rule is sufficiently responsive to wage inflation, the monetary policy expansion needs to be persistent enough for cyclical income inequality to act as an amplifier. Intuitively, if monetary policy is sufficiently aggressive, i.e., $\phi_\pi > \kappa_g^{-1}$, indirect demand effects from future transitory and permanent income changes are weakened and the negative cost of funds effect of countercyclical income inequality dominates.

We now show how scars from cyclical inequality, i.e., the effect of cyclical income inequality on long-run technology growth, depend on the persistence of the monetary tightening.

Proposition 8 (SCARS FROM CYCLICAL INEQUALITY)**.** *There exists a persistence threshold ρ_m^{LR} such that countercyclical income inequality stabilizes technology growth, i.e., $\frac{\partial \mathcal{M}_g}{\partial \chi} > 0$, on $\rho_m \in (0, \rho_m^{LR})$, is irrelevant for technology growth for $\rho_m = \rho_m^{LR}$, and amplifies technology growth, i.e., $\frac{\partial \mathcal{M}_g}{\partial \chi} < 0$, on $\rho_m \in (\rho_m^{LR}, 1)$. If $\phi_\pi > \kappa_g^{-1}$ applies, $\rho_m^{LR} > \rho_m^{SR}$ holds true. We specify ρ_m^{LR} in Appendix B.5.*

Proposition 8 states that countercyclical income inequality amplifies permanent output losses from a monetary policy tightening if the shock persistence is sufficiently large. This result is related to the discussion following Proposition 7. Relative to the output multiplier, the technology multiplier $\mathcal{M}_g$ is augmented in a multiplicative manner by the technology-output elasticity Ω. As the latter depends negatively on the degree of countercyclical income inequality through the cost of funds effect, the market size effect leads to overall amplification only if the shock is persistent enough. The degree of countercyclical inequality stabilizes scars from inequality if $\rho_m < \rho_m^{LR}$, and amplifies them if $\rho > \rho_m^{LR}$.[16] Proposition 8 also states that a monetary tightening needs to be more persistent to induce cyclical inequality to amplify long-run output rather than short-run output.

Based on the Taylor rule responsiveness ϕ_π and the shock persistence ρ_m, it is interesting to discuss the location of a particular model economy within Figure 1 by using Propositions 7 and 8. The economy locates in the bottom-right quadrant for $\phi_\pi \leq \kappa_g^{-1}$. Whether it is placed above or below the scars irrelevance frontier depends on the persistence threshold ρ_m^{LR}. On the contrary, under $\phi_\pi > \kappa_g^{-1}$ there arise three subcases. First, our framework locates in the bottom-left quadrant for $\rho_m < \rho_m^{SR} < \rho_m^{LR}$; second, it locates in the bottom-right quadrant below the SIF for $\rho_m^{SR} < \rho_m < \rho_m^{LR}$; and third, it locates in the bottom-right quadrant above the SIF for $\rho_m^{SR} < \rho_m^{LR} < \rho_m$.

[16]We provide comparative statics of the persistence threshold in Appendix OA1.5. We show that ρ_m^{LR} decreases in the degree of idiosyncratic risk $1 - \tilde{s}$, increases in the slope of the Phillips curve κ_y, and is independent of cyclical income inequality χ. In the light of the empirically observed flattening of the Phillips curve around the Great Recession, our framework may thus provide an additional rationale how countercyclical inequality did not only amplify short-run output, but also long-run permanent output losses.

2.7 The Ambiguous Role of Fiscal Redistribution Policies

In the previous sections, we have shown that (cyclical) income inequality plays a key role in shaping aggregate short- and long-run fluctuations. Subsequently, we theoretically discuss the role of fiscal stabilization policies that *directly* act on the degree of cyclical income inequality. Such policies comprise, for instance, temporary UI benefit extensions during economic downturns that are financed by higher income tax progressivity. Within our stylized analytical framework, we capture those policies through a transitory increase in the tax rate τ_t^D with corresponding persistence $\rho_\tau \in (0, 1)$. Subsequently, we dissect the aggregate short- and long-run effects.

Proposition 9 (Progressive Redistribution)**.** *Consider an economy with $\tau^D > 0$. A transitory increase in redistribution, i.e., $\hat{\tau}_t^D > 0$, affects aggregate outcomes in a HANK model ($\psi = 0$) as follows.*

(a) If wages are fully flexible, or the monetary authority replicates the real interest rate absent nominal rigidity, then redistribution has no effects on output, i.e., $\mathcal{M}_y = 0$.

(b) If wages are sticky, then greater redistribution is expansionary, i.e., $\mathcal{M}_y > 0$.

Moreover, a transitory higher tax redistribution affects aggregate outcomes in a HANK-GS model ($\psi > 0$) with BGP growth rate $g^A > 1 + ((1 - s_c)/(1 + \varphi)s_c)(1 - \beta s(1 - \delta))$ as follows.[17]

(c) If wages are fully flexible, or the monetary authority replicates the real interest rate absent nominal rigidity, then greater redistribution contracts short- and long-run output, i.e., $\mathcal{M}_y < 0$ and $\mathcal{M}_g < 0$.

(d) If wages are sticky, then greater redistribution has ambiguous effects.

(d.1) If $\rho_\tau = 0$ and $\phi_\pi < \kappa_g^{-1}$, then short-run output rises ($\mathcal{M}_y > 0$) and technology falls ($\mathcal{M}_g < 0$).

(d.2) If $\rho_\tau > 0$, then there exist thresholds $\mathcal{D}_g > \mathcal{D}_y$ on the strength of the demand effect $\zeta_\tau - \rho_\tau \zeta_{\tau'}$, above which short-run output $\mathcal{M}_y$ and technology growth $\mathcal{M}_g$ increase. The thresholds are spelled out in Appendix B.6. If $\mathcal{D}_g > \mathcal{D} > \mathcal{D}_y$, there exists T^* such that $\ln Y_{t+T} > \ln Y_{t+T}^P$ for $T < T^*$ and $\ln Y_{t+T} < \ln Y_{t+T}^P$ for $T > T^*$, where $\ln Y_{t+T}^P$ denotes output in the unshocked economy T periods after the shock. This specific time horizon is characterized by

$$T^* = -\frac{\ln\left(1 - (1 - \rho_\tau)\frac{\mathcal{M}_y}{\mathcal{M}_g}\right)}{\ln \rho_\tau}.$$

In the absence of endogenous growth and nominal rigidities, progressive redistribution is irrelevant as output is purely determined by supply factors. Adding wage stickiness makes greater redistribution expansionary. Intuitively, redistribution to high-MPC households raises aggregate

[17] This bound generically applies for reasonable calibrations.

demand and, hence, short-run output. This mechanism is, for instance, explored in Kekre (2022) or Ferriere and Navarro (2022) through UI insurance and progressive income taxation.

In the presence of endogenous growth, without nominal wage rigidity, the effects of transitory redistribution on short-run output and long-run technology are contractionary. In this case, an increase in taxation reduces innovation incentives and thus the long-run production frontier along two margins: a standard behavioral effect due to lower returns to innovation, and the previously described cost of funds effect. As such, permanent income and aggregate demand fall.

When both endogenous growth and aggregate demand interact in the presence of nominal wage rigidity, the effect of progressive redistribution is generally ambiguous. On the one hand, an increase in redistribution induces a *Keynesian Cross* logic by increasing aggregate demand and innovation incentives through the market size effect. This effect acts on the IS equation through $\zeta_\tau - \rho_\tau \zeta_{\tau'}$ (cf. equation (E.1)) and translates to the EG equation. It is consistent with Zidar (2019), who empirically documents significant employment and growth effects from tax cuts on lower-income households. On the other hand, it reduces innovation incentives through $-(1-\rho_\tau)\mathcal{E}_\tau - \rho_\tau \mathscr{M}_\tau$ (cf. equation (E.2)). This effect is empirically consistent with Akcigit et al. (2022), showing that tax increases on higher-income households dampen innovation activity. If the demand effect is sufficiently strong, both output and technology increase. For a medium strength, output increases while technology falls. In this case, there exists a specific time span $t + T^*$ before which the level of output increases relative to a counterfactual unshocked economy, and after which it decreases. Finally, if the demand effect is sufficiently weak, both output and technology fall. Interestingly, Proposition 9 may thus rationalize the inconclusive empirical evidence on the effects of taxation on growth in the medium run (Stokey and Rebelo, 1995; Jaimovich and Rebelo, 2017; Jones, 2022). Within our framework, this ambiguity depends on the strength of demand effects and the persistence of the redistribution shock.

2.8 Model Mechanisms and Scars from Inequality: A Numerical Example

In this section, we use our analytical model to provide an off the shelf numerical assessment of the key mechanisms, which we will investigate in-depth in the quantitative section 3.

The model period is a quarter. We set $\beta = 0.995$ to obtain an annual interest rate of 4%. The mass of hand-to-mouth households is $\lambda = 0.35$, and the probability of staying a saver is $s = 0.965$. We set the inverse Frisch elasticity to $\varphi = 1$. In line with Bilbiie (2020), we choose a degree of countercyclical earnings inequality of $\chi = 1.45$, which maps into an earnings incidence of $\mu = 1.66$.[18] We set the profit tax to $\tau^D = 0.33$ to obtain no steady-state consumption inequality ($\Gamma = 1$), respectively $\tau^D = 0.35$ under exogenous growth.[19] Similar to Fornaro and Wolf (2021),

[18]Empirical estimates of the *matching multiplier* for the US provided by Patterson (2022) indicate a value $cov\,(mpc_{ie}, v_{ie}) > 0$ such that μ plausibly exceeds unity.

[19]Unlike the exogenous growth case, we show in Appendix B.8 that the *first-best* BGP under endogenous growth

we set $\alpha = 0.50$ to match a quarterly share of R&D spending in GDP of 2%. The death probability is $\delta = 0.0125$. The efficacy of R&D spending is $\psi = 1.36$, which implies a quarterly steady-state growth rate of 0.50%. We normalize steady-state hours worked to $L = 1$ by setting the labor disutility to $\nu = 0.61$. Regarding the Phillips curve we set $\epsilon_w = 10$ and choose an adjustment cost parameter of $\theta = 196$, consistent with an average annual wage contract obtained from a Calvo reset probability (Born and Pfeifer, 2020). We set the AR(1) persistence to $\rho_m = 0.65$ and $\rho_\tau = 0.90$. Finally, the Taylor coefficient is $\phi_\pi = 1.80$, ensuring a locally determinate equilibrium.

Decomposing HANK-GS Table 1 decomposes impact multipliers for technology growth, output, inflation, aggregate consumption and consumption inequality for a monetary and a tax shock among three model variants: a representative agent (RA) economy, a permanent heterogeneity two agent (TA) economy, and a heterogeneous agent (HA) economy with idiosyncratic income risk and cyclical inequality. We also compute associated permanent output losses.

Table 1. Impact multipliers and permanent output loss to monetary policy and progressive tax shocks.

GROWTH ENVIRONMENT	MODEL[a]	TAX	IMPACT MULTIPLIERS[b]					PERM. LOSS
		τ^D	$\mathcal{M}_g$	$\mathcal{M}_y$	$\mathcal{M}_\pi$	$\mathcal{M}_c$	$\Delta\mathcal{M}_c^{S-H}$	$\mathscr{L}(\infty)$
A. Monetary shock								
Exogenous	RA	35%	*n.a.*	-0.60	-0.14	-0.60	*n.a.*	0.00
	TA	35%	*n.a.*	-0.75	-0.18	-0.75	0.52	0.00
	HA	35%	*n.a.*	-0.78	-0.19	-0.78	0.54	0.00
Endogenous	RA	33%	-0.94	-1.43	-0.24	-0.52	*n.a.*	-0.67
	TA	33%	-0.83	-2.11	-0.41	-1.31	2.68	-0.60
	HA	33%	-1.10	-2.78	-0.55	-1.73	3.54	-0.79
B. Progressive tax shock								
Exogenous	RA	35%	*n.a.*	0.00	0.00	0.00	*n.a.*	0.00
	TA	35%	*n.a.*	0.41	0.10	0.41	-1.82	0.00
	HA	35%	*n.a.*	0.94	0.23	0.94	-2.19	0.00
Endogenous	RA	33%	-0.33	-0.27	-0.03	0.06	*n.a.*	-0.83
	TA	33%	-0.34	-0.07	0.02	0.27	-0.91	-0.84
	HA	33%	-0.10	0.67	0.18	0.78	-1.76	-0.25

[a] HA model: $s = 0.965$, $\lambda = 0.35$, $\chi = 1.45$; TA model: $s = 1.00$, $\lambda = 0.35$, $\chi = 1.45$; RA model: $s = 1$, $\lambda = 0$, $\chi = 1$.
[b] The monetary shock corresponds to a 25-basis point increase in the quarterly nominal interest rate through ϵ_{mt}. The tax shock reflects a 3% increase, which corresponds roughly to a 1pp increase in τ_t^D. Impact multipliers are denoted in terms of percentage deviation from steady-state. Additionally, inflation and technology growth are expressed in annual terms. Analytical multipliers for progressive tax shocks are provided in Appendix B.6.

While household heterogeneity amplifies short-run output, inflation and consumption inequality to a monetary tightening in an exogenous growth environment, the effects are ambiguous under endogenous growth, in line with the previous theoretical discussion (cf. Proposition 7 and

does not admit *perfect insurance*, i.e., it is characterized by consumption inequality $\Gamma > 1$. However, to ease comparability between exogenous and endogenous model variants, we always ensure a perfect insurance steady state.

8). Under permanent heterogeneity, the technology multiplier $\mathcal{M}_g$ is stabilized relative to the RA case (cf. Definition 1). In contrast, the short-run output multiplier $\mathcal{M}_y$ contracts more strongly, and consumption inequality rises substantially. Idiosyncratic risk under HANK-GS, however, amplifies the reduction in technology growth such that one obtains roughly 20% scars of inequality to a 25-basis points monetary contraction. The corresponding persistence threshold under HANK-GS is $\rho_m^{LR} = 0.49$, whereas the one under permanent heterogeneity is $\rho_m^{LR} = 0.94$. Remarkably, there arises a self-enforcing amplification spiral as endogenous growth itself substantially raises the degree of countercyclical consumption inequality.

Impact multipliers to a progressive tax shock mirror closely Proposition 9. They are expansionary under the exogenous growth environment with household heterogeneity. In contrast, a progressive tax shock contracts short- and long-run output in an endogenous growth RA economy. A TA economy falls into the weak aggregate demand regime, as both short- and long-run output contract, the former however less than in the RA benchmark. Finally, one obtains an increase in short-run output if countercyclical income inequality interacts with idiosyncratic risk. In this case, the HA model falls into the medium aggregate demand regime and generates, perhaps surprisingly, long-run gains from inequality. As such, the tax increase depresses the level of output after only 3 years (cf. Proposition 9 statement (d.2)).

2.9 Discussion and Additional Results

2.9.1 Equilibrium Determinacy The Taylor-principle, i.e., $\phi_\pi > 1$, does not ensure local determinacy in a HANK economy with countercyclical risk or inequality (Acharya and Dogra, 2020; Bilbiie, 2021). In Appendix OA1.1, we state determinacy conditions for our HANK-GS economy. There emerge three aggregate supply (AS) regimes: a benchmark upward sloping AS regime, a flat AS regime, and a downward sloping AS regime. We show that procyclical income risk and inequality, which lead to a discounted Euler-equation, are no longer sufficient to ensure determinacy under an upward sloping AS regime. Intuitively, there arises a complementarity between aggregate demand and permanent income through the market size effect, which may reverse the discounting. Second, we argue how the HANK-GS framework can resolve the *missing (dis)inflation puzzle* during the Great Recession and the *price puzzle* if the long-run Phillips curve becomes relatively flat under the upward sloping AS regime. Under endogenous growth, monetary tightening restrains aggregate demand and technology growth. If the latter channel is sufficiently strong, inflation remains almost insensitive or even rises. Third, we derive conditions under which a fiscal policy that targets high BGP growth restores an upward sloping AS regime.

2.9.2 Forward Guidance Puzzle The *forward guidance puzzle* means that central bank announcements on the future path of interest rates are theoretically extremely powerful, however, their macroeconomic impact is greatly overestimated as indicated by empirical evidence (Del Negro et al., 2015). In Appendix OA1.2 we first show that our model is subject to a "Catch-22", similar to (Bilbiie, 2021); while countercyclical income inequality amplifies short-run output, the forward guidance puzzle on output *and* technology growth is ruled out under procyclical income inequality. We then consider two extensions to resolve this tension. First, additional procyclical income and production risk being orthogonal to cyclical income inequality as in Bilbiie (2021); second, bounded rationality in the form of cognitive discounting (Gabaix, 2020; Pfäuti and Seyrich, 2022). Both extensions act on expectations and, hence, effectively reduce the relative weight on the market size effect. As a result, monetary policy needs to be even more persistent to guarantee that a higher degree of countercyclical inequality amplifies permanent output losses.

2.9.3 Intertemporal Marginal Propensities to Consume Auclert et al. (2018) demonstrate the importance of intertemporal marginal propensities to consume (iMPCs) in disciplining general equilibrium models with heterogeneous agents. We provide iMPCs under HANK-GS in Appendix OA1.3 and show how they compare to HANK depending on the technology output elasticity Ω.

2.9.4 Consumption versus Income Inequality Empirically, consumption inequality reacts more countercyclical than income inequality in response to demand (Coibion et al., 2017; Ampudia et al., 2018) or technology (Gaudio et al., 2021) shocks. In Appendix OA1.4, we first show that income inequality under HANK-GS is allowed to be slightly procyclical to generate countercyclical consumption inequality in response to positive demand and labor-augmenting technology shocks. Second, we show that consumption inequality is more countercyclical than income inequality.

2.9.5 NKPC with Nonuniform Earnings Incidence For reasons of parsimony, our derivation of the wage Phillips curve relies on a hypothetical average household. Thus, we differ from union objectives studied, for instance, in Colciago (2011) or Ascari et al. (2017) extending the setup of Erceg et al. (2000) and Schmitt-Grohé and Uribe (2005) by maximizing a utilitarian sum of utilities. In Appendix OA2, we follow their roots and choose a utilitarian objective that takes explicitly the earnings incidence into account when setting wages. We show that the slope of the Phillips curve depends on the incidence in a U-shaped manner. This is the case as unions provide earnings insurance across households by increasing the slope of the Phillips curve and effectively reducing aggregate fluctuations in hours worked.

3 A Quantitative HANK-GS Economy

The previous section analytically characterized the role of household heterogeneity for the short- and long-run propagation of stabilization policies. We now relax a number of assumptions, which were adopted for the sake of tractability, and refine the setup to a rich quantitative framework. Later on, we use the refined framework to check the robustness of the analytical results and to quantify the short- and long-run effects of monetary policy and social insurance programs.

3.1 Environment

The quantitative framework extends the analytical setup along three dimensions. First, beyond the investor-spender dichotomy, we incorporate multiple layers of household heterogeneity acting on the joint distribution of MPCs and MPIs. Second, we decompose labor income risk into adjustments in hours worked and the risk of becoming unemployed. Third, the welfare state partially insures against idiosyncratic risk through a broader set of social insurance policies, such as tax and transfer programs. The income and wealth distribution endogenously adjust to policy changes. The setup is directly stated in stationary form, where $g^A_{t+1} = A_{t+1}/A_t$ denotes gross technology growth, r^b_t the real net interest rate, and $w_t \equiv \frac{W_t}{A_t P_t}$ the normalized real wage rate.

3.1.1 Households The utility of household i now depends on consumption $c_{it} \equiv \frac{C_{it}}{A_t}$, hours worked ℓ_{it}, and employment status $e_{it} \in \{W, U_S, U_L\}$ according to $U(c_{it}, \ell_{it}, e_{it}) = u(c_{it}) - \nu(\ell_{it}, e_{it})$. The state values of e_{it} represent employment (W), insured short-run unemployment (U_S) and uninsured long-run unemployment (U_L), respectively. We assume a standard CRRA utility, i.e., $u(c_{it}) = \frac{c_{it}^{1-\sigma}}{1-\sigma}$ and $\nu(\ell_{it}, e_{it}) = \mathbb{1}_{e=W} \overline{\nu}_t \frac{\ell_{it}^{1+\varphi}}{1+\varphi} + \mathbb{1}_{e \in \{U_S, U_L\}} \underline{\nu}$. A time-invariant share Λ_E of households are entrepreneurs, who invest to create and accumulate new innovations, while the remaining share $1 - \Lambda_E$ are workers. Entrepreneurs have a relative stock of innovations $a_{it} \equiv \frac{A_{it}}{A_t}$ and are identified with a dummy $d_E = 1$. Moreover, agents are heterogeneous in their discount rate β_{it}, their labor productivity h_{it}, and their wealth $b_{it} \geq \underline{b}$, where $\underline{b}$ is a borrowing limit. The normalized discount rate is $\widetilde{\beta}_{it} = \beta_{it}(1 - \xi)(g^A_{t+1})^{1-\sigma}$, where $\xi \in (0, 1)$ is a death rate. Unless necessary, we subsequently drop (i, t) indexes and indicate next period variables with a prime " $'$ ".

We denote a household's state by $\mathbf{s} \equiv (h, \beta, e, d_E)$. The probability of moving from h to h' and β to β' are denoted by $\mathbb{P}_h(h'|h)$ and $\mathbb{P}_\beta(\beta'|\beta)$, respectively. We denote by $F_h(h)$ the invariant distribution of labor productivity induced by a Markov chain. Unemployment insurance is temporary and the probability of switching from insured short-run unemployment to uninsured long-run unemployment is $\mathbb{P}_e(U_L|U_S) \equiv 1 - \rho^S$, such that $\mathbb{P}_e(U_S|U_S) \equiv \rho^S$. The probability of switching from employment to unemployment depends on the productivity h as well as the economic conditions, and will be specified below. By investing a real amount $x \equiv \frac{X}{A}$, entrepreneurs create innovations with a success probability $p_x(x)$. When an individual dies, a newborn starts with zero net worth and inherits the entrepreneurial type of her parents, whereas β' and h' follows their respective Markov processes. The government fully taxes bequests, and innovations are lost upon death. The distribution of households over the state-space $(a, b, \mathbf{s})$ is denoted by $m_t(a, b, \mathbf{s})$.

We can state the per period budget constraint in real terms as

$$c + (g^A)'b' = y^d + b - \mathcal{T}\Big(y^d\Big) + T - \psi(x)\ , \quad \text{where} \tag{10}$$
$$y^d = wh\ell\mathbb{1}_{e=W} + \min\{\varsigma wh\ell, \overline{\varsigma}\}\mathbb{1}_{e=U_S} + \mathbb{1}_{e\in\{U_L,U_S\}}T^u + r^b b + \Theta\big(a\big)\ .$$

Successful entrepreneurs receive an amount $\Theta(a)$ as returns to innovative investment. Wage payments and unemployment benefits to short-run unemployed workers are given by $wh\ell$ and $\min\{\varsigma wh\ell, \overline{\varsigma}\}$, respectively, where ς is the unemployment insurance (UI) replacement rate and $\overline{\varsigma}$ defines the maximum cap. This specification approximates the US unemployment insurance law. T^u defines minimum transfers that are provided to the needy, i.e. all unemployed households, and captures safety-net programs such as food stamps, cash assistance, or transfers to the disabled. $\mathcal{T}(y^d)$ is the tax schedule on disposable income, and T denotes a lump-sum transfer.

While all households consume c and save an amount b', investment x of entrepreneurs incurs costs $\psi(x) \geq 0$, with $\frac{\psi(x)}{\partial x} > 0$, to improve their *relative* innovation stock a. Upon success, the innovation improves the individual stock by ηA, where $\eta > 0$ denotes the step-size that is linked to the economy-wide innovation stock A.[20] This modelling choice captures the non-rivalry assumption that is, in the presence of increasing returns to scale, an integral part of endogenous growth models. Households' recursive problem writes

$$\mathcal{V}(a, b, \mathbf{s}) = \max_{\{c,b',x\}}\ U(c,\ell) + \widetilde{\beta}\mathbb{E}_{h',\beta',e'}\Big[d_E\mathcal{I}(a',b',\mathbf{s}',x) + (1-d_E)\mathcal{V}(0,b',\mathbf{s}')\Big]\ ,$$
$$s.t.\quad \mathcal{I}(a',b',\mathbf{s}',x) = \int_\iota \Big((1-p_x(x))\mathcal{V}\big(a',b',\mathbf{s}'\big) + p_x(x)\mathcal{V}\big(a'+\eta, b', \mathbf{s}'\big)\Big)dF_\iota(\iota)\ ,$$
$$a' = \iota\left(\frac{1-\delta^A}{(g^A)'}\right)a\ , \quad b' \geq \underline{b}\ , \quad \text{and} \quad (10)\ ,$$

where $\mathcal{I}(a',b',\mathbf{s}',x)$ defines the value associated with innovative investment for a given investment x. According to the law of motion for a', an innovation depreciates relative to the overall quality of innovations through $(g^A)'$ and depreciation δ^A. The shock on the stock of innovation ι captures innovation success or failure orthogonal to the investor's investment.

3.1.2 Labor Market and Earnings Exposure Similar to section 2, nominal rigidity arises from the unions' wage setting. Each employed household i provides $h_{it}\ell_{ikt}$ effective hours of work to a particular union k, which combines them into a union-specific task, i.e., $L_{kt} = \int e_{ikt}h_{it}\ell_{ikt}di$. As before, a competitive labor packer bundles these tasks according to a CES aggregator and sells this service at wage W_t. Each union k sets a wage W_{kt} to maximize the utility of a representative member subject to quadratic adjustment costs $\frac{\theta}{2}\left(\pi^w_{kt} - g^A\right)^2$ that are directly passed onto mem-

[20] This setup is isomorphic to the analytical innovation model from section 3, as the law of motion can be rewritten as $A_{j,t+1} = p\big(A_{j,t} + \eta\big) + (1-p)A_{j,t}$, with $p = (\psi/\eta)I_{j,t}$.

bers' utility. $\pi^w_{kt} = \frac{W_{kt}}{W_{k,t-1}}$ is the gross wage inflation rate. We assume that unions allocate hours worked among their members based on a splitting rule, which captures intensive and extensive margin adjustments. First, off the steady-state total labor across h-types follows

$$\Delta L^h_{kt} \equiv L^h_{kt} - \overline{L}^h_k = \mu(h)(L_{kt} - \overline{L}_k) \,, \quad \text{with} \quad \int_h \mu(h) dF_h(h) = 1 \,,$$

where $\overline{L}_k$ and $\overline{L}^h_k$ denote economy-wide steady-state labor in union k and at productivity h, respectively. The incidence function $\mu(h)$ captures overall labor adjustments across productivity types. Second, for a given h-type, we assume that all members work the same number of hours, i.e., $\ell^h_{ikt} = \ell^h_{kt}$. Third, across all unions, a lottery is organized that determines which households are unemployed. As such, total h-type efficiency weighted hours worked within an union k are given by $L^h_{kt} = h(1 - u^h_{kt})\ell^h_{kt}$, where u^h_{kt} denotes the unemployment rate for a certain h-type. Unions break changes in total labor at productivity h, i.e., ΔL^h_{kt}, into

$$\Delta \ell^h_{kt} = \frac{\phi(h)}{1 - \overline{u}^h_k} \times \frac{\Delta L^h_{kt}}{h} \,, \qquad \text{and} \qquad \Delta u^h_{kt} = -\left(\frac{1 - \phi(h)}{\ell^h_{kt}} \right) \times \frac{\Delta L^h_{kt}}{h} \,, \tag{11}$$

where $\phi(h)$ captures the extent to which total labor fluctuations translate into adjustments in hours worked per worker, and $1 - \phi(h)$ captures adjustments in the employment rate.[21] Fourth, we focus on a steady state in which hours worked are identical across h-type, i.e., $\overline{\ell}^h_k \equiv \overline{\ell}_k$. Finally, the unemployment rate for h-productivity is consistent with flows in and out of employment, i.e.,

$$\Delta u^h_{kt+1} = \sum_{\widetilde{h}} \Big(\underbrace{\widetilde{\mathbb{P}}_{e,t}(\{U_S, U_L\}|W;h)}_{\text{separation rate at } h} (1 - u^h_{kt}) - \underbrace{\mathbb{P}_{e,t}(W|\{U_S, U_L\};h)}_{\text{job-finding rate at } h} u^h_{kt} \Big) \frac{F_h(\widetilde{h})}{F_h(h)} \mathbb{P}_h(h|\widetilde{h}) \,. \tag{12}$$

The sum with respect to all possible states $\widetilde{h}$ comes from the fact that the unemployment rate of a specific h-group evolves with inflows and outflows from other productivity groups.[22] In reality, many short-run unemployed workers do not benefit from UI. To account for this fact, we assume that only a fraction Λ_S of separated workers takes up unemployment benefits, such that $\mathbb{P}_{e,t}(U_S|W;h) = \Lambda_S \widetilde{\mathbb{P}}_{e,t}(\{U_S, U_L\}|W;h)$ and $\mathbb{P}_{e,t}(U_L|W;h) = (1 - \Lambda_S)\widetilde{\mathbb{P}}_{e,t}(\{U_S, U_L\}|W;h)$.

Unions are identical, face the same labor demand, $L_t = L_{kt}$, and set the same wage $W_t = W_{kt}$.

[21] Accordingly, the expression for Δu^h_{kt} follows from the *identity* $\Delta L^h_{kt}/h = \Delta \ell^h_{kt}(1 - \overline{u}^h_k) - \ell^h_{kt} \Delta u^h_{kt}$.

[22] Solving the former equation pins down the separation rate for a given job-finding rate. It requires to solve the linear system $\mathbf{FX} = \mathbf{G}$, with elements in $\mathbf{G}$ such that $g^h_{kt} = \Delta u^h_{kt+1} + u^h_{kt} - \sum_{\widetilde{h}}(1 - \mathbb{P}_e(W|U;\widetilde{h}))u^{\widetilde{h}}_{kt} \frac{F_h(\widetilde{h})}{F_h(h)} \mathbb{P}_h(h|\widetilde{h})$ and elements of $\mathbf{F}$ such that $f_{h,\widetilde{h}} = (1 - u^{\widetilde{h}}_{kt}) \frac{F_h(\widetilde{h})}{F_h(h)} \mathbb{P}_h(h|\widetilde{h})$. A similar equation pins down the job-finding rate given the separation rate.

Wage inflation is then described by the dynamic Phillips Curve

$$\frac{\epsilon_w}{\theta} L_t \int_i \left(v'(L_t) - \frac{\epsilon_w - 1}{\epsilon_w} u'(C_t) w_t \left(1 - \frac{\partial \mathcal{T}(Y^d)}{\partial Y^d} \right) \right) = \pi_t^w \left(\pi_t^w - g^A \right) - \widetilde{\beta}_t \mathbb{E}_t \left[\pi_{t+1}^w \left(\pi_{t+1}^w - g^A \right) \right] ,$$

where C_t and Y_t^d denote aggregate consumption and income, respectively.

3.1.3 Final Production and Innovative Intermediary Goods The specification for production sector closely follows the analytical model. By combining labor and a continuum of intermediary goods X_{jt} of quality A_{jt}, final good producers operate with technology $Y_t^G = (Z_t L_t)^{1-\alpha} \int_0^1 A_{jt}^{1-\alpha} X_{jt}^{\alpha} dj$. They competitively choose labor and intermediary inputs. Final goods are sold at price P_t, intermediary goods are bought at price P_{jt}, and labor pays a wage W_t.

Each intermediary innovative good is produced by a single monopolistic firm, which redistributes its profit to the entrepreneur who owns the firm. As in the analytical model, they set a price $P_{jt} = P_t / \alpha$ at which they sell an amount $X_{jt} = \alpha^{\frac{2}{1-\alpha}} A_{jt} Z_t L_t$. Real profits are given by $\Pi_{jt} = (1 - \tau^D) \Theta_\alpha A_{jt} Z_t L_t$, where τ^D denotes a flat dividend tax. As $A_t \equiv \int_0^1 A_{jt} dj$, aggregate profits and normalized real rents are given by

$$\Pi_t = \int_j \Pi_{jt} dj = (1 - \tau^D) \Theta_\alpha A_t Z_t L_t , \qquad \Theta(a_{jt}) = a_{jt} \frac{\Pi_t}{A_t} .$$

Finally, gross domestic output is $Y_t = Y_t^G - X_t = Y_\alpha A_t Z_t L_t$.

3.1.4 Government and Monetary Policy The government collects revenues by taxing disposable income, profits and dividends and by issuing bonds. Disposable income taxes are specified by an HSV schedule (Benabou, 2002; Heathcote et al., 2017), i.e.,

$$\mathcal{T}(y^d) = \max \left\{ 0, y^d - \vartheta \left(y^d \right)^{1-\varrho} \right\} , \tag{13}$$

where $\varrho \in (-\infty, 1)$ reflects the constant rate of tax progressivity for $y^d > 0$. If $\varrho = 0$, the income tax schedule is linear with constant marginal tax rate $1 - \vartheta$. If $\varrho \in (0, 1)$ the tax schedule is progressive as the ratio of marginal tax rate to the average tax rate is $\mathcal{T}'(y^d) / \left[\mathcal{T}(y^d) / y^d \right] > 1$, whereas $\varrho < 0$ leads to a regressive tax schedule. Total income taxes are $\mathbb{T}_t^y \equiv \int_{(a,b,\mathbf{s})} \mathcal{T}(y^d) \, dm_t(a, b, \mathbf{s})$, dividend taxes $\mathbb{T}_t^D = \tau^D \Theta_\alpha Z_t L_t$, and bequest taxes $\mathbb{T}_t^b = \int_{(a,b,\mathbf{s})} \xi b \, dm_t(a, b, \mathbf{s})$. They are used to finance an exogenous level of government spending G_t, social insurance transfers (i.e., unemployment benefits and safety-net transfers) $\mathbb{S}_t = \int_{(a,b,\mathbf{s})} \left(\mathbb{1}_{e=U_S} \min \{ \varsigma w \ell h, \bar{\varsigma} \} + \mathbb{1}_{e \in \{U_S, U_L\}} T^u \right) dm_t(a, b, \mathbf{s})$, interest payments on bonds $r_t^b B_t^G$ and potential lump-sum transfers to households T_t. The government specifies the level of UI generosity through three margins: a probabilistic duration ρ^S, a replacement rate ς, and a maximum cap of benefits $\overline{\varsigma}$. The government budget constraint is

$$g_{t+1}^A \frac{B_{t+1}^G}{A_{t+1}} + \mathbb{T}_t^y + \mathbb{T}_t^D + \mathbb{T}_t^b = (1+r_t^b)\frac{B_t^G}{A_t} + \frac{G_t}{A_t} + T_t + \mathbb{S}_t \; . \tag{14}$$

Furthermore, monetary policy sets the nominal interest rate according to a Taylor rule, which depends on nominal gross wage inflation π_t^w and the output gap, i.e.,

$$i_t = \bar{r}^b + \phi_\pi \left(\frac{\pi_t^w}{\pi^w} - 1\right) + \phi_Y \left(\frac{Y_t}{Y} - 1\right) + \epsilon_{mt} \; , \tag{15}$$

where $\bar{r}^b$ is the steady-state real interest rate. The Fisher identity implies $1 + r_t^b = \frac{1+i_t}{\mathbb{E}_t[\pi_{t+1}]}$, with $\pi_t \equiv \frac{P_t}{P_{t-1}}$. Finally, price inflation is linked to wage inflation according to $\pi_t = \frac{\pi_t^w}{g_t^A}\frac{Z_{t-1}}{Z_t}$.

Government Budget When studying transitional dynamics, we consider different scenarios on how the government balances its budget: (i) government spending G_t adjusts; (ii) lump-sum taxes T_t adjust; (iii) the degree of tax progressivity $\widetilde{\varrho}_t$ adjusts following the modified schedule $\mathcal{T}(y^d) = \max\left\{0, y^d - \vartheta^{\widetilde{\varrho}_t/\varrho}\left(y^d\right)^{1-\widetilde{\varrho}_t}\right\}$ where the exemption threshold is kept fixed; and finally (iv) debt B_t^G adjusts and the associated deficit is reimbursed over z periods using lump-sum taxes T_t.[23]

3.1.5 Balanced Growth Path Equilibrium

Definition 2. *Given an initial stock of innovations A_{-1}, a nominal wage W_{-1}, government debt B_{-1}^G, an initial distribution $m_{-1}(a, b, \boldsymbol{s})$ and exogenous sequences of taxes and transfers including unemployment benefits and safety net transfers that satisfy the governmental budget constraint (14), a balanced growth path general equilibrium is a sequence of prices $\{P_t, W_t, \pi_t, \pi_t^w, r_t^b, i_t\}$, a growth rate g_{t+1}^A, individual policy functions $\{c(a, b, \boldsymbol{s}), b'(a, b, \boldsymbol{s}), x(a, b, \boldsymbol{s})\}$, a joint distribution over entrepreneurial innovations, assets, and types $m_t(a, b, \boldsymbol{s})$, and aggregates $\{Y_t, L_t, C_t, G_t, B_t, T_t, A_t\}$ such that:*

1. *Households, unions and firms optimally solve their program.*

2. *Monetary and fiscal policy follow their rules.*

3. *Good markets clear and the resource constraint writes $\Psi_t + \frac{G_t}{A_t} + \int_{(a,b,\boldsymbol{s})} c(a, b, \boldsymbol{s})\, dm_t(a, b, \boldsymbol{s}) = \frac{Y_t}{A_t}$, where $\Psi_t \equiv \int \psi(x)\, dm_t(a, b, \boldsymbol{s})$ is aggregate innovative investment.*

4. *Total bond demand equals its supply, i.e., $B_t \equiv \int_{(a,b,\boldsymbol{s})} b \; dm_t(a, b, \boldsymbol{s}) = B_t^G$.*

[23] Specifically, we let lump-sum taxes jump when the shock occurs at time t. Afterwards, they slowly decay with persistence $0 < \rho_T < 1$ according to $T_{t+z} = \rho_T^z T_t$. The jump is a function of the present value of future primary deficits taking into account the path of interest rates and technology growth rates. We characterize the jump in Appendix C.2.1.

5. *On the balanced growth path, variables grow at the endogenous technology growth rate*

$$g^A = (1-\delta^A) + \eta \int_{(a,b,\mathbf{s})} \frac{p_x(x(a,b,\mathbf{s}))}{\Lambda_E} dm_t(a,b,\mathbf{s}) \; .$$

The model has no analytical solution and is solved numerically. We use standard fast-techniques such as the endogenous grid method and non-stochastic simulations, described in Appendix C.2.

3.2 Calibration

The model period is a quarter. We calibrate the model along the balanced growth path to the US economy. Our calibrated parameters and targeted moments are summarized in Table 2.

Table 2. Calibrated parameters.

PARAMETER	SYMBOL	VALUE	SOURCE/TARGET
Preferences			
Discount factor	$\{\Delta^\beta, \beta_M, p^\beta\}$	$\{0.026, 0.97, 0.995\}$	quarterly MPC of 0.18, B^G/Y of 40%
Death rate	ξ	0.005	40 years of average working life
Preference parameters	$\{\sigma, \varphi, \bar{v}\}$	$\{1.5, 1.0, 10\}$	Heathcote et al. (2010), 1/3 time at work
Labor Market			
Permanent component	$\{\rho_h, \sigma_h^2\}$	$\{0.973, 0.04\}$	Storesletten et al. (2004)
Elasticity of labor unions	ϵ	7	Ferriere and Navarro (2022)
Adjustment cost	θ	200	NKPC slope of 0.035
Steady-state unemp. rate	$\bar{u}^h$	Table 3	CPS data
Job-finding/separation rate	$\{\mathbb{P}_e, \phi^s\}$	Table 3	CPS data, Krusell et al. (2017)
UI take-up rate	Λ_S	0.70	IUR of 3.0 (BLS data)
Incidence functions	$\{\mu(h), \phi(h)\}$	Table 3	CPS data
Technology & Innovation			
Share of entrepreneurs	Λ_E	0.10	fraction of US entrepreneurs
Depreciation (quarterly)	δ^A	0.03	assumption
Innovation step parameters	η	0.50	$g^A_{BGP} = 2\%$
Variance of innovation shock	σ_ι^2	0.0225	income Pareto coeff. of 1.7
Success probability	γ	0.44	elasticity of innovation to corp. tax
Returns to scale	α	0.40	share of profits in total income
Government & Monetary Policy			
Tax parameters	$\{\varrho, \vartheta, \tau^D\}$	$\{0.1, 0.7, 0.21\}$	US tax code, G/Y=0.15
Taylor rule	$\{\phi_\pi, \phi_Y\}$	$\{1.5, 0\}$	standard values, e.g., Auclert et al. (2018)
UI benefits	$\{\varsigma, \bar{\varsigma}\}$	$\{0.4, 0.265\}$	US average, 66% of avg. income
UI duration	ρ^S	0.50	expected length of 6 months
Safety-net transfers	T^u	0.102	1.02% of GDP, McKay and Reis (2016)

Households Following Heathcote et al. (2010), we set the inverse EIS to $\sigma = 1.5$ and the inverse Frisch labor supply elasticity to $\varphi = 1$. The latter value lies in between the estimates obtained for male (0.5) and female (1.5) workers. The borrowing limit is specified by $\underline{b} = 0$. To be consistent with the BGP concept, the labor disutility shifter is normalized to $\bar{v}_t = \bar{v}_0(g_t^A)^{1-\sigma}$, where $\bar{v}_0$ is chosen such that hours worked of employed households equal $\bar{\ell} = 1/3$ on the BGP. The death probability is $\xi = 0.005$. We assume that the logarithm of idiosyncratic labor productivity h follows an AR(1) process, i.e., $log(h') = \rho_h log(h) + \epsilon'_h$, where $\epsilon_h \sim \mathcal{N}(0, \sigma_h^2)$, with $\sigma_h^2 = 0.04$ and $\rho_h = 0.973$, which is in the range of plausible values in Storesletten et al. (2004) (Table 2). We discretize the number of labor productivity realizations h into 5 bins, i.e., $h \in \{h_1, ..., h_5\}$.

Households differ in their discount factor component $\beta \in \{\beta_L, \beta_M, \beta_H\}$, which follows a Markov chain with transition probabilities $\mathbb{P}_\beta(\beta'|\beta)$. We calibrate transitions between the "patient", "mid-patient" and "impatient" types such that households can only move to the adjacent states according to $\mathbb{P}_\beta(\beta_M|\beta_L) = \mathbb{P}_\beta(\beta_M|\beta_H) = 1 - p_\beta$, $\mathbb{P}_\beta(\beta_H|\beta_M) = \mathbb{P}_\beta(\beta_L|\beta_M) = (1 - p_\beta)/2$, and $\mathbb{P}_\beta(\beta_i|\beta_i) = p_\beta$, $\forall i$. In our setup, β-heterogeneity generates heterogeneous MPCs across households (Carroll et al., 2017) and takes over the role of idiosyncratic shocks between savers and hand-to-mouth households in the analytical model. We follow Krusell and Smith (1998), Kekre (2022) and Ferriere and Navarro (2022), and assume $p_\beta = 0.995$, $\beta_L = \beta_M - \Delta^\beta$ and $\beta_H = \beta_M + \Delta^\beta$. We set $\beta_M = 0.97$ and $\Delta^\beta = 0.026$ to generate an average MPC out of transitory income of 0.18 and an aggregate debt-to-GDP ratio of 40%, consistent with the US ratio over the past 40 years. Using β-heterogeneity to generate heterogenous MPCs is consistent with Aguiar et al. (2020), who argue that hand-to-mouth households have different *intrinsic* preferences for consumption.[24]

Government and Monetary Policy Following Bayer et al. (2020) and Ferriere and Navarro (2022), the progressivity of the income tax schedule is $\varrho = 0.10$, which closely approximates the US tax code. The value of $\vartheta = 0.7$ is chosen such that the average labor income tax rate is about 23%, which is in the range of plausible values in the literature. For this particular value of ϑ, government spending accounts for 15% of GDP. The corporate income tax is set to the statutory level $\tau^D = 0.21$. We assume a steady-state gross inflation of $\pi = 1$ and a real annual interest rate of $r^b = 4\%$. As such, B^G adjusts at the steady-state mostly through the discount factor β_M.

Unemployment insurance is calibrated as follows. The replacement rate ς is set to 40% of previous labor income, which is the average value across US states between 1990 and 2020. The duration of UI in the US is typically 6 months during normal times. As the model period is a quarter, we set $\rho^S = 0.5$. Following McKay and Reis (2016), the unemployment benefit cap $\overline{\varsigma}$ represents 66% of the average income, and safety-net transfers T^u are set to match 1.02% of GDP. Lump-sum transfers are $T = 0$ in the benchmark economy.

Finally, we follow the literature and assume that the central bank follows a Taylor rule with corresponding weights on inflation $\phi_\pi = 1.5$ and output $\phi_Y = 0$, respectively.

Technology and Innovation The labor share is set to $\alpha = 0.4$ to replicate a labor share in total revenues of roughly 70%. We follow Ferriere and Navarro (2022) and set $\epsilon = 7$, which corresponds

[24] Kaplan and Violante (2022) show that ex-ante heterogeneity through stochastic discount factors mimics wealthy hand-to-mouth households and generates reasonable average MPCs; however, it understates the wealth of households in the middle of the distribution. We show below that our model replicates the wealth distribution reasonably well through the margin of innovation investment and associated returns.

to a union's markup of 16.7%. The wage adjustment cost θ is 200 to match a wage NKPC slope, ϵ/θ, of 0.035. Later on, we will assess the sensitivity of our results when varying the NKPC slope.

A successful innovation arrives at a Poisson rate with $p_x(x) = 1 - exp(-\gamma x)$, where $1/\gamma$ reflects the average time length to create an innovation. The parameter γ is ultimately linked to the elasticity of realized innovations to innovative investment expenditures. According to p. 2241-2242 in Jones (2022), *Akcigit et al. (2022) is currently the most thorough paper estimating this elasticity for inventors, and they robustly find high elasticities. For example, using panel data for US states since 1940, they find macro elasticities of invention (patents, citations, and number of inventors) with respect to top marginal keep rates ranging from 0.5 to 1.5.*[25] We focus on their estimate for corporate taxes as there is a direct mapping to dividend taxes in our model. They report an elasticity of new patents by "corporate inventors" to corporate income taxes of about 0.49. According to their findings, this value constitutes a conservative lower bound on the effects of taxation on overall patenting activities. For example, looking at the elasticity of patents to the personal income net-of-tax, they find an estimate of about 0.8. Finally, we assume that innovation costs are linear, i.e., $\psi(x) = x$.

We set the share of potentially innovative entrepreneurs to $\Lambda_E = 0.10$, which corresponds roughly to the share of entrepreneurs in the US. The quarterly depreciation is $\delta^A = 0.03$, i.e., $\approx$ 12.6% annually. We assume that the innovation shock ι is Gaussian, i.e., $F_\iota(\iota) \sim \mathcal{N}(1, \sigma_\iota^2)$, where the variance $\sigma_\iota^2 = 0.0225$ induces a Pareto law for labor income of about 1.7, consistent with Piketty and Saez (2014). Finally, the step-size $\eta = 0.5$ pins down an annual growth rate of 2%.

Labor Market and Earnings Incidence We use the panel dimension of the *Current Population Survey* (CPS), that includes relevant statistics of the US labor market in waves from 1989 to 2022. We consider all respondents aged between 20 and 65 and do not restrict the analysis to household heads. We classify workers into wage bins, which correspond closely to the productivity bins h in the model. Wages of unemployed individuals are not observed in the CPS. To circumvent this issue, we extrapolate their wages based on the individual's occupation, state, education, sex and age. We compute the long-run steady-state job-finding rate for each h-group, $\overline{\mathbb{P}}_e(W|\{U_S, U_L\}; h)$, and substitute those values for their model counterparts. Steady-state job-separation rates $\overline{\overline{\mathbb{P}}}_e(\{U_S, U_L\}|W; h)$ are pinned down, given the job-finding rates, such that steady-state unemployment rates for each h-group correspond to their long-run average in the CPS. The take-up rate Λ_S is calibrated to match a long-run insured unemployment ratio of 3.0 (total insured unemployed workers over the population of covered workers).

Off the balanced growth path, we parameterize the labor market as follows. As $\int_h h dF_h(h) = 1$, we observe total employment ΔL_t, average hours per h-productivity $\Delta \ell_t^h$, and total hours per h-

[25]Moreover, according to p. 381 in Akcigit et al. (2022), *the majority of the macro effect of personal taxation appears to result from reduced innovation at the individual level, rather than through shifting the location of innovation from one state to another*. That statement reflects that individual elasticities appear to be quite substantial.

productivity $\frac{\Delta L_t^h}{h}$ in the CPS. We use these series to determine adjustments in the average number of hours worked per individual, i.e., $\phi(h) = \frac{\Delta \ell_t^h (1-\bar{u}^h)}{\Delta L_t^h / h}$, due to fluctuations in total amount of hours worked. This measure varies between 0.17 to 0.32 and displays the lowest value at the bottom of the income distribution, meaning that fluctuations in the extensive employment margin are predominant at the bottom of the income distribution. Moreover, we calibrate the incidence according to $\frac{\mu(h)}{h} = \frac{\Delta L_t^h / h}{\Delta L_t}$. Consistent with evidence in Guvenen et al. (2017), Broer et al. (2021b) and Patterson (2022), we find that the total hours at the bottom of the income distribution disproportionally react to fluctuations in aggregate hours. Table 3 below reports the aforementioned statistics.[26]

Table 3. Labor market statistics across h-groups.

		Incidence		Steady-state labor market statistics		
h-group	wage bin	$\mu(h)/h$	$\phi(h)$	$1-\bar{u}^h$	$\mathbb{P}_e(W \vert \{U_S, U_L\}; h)$	$\bar{\tilde{\mathbb{P}}}_e(\{U_S, U_L\} \vert W; h)$
h_1	[0.00 : 0.16]	1.61	0.22	0.107	0.497	0.078
h_2	[0.16 : 0.38]	1.11	0.18	0.069	0.487	0.026
h_3	[0.38 : 0.62]	1.30	0.17	0.053	0.481	0.026
h_4	[0.62 : 0.84]	0.64	0.22	0.041	0.475	0.022
h_5	[0.84 : 1.00]	0.28	0.32	0.028	0.461	0.009

The last two columns of Table 3 concern the extent to which job-separation and job-finding rates react consistently with the observed unemployment rate. Using equation (12), we first solve for the job-separation rates being consistent with fluctuations in the unemployment rate u_t^h, while keeping job-finding rates fixed at their steady-state levels. We label these separation rates $p_{u,t}(h)$. They represent changes in job-separation, which would generate exactly the fluctuations in unemployment rates. To account for the fact that part of unemployment rate fluctuations is due to variations in job-finding rates, we assume that actual separation rates are $\tilde{\mathbb{P}}_{e,t}(\{U_S, U_L\}|W; h) = \phi^s p_{u,t}(h) + (1-\phi^s)\bar{\tilde{\mathbb{P}}}_e(\{U_S, U_L\}|W; h)\ \forall h$, where ϕ^s is the extent to which separation rates account for fluctuations in unemployment rates. Given these actual separation rates, we then solve for the job-finding rates consistent with the unemployment rates using equation (12). Following Krusell et al. (2017), we assume that job separations account for one third of total variation in unemployment rates, i.e., $\phi^s \approx 0.3$. As such, the model captures sufficiently well the unemployment risk and, importantly, its persistence and dynamics.

4 Inspecting the Model's Properties

We shall now investigate the properties of the quantitative model and highlight its performance with respect to targeted and non-targeted moments.

[26]Figure 12 in Appendix C.3 displays cyclical variations of unemployment rates and hours worked across h-groups.

4.1 Long-Run Properties: Income and Wealth Inequality

We first document endogenous moments concerning the distribution of income and wealth in Table 4. The benchmark model generates reasonably high concentrations of income and wealth. This feature can be traced back to the presence of innovators and is amplified by random growth in the innovation process through the idiosyncratic shock ι. Removing the random growth component limits the concentration of wealth by lowering the concentration of income from innovative rents. In this case, and relative to the benchmark economy, the Pareto coefficient of the income distribution would increase from 1.7 to 2.0.[27]

In the model, 10% of the population are entrepreneurs. Innovative entrepreneurs are the main driving force behind the high income and wealth concentration. Absent entrepreneurs ($\Lambda_E = 0$), the model does not generate enough inequality. We define an innovative entrepreneur in the model as an entrepreneur who effectively holds a positive number of innovations, i.e., $a > 0$. Using this definition, it turns out that only 4.5% of the population are innovative entrepreneurs. Therefore, our model implies that only a small proportion of the population is classified as innovator. How does this fact compare to the data? The *Global Entrepreneurship Monitor* documents that among the 11% of the US owner-managed new businesses (in the population aged between 18-64) around 35% indicate that their product or service is new to at least some customers, and that only few or no alternative businesses offer an equivalent product.[28] Extrapolating these numbers to the whole population of entrepreneurs would result in a fraction of innovative entrepreneurs of 3.85%. Interestingly, the 2017 US *Annual Business Survey* (ABS) provides information on 4.6 million active for-profit companies that are publicly or privately held. Around 43% of these companies introduced an innovation during the years 2015-17. Assuming that a single business owner corresponds to a single firm and given that the population of business owners is around 13.3% in the US, according to the Survey of Consumer Finance, this would imply a fraction of business owners of innovative firms of 5.7%. This number is likely an upper bound, as business owners could run multiple innovative firms.

The model also generates a strong endogenous sorting of entrepreneurs at the top of the income and wealth distribution. The share of innovative entrepreneurs within the top 20% and top 5% wealthy households is 27% and 45%, respectively. While not being directly comparable due to the lack of a counterpart for "innovative" entrepreneurs, our results are consistent with the empirically observed sorting of self-employed and/or business owners. Cagetti and De Nardi (2006) document that the top 5% comprises 68% of business owners or self-employed, and 39% if one only considered self-employed business owners. Finally, it is worth mentioning that our model generates a positive sorting between innovative entrepreneurs and income or wealth. In

[27] See Benhabib et al. (2011) for a discussion on how random growth models can generate Pareto distributions.

[28] The data can be retrieved here: `https://www.gemconsortium.org/data`.

Table 4. The distribution of income and wealth.

	Income					Wealth				
	Gini	share going to top (in %)				Gini	share going to top (in %)			
		1%	10%	20%	50%		1%	10%	20%	50%
US data [a]	0.54	17	42	58	85	0.82	38	73	86	98
Benchmark model	0.46	15	38	52	80	0.81	35	71	85	99
No innovation shock ($\sigma_\iota^2 = 0$) [b]	0.45	12	36	51	79	0.72	17	57	75	97
No entrepreneurs ($\Lambda_E = 0$) [c]	0.24	2	19	35	68	0.73	11	56	77	98

[a] Statistics for income and wealth come from the World Inequality Database (WID) and the adjusted Survey of Consumer Finance (1989-2021) from Gaillard and Wangner (2022), respectively.
[b] Recalibrated to match a growth rate of 2% annually using the inverse innovation arrival length γ.
[c] Recalibrated to match the same average MPC and debt-to-GDP ratio using discount factor β_M and Δ_β.

this sense, our model is consistent with the causal link put forward in Aghion et al. (2019).

4.2 Short-Run Properties: Consumption, Investment, and Income Exposure

We now focus on the joint distribution of MPCs and the earnings incidence over the business cycles, which is a key driver of aggregate demand amplification. Additionally, we assess the distribution of MPIs and its sensitivity to tax changes, which affects the long-run transmission of transitory stabilization policies. We compute MPCs and MPIs as follows

$$\text{MPC}(a, b, \mathbf{s}) = \frac{\partial c(a, b, \mathbf{s})}{\partial b}, \qquad \text{MPI}(a, b, \mathbf{s}) = \mathbb{1}_{d_E=1} \frac{\partial x(a, b, \mathbf{s})}{\partial b}.$$

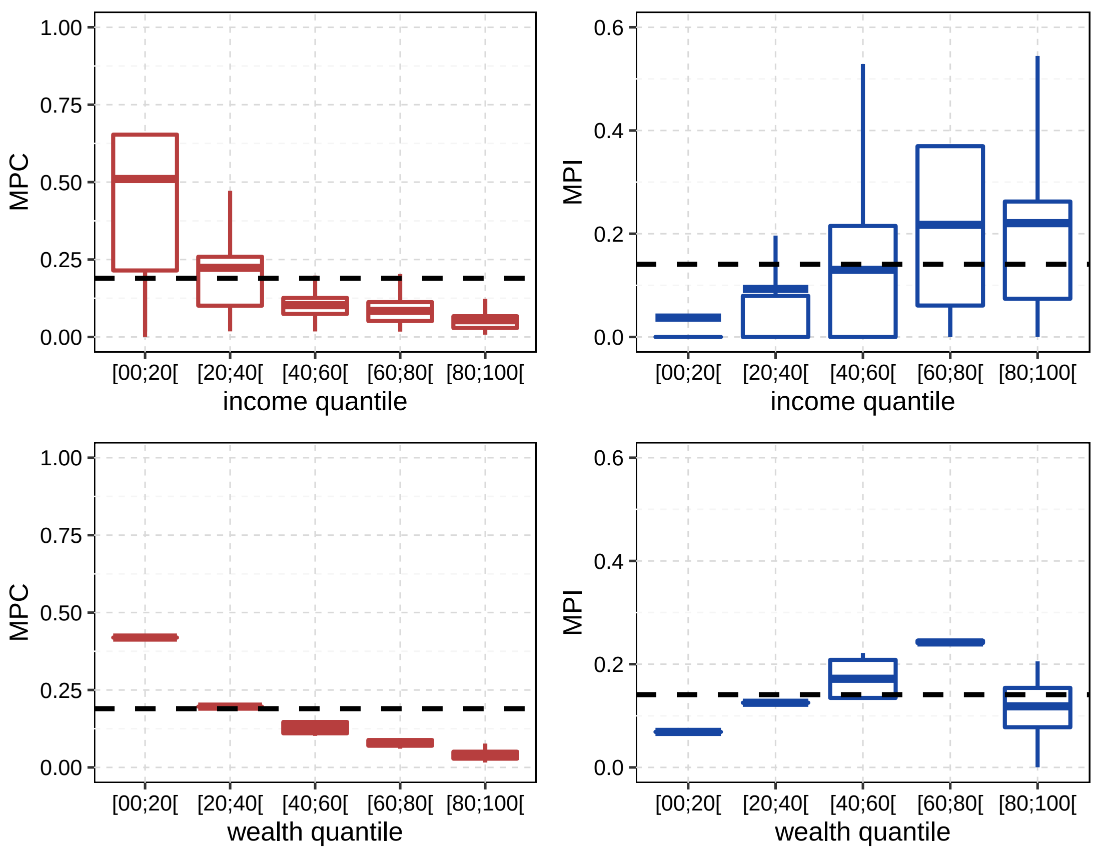
Table 5 displays the model-implied MPCs for several groups of individuals. We find that unemployed individuals have the highest MPCs. This finding is in line with the view of long-run unemployment as a "tag" for high MPCs, as highlighted in Kekre (2022). Long-run unemployed workers exhibit an MPC which is, on average, three times larger than the average MPC. In contrast, innovative entrepreneurs have the lowest MPC, about four times smaller than the average. As such, the implied fraction of hand-to-month households – defined as households at the borrowing limit $b = 0$ – is roughly 25%.

Table 5. Model-generated quarterly MPC and mean wealth across household groups.

Group	Quaterly MPC	Mean Wealth
Overall Population	0.18 (targeted)	1.0 (normalized)
Unemployed workers	0.38	0.76
Short-run insured unemployment	0.21	0.80
Long-run uninsured unemployment	0.57	0.71
Employed workers	0.16	1.01
Innovative entrepreneurs	0.04	10.15

How can we interpret the distribution of MPCs among the different groups? In line with Carroll et al. (2017) and Ganong and Noel (2019), we obtain that the average MPC is much higher among unemployed individuals. This is the case as MPCs are decreasing in wealth due to the presence of a borrowing limit, and unemployed households are more likely to be of low labor productivity with little wealth. In Figure 2, we display the distribution of MPCs and MPIs across the income and the wealth distribution. Overall, MPCs fall along the income and wealth distribution. As argued in section 3, this feature is highly relevant for the strength of the market size effect.

Figure 2. Model generated MPCs and MPIs across the income and wealth distributions.



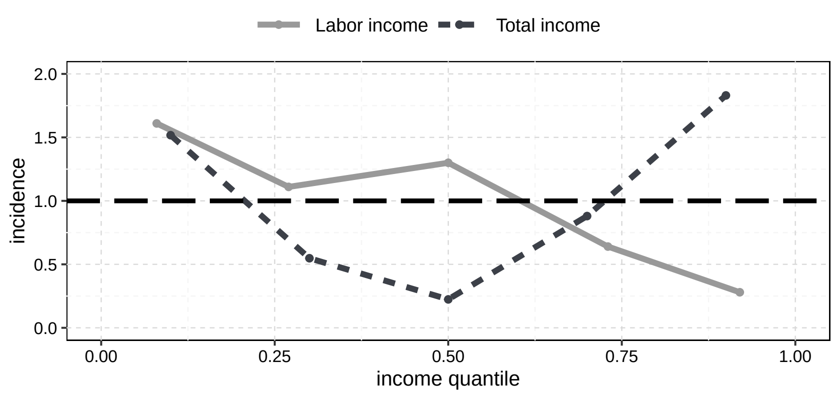
Legend (Boxplot): Rectangle borders delimit the 1st and 3rd quartile, and the bold bar displays the mean.

We find that MPIs follow a hump-shaped pattern, i.e., they are the highest at the middle-top of the wealth and income distribution. On the one hand, due to the selection of innovative entrepreneurs at higher income and wealth levels, MPIs tend to increase with wealth and income in the cross-section. On the other hand, the step innovation process generates a form of decreasing returns to scale at the top of the wealth distribution. Intuitively, for a given innovative investment, gains from additional investments decrease with wealth as $\frac{\partial \mathcal{I}(a,b,\mathbf{s},x)}{\partial b} < 0$. As such, MPIs decrease beyond a certain income and wealth threshold.

4.2.1 Income Incidence As shown in Table 3, the model-implied earnings incidence decreases along earning bins, consistent with recent empirical evidence in Broer et al. (2020) for monetary policy shocks based on German administrative data. Because of returns to innovative entrepreneurial activities, the total income incidence may however differ from the labor income incidence. To measure this discrepancy, we compute the incidence of aggregate income deviations on different individual total income bins by $\beta(\varkappa) = \frac{\Delta y_{\varkappa}}{\Delta y}$, where $\Delta y_{\varkappa}$ denotes the logarithm of income growth in bin $\varkappa$ of total income, and Δy denotes the logarithm of total income variations. By construction, $\int_{\varkappa} \Delta y_{\varkappa} dF_{\varkappa}(\varkappa) = \Delta y$ holds true.

Figure 3 shows that the total income incidence is U-shaped; while the decrease at the bottom is driven by a declining labor income incidence, the increase at the top is driven by the procyclical nature of innovative rents together with the self-selection of innovative entrepreneurs at the top of the distribution.[29] The U-shaped pattern is specific to the usage of nominal wage rigidity. Under sticky prices, the upper incidence on total income would decrease as profits are countercyclical.

Figure 3. Household *betas*: labor income and total income incidence.



Remark: The incidence is computed at impact from a 25-basis point surprise in the nominal interest rate.

4.2.2 Tax-Elasticity of Innovative Investment Finally, we investigate how innovative investment reacts to tax changes. Table 6 considers the partial equilibrium elasticity of growth, innovative investment and top income inequality with respect to the dividend tax rate τ^D and the personal income tax progressivity ϱ. Higher taxes substantially lower incentives to innovate and the resulting income inequality. A 1% increase in dividend taxation decreases the top 1% income share by 0.064%. In terms of growth, this increase lowers annual growth by 0.12 percentage points. Notice that we do not distinguish between labor income taxes or capital income taxes, but rather focus on the overall income tax progressivity or profit taxes. This assumption raises the question on how entrepreneurs report their taxable income. According to Smith et al. (2019), top earners

[29] The latter feature is, for instance, emphasized in Moll et al. (2022). They notably show that gains from automation overproportionally benefit households at the top of the income distribution.

derive most of their income from entrepreneurial business income (e.g., pass-through profits) that is taxed as ordinary income rather than from financial capital, which supports the approach taken in our paper. Our results is also closely linked to recent contribution by Jones (2022), who shows that taxation affects income inequality to the extent that it lowers the accumulation of innovative investments and, thus, the tail of distributed rents from innovation.

Table 6. Model response to dividend and income taxation in partial equilibrium [a].

INDICATOR ELASTICITY	RESPONSE TO AN INCREASE OF TAXATION	
	Dividend taxation τ^D	Income tax progressivity ϱ
Annual net growth rate g^A_{t+1}	-1.19	-0.79
Innovative expenditure Ψ_t	-0.47^b	-0.33
Top 1% income share	-0.06	-0.04
Fraction of innovative entrepreneur	-0.18	-0.02

[a] The elasticity is computed based on a one percentage point increase in τ^D from 0.20 to 0.21, and an increase in ϱ from 0.10 to 0.11.
[b] Targeted to generate the same empirical elasticity as in Akcigit et al. (2022) using patenting as proxy for innovation expenditures.

5 Quantitative Analysis of Macroeconomic Stabilization Policies

The central argument of this paper is that the income incidence of macroeconomic stabilization policies across households with heterogeneous MPCs and MPIs shapes aggregate short- and long-run outcomes. In this section, we use our analytical results to shed light on the efficacy of stabilization policies through the lens of the quantitative environment. We begin by analyzing the transmission of monetary policy in section 5.1, and then study an often implemented form of fiscal redistribution through discretionary extensions in unemployment benefits in section 5.2.

5.1 The Transmission of Monetary Policy

To evaluate the role of household heterogeneity for the transmission of a monetary policy, we compare the aggregate effects in the full HANK-GS economy to the same economy with: (1) household heterogeneity but a uniform earnings and unemployment incidence, i.e. $\mu(h) = \phi(h) = 1 \ \forall h$; (2) household heterogeneity in earnings and unemployment but not in the entrepreneurial status, i.e. $\Lambda_E = 1.0$; (3) a representative household.[30] The first comparison is designed to measure the effects of a higher degree of cyclical income inequality, while keeping the structure and parameters of the model identical. The second comparison isolates the cost-of-funds channel for innovative invest-

[30] Our representative household economy follows the exact same macro-block as the quantitative HANK-GS with two differences. First, we set the idiosyncratic components to a unique value, i.e. $h = \overline{h}$, $\beta = \overline{\beta}$, and $e = W$. Moreover, all individuals are entrepreneurs, i.e. $d_I = 1$. Second, we recalibrate β and γ to match the same aggregate moment for the steady state debt-to-GDP ratio and the technology growth rate. Our results are robust to using different re-calibration strategies, including for example fixing γ but changing η to match the same aggregate growth rate.

ment. Finally, the third comparison aims at quantifying the effects arising from the presence of inequality *per se*. Throughout, we consider a monetary surprise, that all else equal, would increase the nominal interest rate on liquid bonds by 25-basis points in the first quarter, i.e. $\epsilon_{mt} = 0.0025$. The persistence of the shock is set to $\rho_m = 0.65$. Until further notice, we assume that lump-sum transfers adjust to balance the government budget constraint.

Result 1. *The monetary policy transmission in the quantitative economy has the following properties.*

(a) The benchmark HANK-GS economy amplifies short-run output Y_t / A_t but stabilizes technology growth g^A_{t+1} compared to the representative household economy. The result is driven by different magnitudes in the average MPC and MPI.

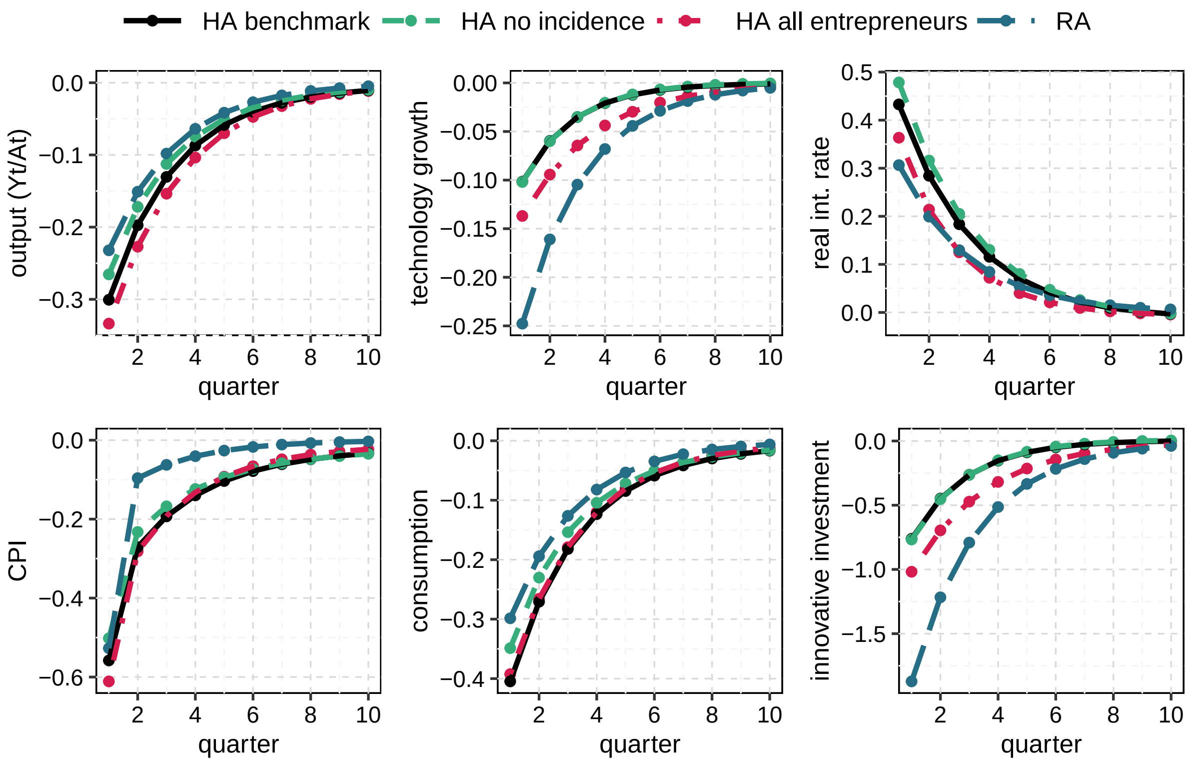
(b) A higher degree of cyclical income inequality amplifies short-run output Y_t / A_t, and is close to be neutral for technology growth g^A_{t+1}. The result is robust to a wide range of shock persistence levels, the dynamic structure of the NKPC, and the government financing scheme.

5.1.1 HANK-GS versus RANK-GS We begin with statement (a) of Result 1 and measure potential *scars from inequality*, as defined in Definition 1. Figure 4 displays the impulse responses for key aggregate variables under both the baseline HANK-GS economy and its RANK-GS counterpart. We find that household heterogeneity amplifies short-run output while it stabilizes technology growth. Computing the output and technology growth multiplier, $\mathcal{M}_y$ and $\mathcal{M}_g$, our results indicate a short-run output amplification from inequality of about 0.07 percentage points and a long-run stabilization from inequality of about 0.15 annualized percentage points at impact.

To understand the amplification of technology growth in the representative agent economy, it is useful to compare the propagation to a version of the HANK-GS economy in which all agents are entrepreneurs (only 75% of them are actually innovative entrepreneurs). In this case, we find that technology growth is amplified relative to the baseline economy. The result is driven by the fact that a variation in the overall returns to innovative investment goes along with a stronger cost of funds effect. Innovative entrepreneurs are now relatively poorer and are thus more prone to adjust investments following an income shock in order to smooth consumption. In other words, the responsiveness of investment increases in this economy. In the RANK-GS economy, this effect generates – despite a lower market size effect due to a low average MPC – short-run stabilization but long-run amplification.

5.1.2 The Effects of Cyclical Income Inequality The previous result that the benchmark HANK-GS economy amplifies short-run output but stabilizes technology growth relative to the RANK-GS economy may be driven by key structural differences between those two economies. We therefore perform a second experiment in which we isolate the role of cyclical income inequality for the transmission of monetary policy. The advantage of this approach is that structural parameters of

Figure 4. The transmission of monetary policy under HANK-GS.



Note: Output, consumption and investment are expressed in percentage deviation from steady state. The real interest rate, inflation, and technology growth are in annualized percentage point deviation.

the steady-state are preserved, while only specific components regarding the heterogeneous incidence in aggregate income fluctuations vary. Conceptually, this experiment is linked to section 2.6, as it amounts to trace out the role of varying cyclical earnings inequality and to locate the economy within the diagram of Figure 1. Table 7 displays the results under different scenarios.

In the light of section 2.6, countercyclical inequality has an ambiguous effect on technology growth. On the one hand, investors are, on average, less exposed to aggregate fluctuations through their labor income and keep investing into innovation, which stabilizes technology growth. On the other hand, the amplification of the output response pushes towards a stronger market size effect. We find that the latter effect dominates. A higher incidence, through both hours worked and unemployment fluctuations, amplifies the short-run output multiplier but increases the long-run technology loss. The interaction between heterogeneous unemployment risk and an unequal incidence on hours worked is shown to largely amplify the short-run output response. This finding is robust to the dynamic properties of the NKPC, i.e. whether we consider a static NKPC as in section 2 or a forward-looking NKPC. Nevertheless, our results indicate that a higher degree of countercyclical income inequality plays a minor role for the long-run transmission, given the benchmark level of shock persistence.

Table 7. Decomposition of impact multipliers to an anticipated monetary policy shock.

MODEL	Output	Technology
Benchmark economy	−0.824	−0.114
(a) No incidence with unemployment	−0.797	−0.117
(b) No incidence without unemployment	−0.565	−0.109
Benchmark economy with static NKPC	−0.788	−0.109
(a) No incidence with unemployment	−0.759	−0.112
(b) No incidence without unemployment	−0.555	−0.108

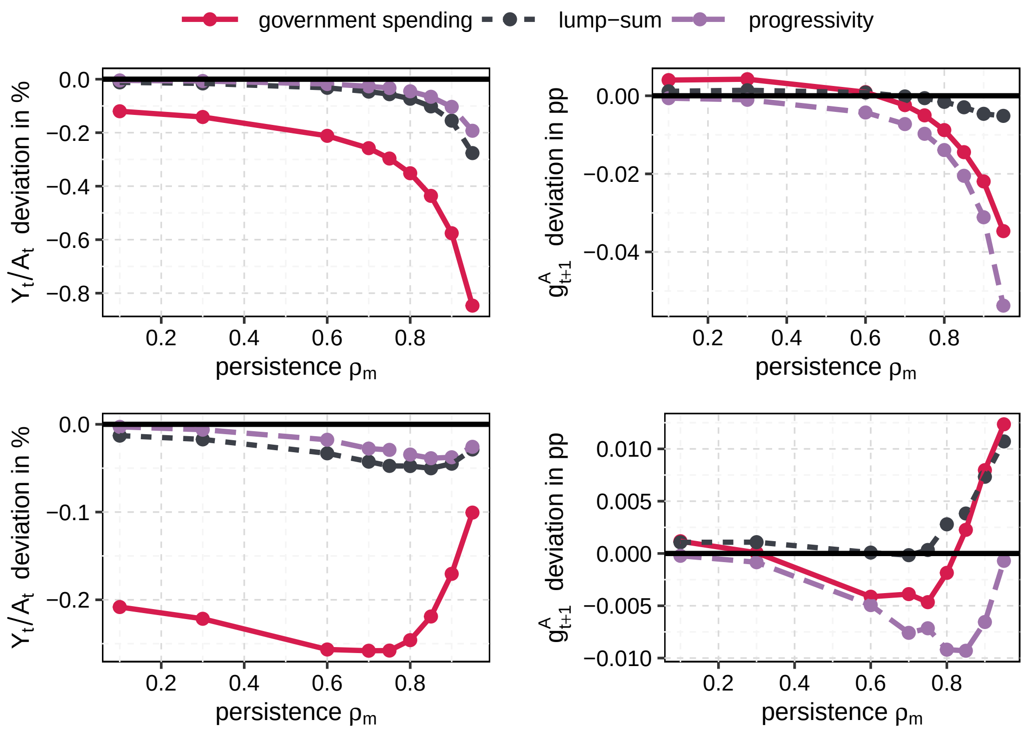
Note: Output multipliers are expressed in %-deviation from the steady-state. Technology multipliers are additionally denoted in annualized pp-deviation. Government budget closed by adjusting G_t. Case (a) ensures a uniform incidence according to $\mu(h) = 1.0, \phi(h) = 0.2$, while case (b) uses $\mu(h) = \phi(h) = 1.0$.

5.1.3 The Interaction between Shock Persistence and Cyclical Inequality We have thus far analyzed the role of cyclical inequality while keeping the magnitude and the persistence of the monetary surprise fixed. In Propositions 7 and 8, however, we have argued that the effects of cyclical income inequality on the short- and long-run propagation of monetary shocks crucially depend on the persistence. In Figure 5, we recompute the transition path under a static and a forward-looking NKPC and under differing financing schemes of the government budget constraint, i.e. lump-sum taxes, government spending, and tax progressivity. We compare our benchmark economy to the one without earnings incidence and no unemployment ($\mu(h) = \phi(h) = 1.0$).

Under the static NKPC (*top row*), a higher persistence induces countercyclical income inequality to decrease the impact multipliers of output unambiguously. The effect is nonlinear and the strongest if the budget constraint is balanced by cutting government spending, and weakest under rising tax progressivity. The former arises as cutting government spending reduces aggregate demand even further and, thus, amplifies the initial contraction. The latter arises as tax progressivity lowers the tax burden on high-MPC households and, thus, reduces the short-run contraction. Moreover, there arises a unique threshold on the persistence below (resp. above) which countercyclical income inequality stabilizes (resp. amplifies) technology growth. At this point, the redistributive effects of monetary policy though the cost of funds and the market size effect exactly offset each other. While there is no threshold for tax progressivity financing, the threshold is high for lump-sum and government spending adjustments, at a level of $\rho_m^{LR} = 0.60$ and $\rho_m^{LR} = 0.70$, respectively. Intuitively, higher tax progressivity generates an additional behavioral tax effect by lowering innovative investment returns – which relatively strengthens the market size effects – such that the counterbalancing cost of funds effect is small.

In the case of a forward-looking NKPC (*bottom row*), the effects of a higher degree of countercyclical income inequality depend non-trivially on the persistence of the monetary tightening. A higher persistence generates a U-shaped pattern; countercyclical income inequality decreases

Figure 5. The role of persistence for the short- and long-run transmission of monetary policy.



Note: The top row displays results for a static NKPC and the bottom row for a forward-looking NKPC. Results are expressed in percentage deviation from the economy with no incidence for output and in annualized percent point deviation from the economy with no incidence for technology growth.

output for low persistent levels and increases it for high persistent levels. This effect is strongest when government spending adjusts. The U-shape arises as wage inflation decreases that strongly for persistence levels above a certain point such that the Taylor rule endogenously offsets parts of the initial contraction by lowering nominal and real interest rates. This adjustment generates a complex response of technology growth as it limits the crowding out of private investment. For low levels of persistence, the cost of funds effect dominates and stabilizes technology growth. For moderate values, the market size effect dominates and countercyclical income inequality amplifies technology growth. Finally, the interest rate response dominates for high persistence levels, which stabilizes technology growth.

Our results highlight a non-trivial dimension through which the persistence of monetary policy may affect both the short- and long-run output response, depending on how the government budget is balanced. More generally, while the effects of an unequal income incidence are limited for moderate persistence levels, they are substantial for high persistence levels.[31]

[31] Our findings are in line with Alves et al. (2020) who document in a two-asset HANK economy that an unequal income incidence has only small effects on the propagation of monetary policy shocks. Contrary to them, we highlight that there arise substantial and nonlinear effects of an unequal income incidence for high persistence levels.

5.2 The Role of Social Insurance and Tax-and-Transfer Programs

We now shift our focus to assess the short- and long-run transmission of discretionary social insurance and tax-and-transfer programs as macroeconomic stabilization policies. Such policies reduce countercyclical consumption and income inequality by alleviating individual hardship.

As our leading example, we study the aggregate effects of temporary UI duration extensions. This policy is one of the most frequently implemented and analyzed macroeconomic stabilizers in the United States (see Nakajima (2012), McKay and Reis (2016), Gaillard and Kankanamge (2022) or Kekre (2022) among many others). This choice facilitates to discuss our results in light of previous findings and unpack how the joint analysis of household heterogeneity, aggregate demand and innovative investment alters the efficacy of such a policy.

Policy Experiment Over the past decades, the US Congress has passed several UI extensions that were automatically activated in periods of economic downturns, contingent on the economy-wide level of unemployment. At the federal level, the Temporary Extended Unemployment Compensation (TEUC) program was enacted in March 2002 and the Emergency Unemployment Compensation Act (EUC08) in June 2008. The EUC08 has been complemented by extended unemployment benefits (EB) programs at the state level that provided further UI extensions.[32] Following the former two programs, the maximum duration of eligible unemployment insurance benefits was massively prolonged, far beyond the standard coverage of 26 weeks during normal times. As such, UI durations reached effectively up to 99 weeks by the end of 2009.

To account for the extension in UI durations within our setup, we assume that the probability to stay in insured short-run unemployment (i.e., ρ^S) mimics the unemployment benefits duration during the Great Recession. This corresponds to an increase from two quarters (26 weeks) in the model to almost eight quarters (99 weeks), i.e., we temporarily increase ρ^S from 0.50 to 0.87 with a corresponding persistence of 0.85. As before, we consider different financing instruments, i.e., lump-sum taxes, government spending, increasing income tax progressivity and additional government debt. In reality, the ratio of total federal public debt over GDP went up by 20 percentage points after the Great Recession. Additionally, Bayer et al. (2020) provide empirical evidence that the degree of income tax progressivity has increased in between five to ten percent.

We run three main experiments. First, we investigate the aggregate short- and long-run effects to a temporary extension in the UI duration. We show that the respective financing instrument of this stabilization policy has drastic repercussions on long-run technology growth. Second, we quantify the forces put forward in Proposition 9 and focus on income tax policies to finance the UI.

[32] Moreover, the American Recovery and Reinvestment Act of 2009 granted further transfers to low income earners and unemployed individuals (such as food stamps), in addition to the unemployment insurance extensions. We focus on the case of UI extensions only in this experiment. In a sensitive analysis, we show similar findings for safety-net transfers.

We show how the degree of wage stickiness and the tax incidence along the income distribution substantially alters the propagation in this case. Our results are summarized below.

Result 2. *The UI transmission in the quantitative economy has the following properties.*

(a) When extensions in UI duration are financed by lump-sum transfers or debt, short-run output and technology growth increase. Financing via cutting government spending is close to be neutral for short-run output and long-run technology growth.

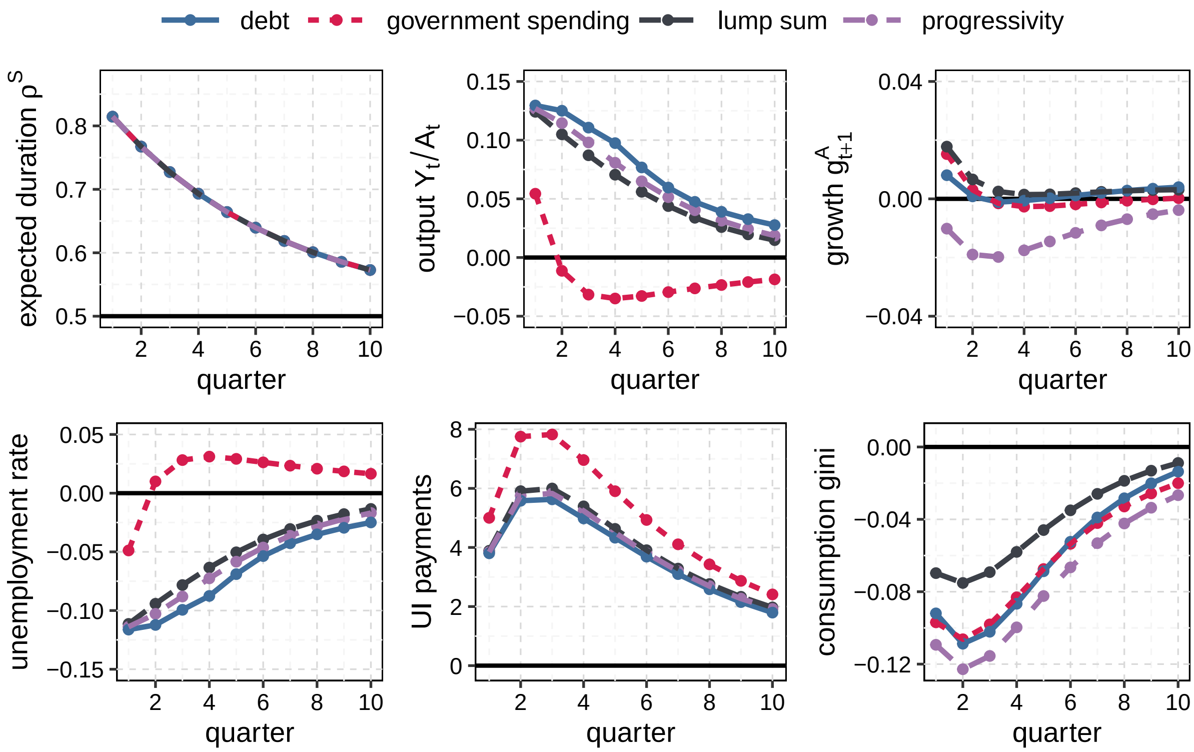
(b) In the benchmark economy, financing via higher income tax progressivity is beneficial for short-run output but harms long-run technology growth such that the policy becomes contractionary after two years. If demand effects are strong through a high wage stickiness with NKPC slope below $\epsilon/\theta = 0.023$*, progressive redistribution increases short-run output and long-run technology growth. On the contrary, if demand effects are weak with NKPC slope above* $\epsilon/\theta = 0.23$*, progressive redistribution decreases short-run output and long-run technology growth.*

(c) The joint distribution of MPCs and MPIs provides a rationale for taxing middle-class incomes. The result is in sharp contrast to economies with exogenous growth or flexible wages. Taxing high-incomes is always beneficial in economies with exogenous growth and sticky wages. Taxing low-incomes is always beneficial in economies with endogenous growth and flexible wages.

5.2.1 The Importance of the UI Financing Scheme. Proposition 9 highlighted that there arise three regimes after an increase in tax and transfer generosity: a regime in which short-run output and technology growth increase; a regime in which short-run output increases but technology growth decreases; and a regime in which both short-run output and technology growth decrease. In which of these regimes does a temporary extension in UI duration fall? How does the answer to the previous question depend on the financing scheme in place?

We gather the impulse responses to a temporary extension in UI duration in Figure 6. If the policy is financed by reducing government spending, we find that the reduction in the demand from lower government spending outweighs the increase in demand from private consumption. As such, normalized output Y_t/A_t falls and unemployment increases after two quarters. When the UI shock is financed by raising debt, lump-sum or progressive taxes, output increases. The strongest boost arises under progressive taxation, as the bottom of the income distribution with high MPCs does not bear any additional tax incidence in this case. Higher income tax progressivity, however, is at the cost of a reduction in technology growth as incentives to innovate are distorted. As a result, progressive income taxes generate a trade-off between short-run stabilization and long-run growth. Finally, under debt financing, the aggregate effects on output and technology growth are the highest. Our results, thus, provide a rationale for debt financing.

Figure 6. Aggregate effects of temporary extension in duration of UI benefits.



Remark: The anticipated shock in UI duration follows an AR(1)-process, i.e., $\rho_t^S = 0.85\rho_{t-1}^S + 0.15\rho_{ss}^S + \epsilon_t^S$, where the initial shock size is given by $\epsilon_0^S = 0.87$. Output and UI payment are expressed in percentage deviation from steady state. The growth rate and unemployment rate are in annualized percentage point deviation. The Gini coefficient is in deviation from steady-state.

How do short- and long-run effects compare? To put our results into perspective, we compute the output multiplier in the non-stationary economy after one year, and the long-run output multiplier relative to the counterfactual unshocked economy.[33] We present our results in Table 8. With respect to progressive income taxes there are several insights worth mentioning. First, there arises a strong short-run complementarity between aggregate demand and technology growth, as the one year multiplier under the benchmark economy is larger than the one without endogenous growth. Second, the presence of nominal wage rigidity gives rise to a short-run expansion in output, which significantly mitigates the long-run output loss relative to a counterfactual flexible wage economy. As a result, the long-run multiplier is in absolute value four times smaller in the sticky wage economy, and the temporary extension in the duration of UI benefits becomes

[33]Let us denote $\Delta\text{policy}_{t+s} = \text{UI payment}_{t+s} - \text{UI payment}$ and output in the counterfactual unshocked economy by Y_t^P. The short-run multiplier after one year ($T = 4$) and the annualized long-run multiplier ($T = \infty$) are defined by

$$\text{SR-output-multiplier} \equiv \frac{\sum_{s=0}^{T=4} Y_{t+s} - Y_{t+s}^P}{\sum_{s=0}^{T=4} \Delta\text{policy}_{t+s}}, \quad \text{and} \quad \text{LR-output-multiplier} \equiv \frac{4 \times (Y_{t+\infty} - Y_{t+\infty}^P)}{\sum_{s=0}^{\infty} \Delta\text{policy}_{t+s}}.$$

contractionary only after two years (i.e., 8.5 quarters).

We compare these results to the ones obtained under alternative financing schemes. Under lump-sum taxes, positive demand effects are relative weakened and the short-run output multiplier raises sightly less. As innovation decisions are, however, less distorted, the rise in demand generates positive long-run gains from this policy. In this case, a short- versus long-run stabilization tradeoff does not emerge. Finally, if the government cuts spending to finance the UI extension, overall aggregate demand contracts and output falls after two quarters. As income inequality is countercyclical, this policy redistributes towards entrepreneurs such that market size and cost of funds effect go in opposite directions and technology growth is almost insensitive to the policy. Finally, debt financing generates the strongest long-run technology growth effects.

Table 8. Decomposition of aggregate effects from temporary extension in duration of UI benefits.

Financing-Instrument	Short and Long-Run Output Multiplier								Time T^*
	Benchmark		Flex. Wage		Exo. Growth		Flex. + Exo.		
	1 year	$t \to \infty$	1 year	$t \to \infty$	1 year	$t \to \infty$			
Progressive tax	1.13	−0.17	−0.53	−0.70	0.93	0.00	−0.06	0.00	8.50
Lump-sum tax	0.89	0.12	−0.21	−0.13	0.63	0.00	−0.05	0.00	n.a.
Government spending	−0.02	0.03	−1.15	−0.13	−0.30	0.00	−0.86	0.00	n.a.
Debt	0.74	0.22	−0.31	−0.25	0.90	0.00	−0.04	0.00	n.a.

Remark: When there arises a trade-off between short-run output gains and long-run losses, T^* denotes the time horizon at which short-run gains are neutralized.

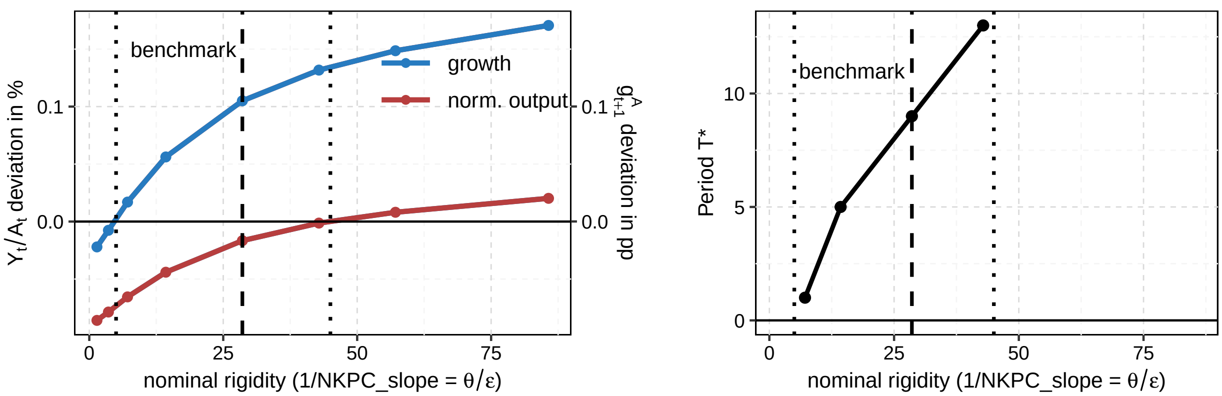
5.3 A Detailed Look at the Incidence of Stabilization Policies

Our quantitative findings provide a novel perspective on the efficacy of progressive stabilization policies. When neglecting the long-run propagation through technology growth, transitory progressive income redistribution turns out to be a successful stabilization policy in HANK economies. It does not only alleviate individual hardship during economic downturns, but also effectively stimulates aggregate demand by redistributing income to households with a high MPC. Kekre (2022), for instance, documents that a longer UI duration, that is financed with lump sum taxes on employed workers, increases aggregate demand, reduces unemployment and stabilizes the economy during recessions. Related, Ferriere and Navarro (2022) show that government spending multipliers are larger when they are financed by progressive income taxes. Our results confirm their findings regarding the short-run effectiveness of such policies. In contrast to them, another literature strand emphasizes that progressive income taxes lower innovation incentives and long-run growth (Jaimovich and Rebelo, 2017; Jones, 2022). Our analysis synthesizes both strands and points toward a short- versus long-run stabilization tradeoff as higher progressivity increases aggregate demand but distorts innovative investment decisions.

5.3.1 The Role of Nominal Rigidities

As shown above, one of the key forces through which fiscal redistribution matters for the short- and long-run propagation of UI extensions is the strength of demand effects. In light of Proposition 9, we now ask which degree of wage stickiness rationalizes each of the aforementioned three regimes. Figure 7 (left panel) displays the impact response of technology growth and normalized output depending on the NKPC slope. The right panel displays the implied time horizon T^* beyond which the policy implies a permanently lower output level relative to the unshocked economy.

Figure 7. Nominal rigidity and associated regime selection under progressive income tax financing.



When nominal rigidities are small, an increase in UI generates both a short- and a long-run output loss. In this case, demand effects are weak and returns to innovation decrease as the market size decreases and tax distortions increase. When nominal rigidities are sufficiently strong, short-run output and technology growth increase. This case arises under our benchmark calibration and generates a tradeoff between short- and long-run output gains. This tradeoff is shown in the right subfigure, which displays the time horizon T^* at which the UI policy generates neither output gains nor losses relative to the unshocked economy. Importantly, T^* increases in the degree of nominal wage rigidity as the latter strengthens demand effects. For high enough levels, it may even outweigh the disincentive effects on innovation induced by higher progressive income taxes. In this case, the UI benefit extension increases both short- and long-run output gains.

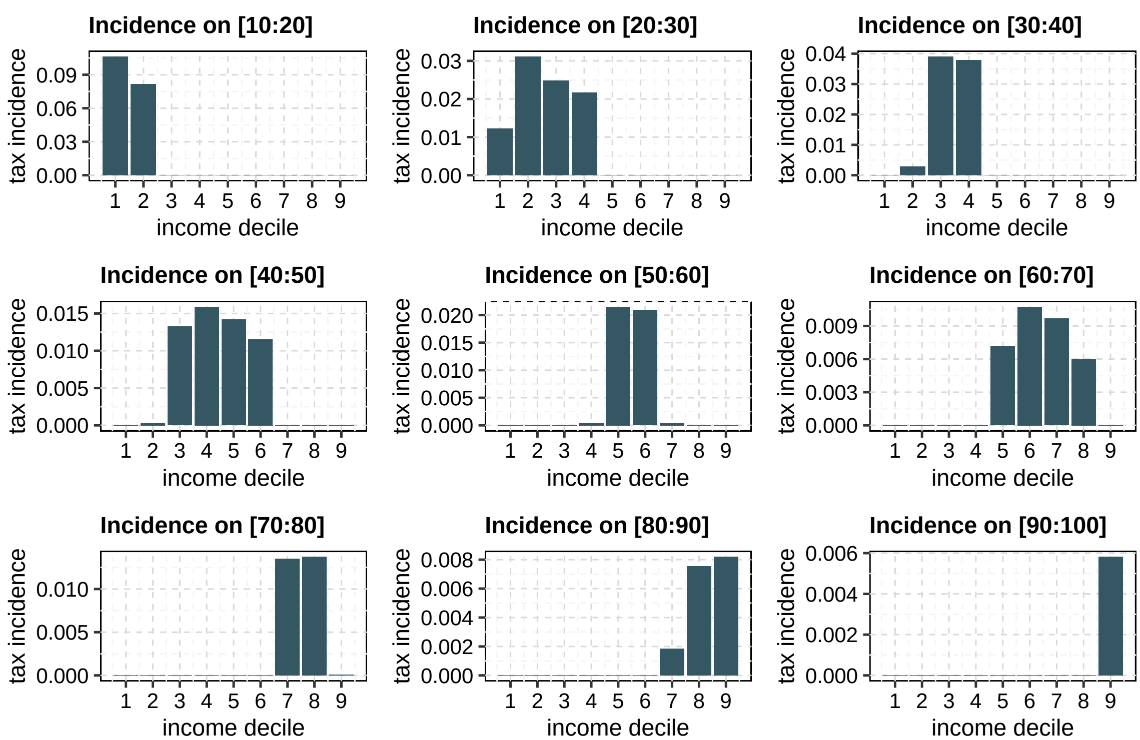
5.3.2 The Role of a Heterogeneous Tax Incidence

In Figure 2, we have documented that MPCs are, on average, high at the bottom of the income distribution, while MPIs are, on average, high at the middle and the top of the income distribution. This suggests that the tax incidence of additional costs from the UI extension may have large consequences on aggregate dynamics. To validate this hypothesis, we modify the tax schedule from the benchmark economy to target specific income groups, i.e.,

$$\mathcal{T}(y^d) = \max\left\{0,\ y^d - \vartheta\left(y^d\right)^{1-\varrho},\ \log(y^d - \underline{y}^1) + \underline{y}^2\right\},$$

where $\underline{y}^1$ is a scale parameter and $\underline{y}^2$ is a time-varying constant that adjusts to balance the government budget constraint in every period. In case of a primary surplus, we let lump-sum transfers adjust.[34] When $\underline{y}^1$ and $\underline{y}^2$ both increase, the additional tax incidence of the UI extension gradually falls on higher income deciles. In Figure 8, we vary the scale parameter $\underline{y}^1$ to target particular income deciles and display the resulting additional tax incidence $= \frac{\text{tax}_{\text{after change}} - \text{tax}_{\text{benchmark}}}{\text{tax}_{\text{benchmark}}}$.[35] Figure 9 gathers the aggregate effects when the tax incidence falls on a particular income decile.[36] We display normalized output Y_t/A_t, technology growth g^A_{t+1}, and the dynamics of output Y_t at different horizons. In all figures, the direction of the arrow refers to a tax incidence on a higher income decile, i.e., in ascending order from the first to the ninth decile.

Figure 8. Heterogeneous tax incidence as a function of the scale parameter $\underline{y}^1$.



Legend: The subfigures display the tax incidence at different income deciles when varying the scale parameter $\underline{y}^1$. In all cases, $\underline{y}^2$ adjusts to balance the government budget.

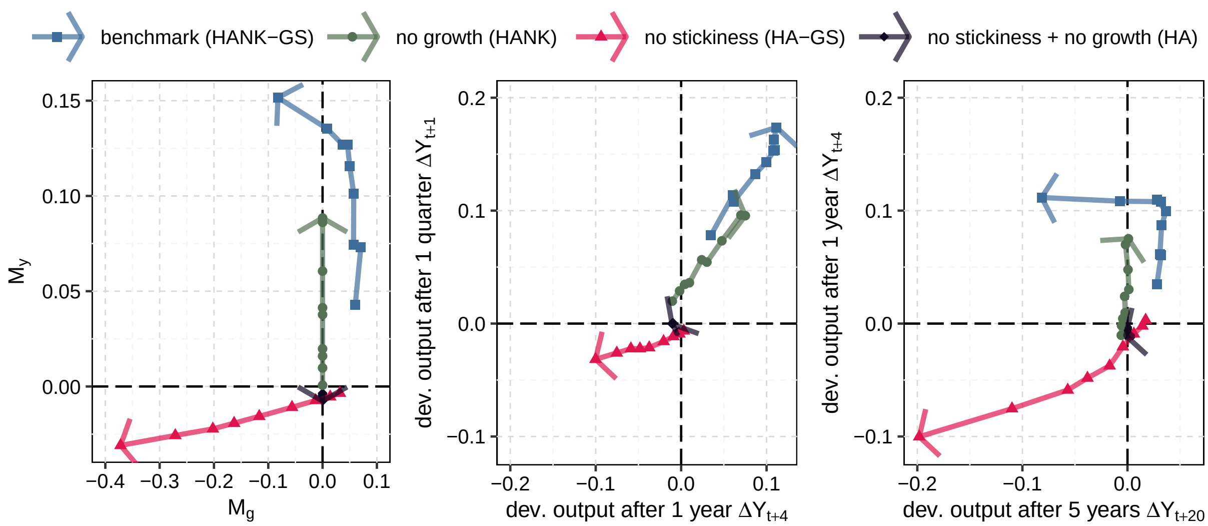
First, in a heterogeneous agent economy without endogenous growth and nominal wage rigid-

[34] Notice that the UI extension leads for the bulk of calibrations to a primary deficit.

[35] There are many other ways to generate a heterogeneous tax incidence, including non-linear piece-wise functions. We illustrate our case in the Appendix C.2.2.

[36] In Appendix C.5, we report additional results when the tax incidence falls on the top 1% income households.

Figure 9. Heterogeneous tax incidence and the short- and long-run effects of a temporary UI extension.



Legend: The left panel displays the annual multipliers for normalized output Y_t / A_t and technology growth g^A_{t+1}. The middle and right panels show the dynamics of the economy at different horizons. Each dot represents the tax incidence on a targeted income decile, in accordance to Figure 8 and applied for each model class. The arrow indicates the direction of the tax incidence toward higher income deciles. The y-axis represents short-run output gains/losses, i.e., output deviations from the unshocked economy after one quarter (left and middle panel) and after one year (right pannel). The x-axis represents longer-run output gains/losses, i.e., output deviations from the unshocked economy after one quarter (left panel), after one year (middle panel) and after five years (right panel).

ity (HA model) it is almost irrelevant whether the tax incidence falls on the bottom or the top of the income distribution. In this case, real wages directly adjust to the UI extension and MPC heterogeneity is inconsequential. Second, in an endogenous growth economy without nominal wage rigidity (HA-GS model), a higher tax incidence on the top of the income distribution decreases both short- and long-run output. Both responses result from lower innovative investment. After five years, the total loss relative to the unshocked economy is substantial and corresponds to an output loss of roughly one BGP quarter. In contrast, in an exogenous growth economy with nominal wage rigidity (HANK model), a higher tax incidence on the top of the income distribution increases short-run output gains, and is, however, irrelevant for long-run dynamics. In this case, the sole presence of MPC heterogeneity pushes toward lower taxes at the bottom of the income distribution in order to maximize output gains.

What happens when the economy features jointly nominal wage rigidity and endogenous growth (benchmark HANK-GS model)? In this case, more redistribution increases aggregate demand, which pushes toward higher innovative investments. At the same time, however, more redistribution tends to reduce innovation incentives such that the overall effect becomes ambiguous. Quantitatively, we find that moving the tax incidence from the bottom to the top of the income distribution generates *nonlinear* effects. It first increases short-run output (after one year) without affecting technology growth. The marginal effect on technology growth is close to zero because the extra market size offsets the effects from higher taxes on innovation. Moreover, when the incidence falls on the majority of households at the middle and the top of the income distribution, technology growth becomes negative. Last, when the tax incidence falls mainly on innovative entrepreneurs, both short- and long-run output fall.

Overall, our analysis suggests that neglecting either nominal rigidities or endogenous growth leads to a misleading view on the desirability of stabilization policies that are financed by taxes. Our results emphasize that the joint distribution of MPCs and MPIs together with the tax incidence are necessary to fully understand the transmission of such policies. As such, we stress that a policy which induces a tax incidence on middle-class incomes rather than the bottom-class or middle-top-class incomes generates both short- and long-run output gains. For example, the right subplot of Figure 9 indicates that a tax incidence on the fifth income decile maximizes output gains five years after the UI extension. Those gains are superior to the ones achieved under lump-sum taxes. Moreover, as seen earlier, debt-financing that is coupled to future lump-sum repayments generates the highest long-run output gains. Hence, debt financing and the taxation of middle-class incomes stand out as effective tools to finance stabilization policies under HANK-GS.

5.4 Robustness

In addition to the previous analysis, we have explored the robustness of our findings along several dimensions. First, we account for the possibility that discretionary variations in the duration of UI generosity may alter the job-posting behavior of firms and the incentives to work by lowering job-finding rates. To integrate these features, we assumed that the unemployment adjustment margin $\phi(h)$ increases in the duration of the UI benefits ρ_S. This modeling choice captures the fact that firing costs increase under the UI policy such that fewer jobs are opened. Moreover, we assumed that job-finding rates decrease in the UI duration in order to capture possible moral hazard incentives. Under this alternative specification, short-run output still increases for a reasonable range of job-posting and job-finding disincentive effects. Overall, the inclusion of these elements weakens the market size effect under all financing scheme, which reduces the technology growth. Second, we compared the relative efficacy between extensions in the UI duration, extensions in the amount of UI benefits and an increase in safety-net transfers T^u. Our results remain qualitatively valid and the relevant tradeoffs are preserved. Finally, we also considered a model extension in which we allow for an extensive margin of entering entrepreneurship in the spirit of Jaimovich and Rebelo (2017). In such a version, entry into innovative investment incurs an additional fixed cost. This version provided qualitatively similar results, such that we focused in the main body of the paper on a simpler setup without entry into and exit from entrepreneurship.

6 Conclusion

Consistent with recent empirical evidence, we develop a unified HANK-GS framework in which household heterogeneity, business cycles and long-run growth jointly interact. We apply this framework to revisit the efficacy of discretionary macroeconomic stabilization policies. We show that cyclical variations in income inequality, which are endogenously engendered by such policies, do no only affect output in the short-run, but may also have substantial effects on long-run technology growth.

To derive these findings, we first laid down an analytical model that enriches a limited heterogeneity variant of the HANK model by endogenous growth. We identified three key statistics to understand the role of cyclical income inequality for the long-run propagation of macroeconomic stabilization policies: (i) the income exposure of high-MPI households, (ii) the income exposure of high-MPC households, and (iii) the persistence of the discretionary policy. We then showed that a regime arises in which discretionary progressive redistribution policies stabilize output and inequality in the short-run but slow down the long-run recovery from economic downturns by generating permanent output losses. Second, we quantitatively assessed these findings within a full-blown HANK-GS model of the US economy. While countercyclical income inequality has only small effects on long-run technology growth in the transmission of a monetary surprise, household heterogeneity *per se* stabilizes long-run technology growth relative to a representative household benchmark. Finally, we evaluated temporary extensions in the duration of unemployment benefits during the Great Recession and showed that their long-run propagation largely depends on the financing instrument used by the government.

This paper provides a first step to jointly assess the complex interaction between household heterogeneity, business cycles and growth. Our analysis abstracted from numerous factors that shape this nexus, part of which we plan to address in our own research agenda. We consider heterogeneity in the composition of household consumption bundles (Beraja and Wolf, 2022; Laibson et al., 2022) as a fruitful extension, that may have important repercussions on innovative investment. A further natural direction is to explore the normative features of our framework, in particular the design of optimal tax and social insurance systems. Finally, much more empirical work is needed to structurally identify the short- and long-run effects of (cyclical) income inequality and stabilization policies. We leave the answers to these exciting questions to future research.

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Appendix

The Appendix is organized as follows. Appendix A contains derivations of the analytical heterogeneous agent New Keynesian growth economy, while Appendix B presents proofs to section 2. Appendix OA1 discusses additional results and extensions. Finally, Appendix C presents details on the computational algorithm as well as the model calibration of our quantitative analysis.

A Theoretical Appendix - Analytical Derivations

A.1 Competitive Equilibrium

A.1.1 Household Problem The derivation of the household problem is similar to Bilbiie et al. (2022). The first order conditions to the maximization objective of savers are

$$
\begin{aligned}
\mathcal{U}_c(C_t^S, L_t^S) &= \beta \mathbb{E}_t \left[(1-\lambda) s \frac{\partial V^S(B_{t+1}^S, \omega_{t+1}^S)}{\partial B_{t+1}^S} + \lambda (1-s) \frac{\partial V^H(B_{t+1}^H, \omega_{t+1}^S)}{\partial B_{t+1}^H} \right] + \Xi_t^S \, , \\
\mathcal{U}_c(C_t^S, L_t^S) \frac{q_t}{1-\lambda} &= \beta \mathbb{E}_t \left[\frac{\partial V^S(B_{t+1}^S, \omega_{t+1}^S)}{\partial \omega_{t+1}^S} \right] , \\
\Xi_t^S b_{t+1}^S &= 0 \, ,
\end{aligned}
$$

where the first equation is the optimality condition for bond holdings b_{t+1}^S, the second equation the optimality condition for stock holdings ω_{t+1}^S of intermediary firms, while the third equation denotes the complementary slackness condition with Lagrange multiplier Ξ_t^S. Similarly, the first order conditions of hand-to-mouth households read

$$
\begin{aligned}
\mathcal{U}_c(C_t^H, L_t^H) &= \beta \mathbb{E}_t \left[\lambda h \frac{\partial V^H(B_{t+1}^H)}{\partial B_{t+1}^H} + (1-\lambda)(1-h) \frac{\partial V^S(B_{t+1}^S, \omega_{t+1}^S)}{\partial B_{t+1}^S} \right] + \Xi_t^H \, , \\
\Xi_t^H b_{t+1}^H &= 0 \, ,
\end{aligned}
$$

where the first equation denotes the condition for bond holdings b_{t+1}^H, and the second one the complementary slackness condition with corresponding Lagrange multiplier Ξ_t^H. The Envelope condition to the saver Bellman equation regarding bond holdings is

$$
\begin{aligned}
\frac{\partial V^S(B_t^S, \omega_t^S)}{\partial B_t^S} &= \frac{\mathcal{U}_c^S R_{t-1}}{(1-\lambda)\pi_t} + \left(-\frac{\mathcal{U}_c^S - \Xi_t^S}{(1-\lambda)s} + \beta \mathbb{E}_t \left[\frac{\partial V^S(B_{t+1}^S, \omega_{t+1}^S)}{\partial B_{t+1}^S} + \frac{\lambda}{1-\lambda} \frac{\partial V^H(B_{t+1}^H)}{\partial B_{t+1}^H} \frac{\partial B_{t+1}^H}{\partial B_{t+1}^S} \right] \right) \frac{\partial B_{t+1}^S}{\partial B_t^S} \\
&= \frac{\mathcal{U}_c^S R_{t-1}}{(1-\lambda)\pi_t} \, ,
\end{aligned}
$$

where the last equality is due to $\frac{\partial B^H_{t+1}}{\partial B^S_{t+1}} = \frac{1-s}{s}$. Analogously, the saver Envelope condition regarding stock holdings is given by

$$\begin{aligned}\frac{\partial V^S(B^S_t, \omega^S_t)}{\partial \omega^S_t} &= \frac{\mathcal{U}^S_c(q_t + D_t)}{1-\lambda} + \left(-\frac{\mathcal{U}^S_c q_t}{1-\lambda} + \beta \mathbb{E}_t\left[\frac{\partial V^S(B^S_{t+1}, \omega^S_{t+1})}{\partial \omega^S_{t+1}}\right]\right)\frac{\partial \omega^S_{t+1}}{\partial \omega^S_t} \\ &= \frac{\mathcal{U}^S_c(q_t + D_t)}{1-\lambda} .\end{aligned}$$

Following similar steps, the Envelope condition for the hand-to-mouth Bellman equation is

$$\begin{aligned}\frac{\partial V^H(B^H_t)}{\partial B^H_t} &= \frac{\mathcal{U}^H_c R_{t-1}}{\lambda \pi_t} + \left(-\frac{\mathcal{U}^H_c - \Xi^H_t}{\lambda h} + \beta \mathbb{E}_t\left[\frac{\partial V^H(B^H_{t+1})}{\partial B^H_{t+1}} + \frac{1-\lambda}{\lambda}\frac{\partial V^S(B^s_{t+1}, \omega^S_{t+1})}{\partial B^S_{t+1}}\frac{\partial B^S_{t+1}}{\partial B^H_{t+1}}\right]\right)\frac{\partial B^H_{t+1}}{\partial B^H_t} , \\ &= \frac{\mathcal{U}^H_c R_{t-1}}{\lambda \pi_t} ,\end{aligned}$$

where the last equality follows because of $\frac{\partial \boldsymbol{B}^S_{t+1}}{\partial \boldsymbol{B}^H_t} = \frac{1-h}{h}$. Imposing the insurance Equilibrium Properties 1 leaves us due to $\Xi^H > 0$ and $\Xi^S = 0$ with the Euler equation for saver households

$$\frac{1}{C^S_t} = \beta \mathbb{E}_t\left[\frac{R_t}{\pi_{t+1}}\left(s\frac{1}{C^S_{t+1}} + (1-s)\frac{1}{C^H_{t+1}}\right)\right] .$$

A.1.2 Union Wage Setting In this section, we provide the details regarding the derivation of the static New Keynesian Wage Phillips curve. Differentiated labor inputs are bundled according to a CES aggregator of the form

$$L_t = \left(\int_0^1 L_t(l)^{\frac{\epsilon_w - 1}{\epsilon_w}} dl\right)^{\frac{\epsilon_w}{\epsilon_w - 1}} , \quad \text{with} \quad \epsilon_w > 1 .$$

The firm's minimization problem is written as

$$\min_{\{L_t(l)\}_l} \int_0^1 W_t(l) L_t(l) \quad s.t. \quad L_t = \left(\int_0^1 L_t(l)^{\frac{\epsilon_w - 1}{\epsilon_w}} dl\right)^{\frac{\epsilon_w}{\epsilon_w - 1}} .$$

The corresponding first order condition is given by

$$-W_t(l) + \Xi_w L_t^{\frac{1}{\epsilon_w}} L_t(l)^{-\frac{1}{\epsilon_w}} = 0 ,$$

which holds for each variety l and where Ξ_w denotes the Lagrange multiplier on the CES aggregator. Considering with a slight abuse of notation two distinct varieties l and l', we get

$$L_t(l) = L_t(l') \left(\frac{W_t(l)}{W_t(l')} \right)^{-\epsilon_w} .$$

Substituting the former expression back into the CES aggregator yields

$$L_t = \left(\int_0^1 \left(L_t(l') \left(\frac{W_t(l)}{W_t(l')} \right)^{-\epsilon_w} \right)^{\frac{\epsilon_w - 1}{\epsilon_w}} dl \right)^{\frac{\epsilon_w}{\epsilon_w - 1}} = L_t(l') W_t(l')^{\epsilon_w} \left(\int_0^1 W_t(l)^{1-\epsilon_w} dl \right)^{-\frac{\epsilon_w}{1-\epsilon_w}} .$$

Given that the aggregate wage index is defined by

$$W_t \equiv \left(\int_0^1 W_t(l)^{1-\epsilon_w} dl \right)^{\frac{1}{1-\epsilon_w}} ,$$

the above equation can be rewritten by replacing l' with a generic l as

$$L_t(l) = L_t^d \left(\frac{W_t(l)}{W_t} \right)^{-\epsilon_w} .$$

The maximization objective of union l is thus given by

$$\begin{aligned}
&\max_{\{W_t(l)\}} \ \ln C_t(l) - \nu \frac{(L_t(l))^{1+\varphi}}{1+\varphi} \quad s.t. \\
&L_t(l) = \left(\frac{W_t(l)}{W_t} \right)^{-\epsilon_w} L_t^d , \\
&C_t(l) = \frac{W_t(l)}{P_t} L_t(l) + D_t(l) + T_t^H(l) - \frac{\theta}{2} \left(\frac{W_t(l)}{W_{t-1}} - g^A \right)^2 Y_t ,
\end{aligned}$$

where we have directly imposed the equilibrium condition for shares, i.e. $\omega_{t+1} = \omega_t = 1 \ \forall t$, respectively bonds $b_{t+1}^S = b_{t+1}^H = 0$. Notice that nominal labor income of the average household is given by

$$W_t(l) L_t(l) = L_t^d \frac{W_t(l)^{1-\epsilon_w}}{W_t^{-\epsilon_w}} .$$

The first order condition to the maximization problem is thus given by

$$\frac{1}{C_t(l)} \left[\frac{(1-\epsilon_w) L_t(l)}{P_t} - \theta \left(\frac{W_t(l)}{W_{t-1}} - g^A \right) \frac{Y_t}{W_{t-1}} \right] + \nu \epsilon_w \frac{L_t(l)^{1+\varphi}}{W_t(l)} = 0 .$$

As unions face identical first order conditions, we presuppose a symmetric equilibrium, i.e. $W_t(l) = W_t$ and thus $L_t = L_t(l)$. Multiplying through with W_t and defining gross nominal wage inflation as $\pi_t^w \equiv \frac{W_t}{W_{t-1}}$ provides us with

$$\left(\pi_t^w - g^A\right) \pi_t^w Y_t = \frac{\epsilon_w}{\theta} L_t \left(\nu L_t^{\varphi} C_t - \frac{\epsilon_w - 1}{\epsilon_w} \frac{W_t}{P_t}\right) .$$

A.1.3 Final and Intermediate Good Production Final Good. The objective of the competitive final good producer is given by

$$\max_{\{L_t, X_{j,t}\}} \; P_t (Z_t L_t)^{1-\alpha} \int_0^1 A_{j,t}^{1-\alpha} X_{j,t}^{\alpha} dj - P_{j,t} X_{j,t} - W_t L_t \, ,$$

with corresponding first order conditions

$$W_t = (1-\alpha) P_t Z_t^{1-\alpha} L_t^{-\alpha} \int_0^1 A_{j,t}^{1-\alpha} X_{j,t}^{\alpha} dj \, ,$$

$$P_{j,t} = \alpha P_t (Z_t L_t)^{1-\alpha} A_{j,t}^{1-\alpha} X_{j,t}^{\alpha-1} \, .$$

Intermediate Goods. Using the previous two conditions, one can write the price setting problem of intermediate firm profits as

$$\max_{\{P_{j,t}\}} \; \Theta_{j,t}^n = (P_{j,t} - P_t) X_{j,t}^d = (P_{j,t} - P_t) \left(\frac{P_{j,t}}{\alpha P_t}\right)^{\frac{1}{\alpha-1}} A_{j,t} Z_t L_t$$

The first order condition to this problem is

$$\left(\frac{P_{j,t}}{\alpha P_t}\right)^{\frac{1}{\alpha-1}} + \frac{1}{\alpha-1} (P_{j,t} - P_t) \left(\frac{P_{j,t}}{\alpha P_t}\right)^{\frac{1}{\alpha-1}-1} \frac{1}{\alpha P_t} = 0 \, ,$$

which can be rearranged to

$$1 + \frac{1}{\alpha-1} \frac{P_{j,t} - P_t}{P_{j,t}} = 0 \quad \Leftrightarrow \quad \alpha P_{j,t} = P_t \, .$$

As a result, one obtains

$$X_{j,t} = \alpha^{\frac{2}{1-\alpha}} A_{j,t} Z_t L_t \, ,$$

and consequently

$$Y_t^G = \alpha^{\frac{2\alpha}{1-\alpha}} A_t Z_t L_t \, .$$

Finally, nominal profits are given by

$$\Theta^n_{j,t} = \alpha^{-1}\left(1-\alpha\right)\alpha^{\frac{2}{1-\alpha}} P_t A_{j,t} Z_t L_t = \Theta_\alpha P_t A_{j,t} Z_t L_t \ ,$$

where $\Theta_\alpha \equiv \alpha^{-1}\left(1-\alpha\right)\alpha^{\frac{2}{1-\alpha}}$. Thus, real profits are given by $\Theta_{j,t} = \Theta_\alpha A_{j,t} Z_t L_t$. Using the results from the price setting problem, one can restate the innovation objective as

$$\begin{aligned}
&\max_{\{A_{j,t+1}\}} \ \mathbb{E}_0\left[\sum_{t=0}^{\infty}\frac{(\beta s(1-\delta))^t}{C^S_t}\left((1-\tau^D_t)\Theta_{j,t} - s(1-\delta)I_{j,t}\right)\right] \ , \\
&s.t. \qquad \Theta_{j,t} = \Theta_\alpha A_{j,t} Z_t L_t \ , \\
&\qquad\qquad A_{j,t+1} = A_{j,t} + \psi I_{j,t} \ , \quad A_{j,0} > 0 \ .
\end{aligned}$$

The corresponding Lagrangian becomes

$$\mathcal{L} = \mathbb{E}_0\left[\sum_{t=0}^{\infty}\frac{(\beta s(1-\delta))^t}{C^S_t}\left((1-\tau^D_t)\Theta_\alpha A_{j,t} Z_t L_t - s(1-\delta)\frac{A_{j,t+1}-A_{j,t}}{\psi}\right)\right] \ .$$

Taking the first order condition with respect to $A_{j,t+1}$ results in

$$-\frac{(\beta s(1-\delta))^t}{C^S_t}\frac{s(1-\delta)}{\psi} + \mathbb{E}_t\left[\frac{(\beta s(1-\delta))^{t+1}}{C^S_{t+1}}\left((1-\tau^D_{t+1})\Theta_\alpha Z_{t+1} L_{t+1} + \frac{s(1-\delta)}{\psi}\right)\right] = 0 \ .$$

Rearranging results in

$$1 = \beta\mathbb{E}_t\left[\frac{C^S_t}{C^S_{t+1}}\left((1-\tau^D_{t+1})\psi\Theta_\alpha Z_{t+1} L_{t+1} + s(1-\delta)\right)\right] \ .$$

A.1.4 Inflation Dynamics Substituting the expression for $X_{j,t}$ into the final producer labor first order condition, one obtains

$$P_t = \frac{1}{(1-\alpha)\alpha^{\frac{2\alpha}{1-\alpha}}}\frac{W_t}{Z_t A_t} \ ,$$

such that one obtains

$$\pi_t = \pi^w_t \frac{Z_{t-1}}{Z_t}\frac{1}{g^A_t} \ .$$

A.2 Stationary Equilibrium

The stationary equilibrium is obtained by normalizing trending variables by aggregate endogenous technology growth A_t. As a result, it is described by the following equations.

1. Saver Euler equation	$\frac{g^A_{t+1}}{c^S_t} = \beta \mathbb{E}_t \left[\frac{R_t}{\pi_{t+1}} \left(s \frac{1}{c^S_{t+1}} + (1-s) \frac{1}{c^H_{t+1}} \right) \right]$
2. Growth equation	$g^A_{t+1} = \beta \mathbb{E}_t \left[\frac{c^S_t}{c^S_{t+1}} \left((1-\tau^D_{t+1}) \psi \Theta_\alpha Z_{t+1} L_{t+1} + s(1-\delta) \right) \right]$
3. Real wage	$w_t = (1-\alpha)\alpha^{\frac{2\alpha}{1-\alpha}} Z_t$
4. Wage Phillips curve	$\left(\pi^w_t - g^A\right) \pi^w_t y_t = \frac{\epsilon_w}{\theta} L_t \left(\nu L^\varphi_t c_t - \frac{\epsilon_w - 1}{\epsilon_w} (1-\alpha) \alpha^{\frac{2\alpha}{1-\alpha}} Z_t \right)$
5. Fiscal transfers	$t^H_t = \tau^D_t \Theta_\alpha Z_t L_t$
6. Hours worked	$L^H_t = (1-\mu)L + \mu L_t\ , \quad L^S_t = \frac{\lambda(\mu-1)}{1-\lambda} L + \frac{1-\lambda\mu}{1-\lambda} L_t\ ,$ $L_t = \lambda L^H_t + (1-\lambda) L^S_t$
7. HtM consumption	$c^H_t = w_t L^H_t + \frac{\tau^D_t}{\lambda} \Theta_\alpha Z_t L_t$
8. Saver consumption	$c^S_t = w_t L^S_t + \frac{d_t}{1-\lambda}$
9. Dividends	$d_t = (1-\tau^D_t)\alpha^{-1}(1-\alpha)x_t - \iota^A_t$
10. Consumption	$\lambda c^H_t + (1-\lambda) c^S_t = c_t$
11. LOM productivity	$g^A_{t+1} = 1 + \psi \iota^A_t$
12. Monetary policy rule	$r_t = r + \phi_\pi \ln\left(\frac{\pi^w_t}{\pi^w}\right) + \epsilon_{mt}$
13. Intermediary inputs	$x_t = \alpha^{\frac{2}{1-\alpha}} Z_t L_t$
14. Gross output	$y^G_t = \alpha^{\frac{2\alpha}{1-\alpha}} Z_t L_t$
15. Output	$y_t = \Upsilon_\alpha Z_t L_t$
16. Resource constraint	$y_t = c_t + \iota^A_t + \frac{\theta}{2}\left(\pi^w_t - g^A\right)^2 y_t$
17. Inflation dynamics	$\pi_t = \pi^w_t \frac{Z_{t-1}}{Z_t} \frac{1}{g^A_t}$

A.3 Steady State

As stated in the main body of the text, we consider a steady state that is characterized by $Z = 1$ and $\pi = 1$. Based on these assumptions, one obtains $\pi^w = g^A$ such that the steady state of the HANK-GS economy is determined recursively. The subsequent three equations determine steady state hours worked, based on which one can recursively determine the remaining values.

$$\begin{aligned} g^A &= \beta\left((1-\tau^D)\psi\Theta_\alpha L + s(1-\delta)\right) \,, \\ \nu L^\varphi c &= \frac{\epsilon_w - 1}{\epsilon_w}(1-\alpha)\alpha^{\frac{2\alpha}{1-\alpha}} \,, \\ Y_\alpha L &= c + \frac{g^A - 1}{\psi} \,. \end{aligned}$$

As a result, steady state hours worked are implicitly characterized by

$$\nu L^\varphi \left(\left(Y_\alpha - \beta(1-\tau^D)\Theta_\alpha\right) L + \frac{1-\beta s(1-\delta)}{\psi}\right) = \frac{\epsilon_w - 1}{\epsilon_w}(1-\alpha)\alpha^{\frac{2\alpha}{1-\alpha}} \,. \tag{16}$$

To see whether there exists a unique $L > 0$ that satisfies the previous equation, notice that

$$\begin{aligned} Y_\alpha - \beta(1-\tau^D)\Theta_\alpha &= \alpha^{\frac{2\alpha}{1-\alpha}}(1-\alpha^2) - \beta(1-\tau^D)\alpha^{-1}(1-\alpha)\alpha^{\frac{2}{1-\alpha}} \\ &= \alpha^{\frac{2\alpha}{1-\alpha}}(1-\alpha^2)\left(1 - \beta(1-\tau^D)\frac{\alpha(1-\alpha)}{1-\alpha^2}\right) \\ &= \alpha^{\frac{2\alpha}{1-\alpha}}(1-\alpha^2)\left(1 - \beta(1-\tau^D)\frac{\alpha}{1+\alpha}\right) \,. \end{aligned} \tag{17}$$

It can be seen that the previous expression is increasing in τ^D. Hence, if it is positive for $\tau^D = \underline{\tau}^D$, then it is also positive for all τ^D. Notice that $\underline{\tau}^D$ is determined by ensuring positive steady state consumption of hand-to-mouth households, which is the case if

$$\begin{aligned} c^H &= (1-\alpha)\alpha^{\frac{2\alpha}{1-\alpha}} L + \frac{\tau^D}{\lambda}\Theta_\alpha L = \left((1-\alpha)\alpha^{\frac{2\alpha}{1-\alpha}} + \alpha^{-1}(1-\alpha)\alpha^{\frac{2}{1-\alpha}}\frac{\tau^D}{\lambda}\right) L \\ &= (1-\alpha)\alpha^{\frac{2\alpha}{1-\alpha}} L \left(1 + \frac{\alpha\tau^D}{\lambda}\right) \,, \end{aligned}$$

which is positive if $\tau^D > \underline{\tau}^D \equiv -\frac{\lambda}{\alpha}$. Substituting $\underline{\tau}^D$ into (17), we obtain

$$Y_\alpha - \beta(1-\tau^D)\Theta_\alpha = \alpha^{\frac{2\alpha}{1-\alpha}}(1-\alpha^2)\left(1 - \beta\frac{\lambda+\alpha}{1+\alpha}\right) > 0 \,,$$

which is strictly positive because of $\lambda \in [0, 1)$ and $\beta < 1$. As a result, the left hand side of (16) is strictly increasing in L. Moreover, it is zero for $L = 0$. As a result, there exists a unique positive L

that satisfies (16). Based on this value, one can then determine all remaining values. Steady state saver consumption, for instance, is given by

$$
\begin{aligned}
c^S &= (1-\alpha)\alpha^{\frac{2\alpha}{1-\alpha}}L + \frac{(1-\tau^D)}{1-\lambda}\alpha^{-1}(1-\alpha)\alpha^{\frac{2}{1-\alpha}}L - \frac{g^A-1}{(1-\lambda)\psi} \\
&= (1-\alpha)\alpha^{\frac{2\alpha}{1-\alpha}}L\left(1+\alpha\frac{1-\tau^D}{1-\lambda}\right) - \frac{g^A-1}{(1-\lambda)\psi}\,.
\end{aligned}
$$

Steady state saver consumption is positive if

$$
\tau^D < \overline{\tau}^D \equiv 1 + \frac{1-\lambda}{\alpha(1-\beta)} + \frac{1-\beta s(1-\delta)}{\psi(1-\beta)\Theta_\alpha L}\,.
$$

Additionally, hand-to-mouth income is given by $y^H = c^H$, whereas saver income is given by $y^S = c^S + \frac{g^A-1}{(1-\lambda)\psi}$. Importantly, notice that there exists an amount of redistribution $\tau^{eq,D}$ such that there is no consumption inequality in the steady state, i.e. $\Gamma = 1$. Defining $\xi_c \equiv (1-\alpha)\alpha^{\frac{2\alpha}{1-\alpha}}L$, the redistribution level $\tau^{eq,D}$ is implicitly determined by

$$
\begin{aligned}
\xi_c\left(1+\alpha\frac{\tau^{eq,D}}{\lambda}\right) &= \xi_c\left(1+\alpha(1-\beta)\frac{1-\tau^{eq,D}}{1-\lambda}\right) + \frac{1-\beta s(1-\delta)}{(1-\lambda)\psi} \\
\Leftrightarrow \tau^{eq,D}\alpha\xi_c\left(\frac{1}{\lambda}+\frac{1-\beta}{1-\lambda}\right) &= \alpha\xi_c\frac{1-\beta}{1-\lambda} + \frac{1-\beta s(1-\delta)}{(1-\lambda)\psi} \\
\Leftrightarrow \tau^{eq,D}\alpha\xi_c\frac{1-\beta\lambda}{\lambda(1-\lambda)} &= \alpha\xi_c\frac{1-\beta}{1-\lambda} + \frac{1-\beta s(1-\delta)}{(1-\lambda)\psi} \\
\Leftrightarrow \tau^{eq,D} &= \lambda\frac{1-\beta}{1-\beta\lambda} + \lambda\frac{1-\beta s(1-\delta)}{1-\beta\lambda}\frac{1}{\psi\alpha(1-\alpha)\alpha^{\frac{2\alpha}{1-\alpha}}L} > 0\,.
\end{aligned}
$$

Finally, steady state growth is positive, i.e. $g^A > 1$, if

$$
\beta\left((1-\tau^D)\psi\Theta_\alpha L + s(1-\delta)\right) > 1 \quad \Leftrightarrow \quad \tau^D < 1 - \frac{\beta^{-1}-s(1-\delta)}{\psi\Theta_\alpha L}\,.
$$

A.4 Log-Linear Stationary Equilibrium

By log-linearizing the equilibrium conditions stated in Appendix A.2 around its non-stochastic steady state, we obtain

1. Saver Euler equation	$\hat{c}_t^S = -(\hat{i}_t - \mathbb{E}_t[\hat{\pi}_{t+1}]) + \tilde{s}\mathbb{E}_t[\hat{c}_{t+1}^S] + (1-\tilde{s})\mathbb{E}_t[\hat{c}_{t+1}^H] + \hat{g}_{t+1}^A$
2. Growth equation	$\hat{g}_{t+1}^A = \hat{c}_t^S - \mathbb{E}_t[\hat{c}_{t+1}^S] + \mathscr{M}\mathbb{E}_t[\hat{z}_{t+1} + \hat{L}_{t+1}] - \mathscr{M}_\tau\mathbb{E}_t[\hat{\tau}_{t+1}^D]$
3. Real wage	$\hat{w}_t = \hat{z}_t$
4. Wage Phillips curve	$\hat{\pi}_t^w = \kappa\left(\varphi\hat{L}_t + \hat{c}_t - \hat{z}_t\right)$
5. Fiscal transfers	$\hat{t}_t^H = \hat{\tau}_t^D + \hat{z}_t + \hat{L}_t$
6. Hours worked	$\hat{L}_t^H = \mu\hat{L}_t\,, \quad \hat{L}_t^S = \frac{1-\lambda\mu}{1-\lambda}\hat{L}_t\,, \quad \hat{L}_t = \lambda\hat{L}_t^H + (1-\lambda)\hat{L}_t^S$
7. HtM consumption	$\hat{c}_t^H = \frac{wL}{c^H}\left(\hat{w}_t + \hat{L}_t^H\right) + \frac{\tau^D}{\lambda}\frac{\Theta_\alpha L}{c^H}\left(\hat{\tau}_t^D + \hat{z}_t + \hat{L}_t\right)$
8. Saver consumption	$\hat{c}_t^S = \frac{wL}{c^S}\left(\hat{w}_t + \hat{L}_t^S\right) + \frac{d}{(1-\lambda)c^S}\hat{d}_t$
9. Dividends	$\hat{d}_t = -\tau^D\alpha^{-1}(1-\alpha)\frac{x}{d}\hat{\tau}_t^D + (1-\tau^D)\alpha^{-1}(1-\alpha)\frac{x}{d}\hat{x}_t - \frac{\iota^A}{d}\hat{\iota}_t^A$
10. Consumption	$\lambda c^H\hat{c}_t^H + (1-\lambda)c^S\hat{c}_t^S = c\hat{c}_t$
11. LOM productivity	$\hat{g}_{t+1}^A = \frac{g^A-1}{g^A}\hat{\iota}_t^A$
12. Monetary policy rule	$\hat{i}_t = \phi_\pi\hat{\pi}_t^w + \epsilon_{mt}$
13. Intermediary inputs	$\hat{x}_t = \hat{z}_t + \hat{L}_t$
14. Gross output	$\hat{y}_t^G = \hat{z}_t + \hat{L}_t$
15. Output	$\hat{y}_t = \hat{z}_t + \hat{L}_t$
16. Resource constraint	$\hat{y}_t = s_c\hat{c}_t + (1-s_c)\hat{\iota}_t^A$
17. Inflation dynamics	$\hat{\pi}_t = \hat{\pi}_t^w + \hat{z}_{t-1} - \hat{z}_t - \hat{g}_t^A$

where we have used the following auxiliary parameters:

$$\tilde{s} \equiv \frac{s}{s+(1-s)\Gamma}\,, \quad \mathscr{M} \equiv \frac{g^A - \beta s(1-\delta)}{g^A}\,, \quad \mathscr{M}_\tau \equiv \frac{\beta\tau^D\psi\Theta_\alpha L}{g^A}\,, \quad \kappa \equiv \frac{\epsilon_w - 1}{\theta}\frac{L}{(\pi^w)^2 y}(1-\alpha)\alpha^{\frac{2\alpha}{1-\alpha}}\,, \quad s_c \equiv \frac{c}{y}\,.$$

B Theoretical Appendix - Proofs

Remark: Subsequently, we impose $\chi \geq 0$ throughout the proofs below.

B.1 Balanced Growth Path Steady State

B.1.1 Proof Proposition 1

Proof. The first part of the proof is similar to the proof of Proposition 1 in Fornaro and Wolf (2021). Notice that a necessary condition for individual saver and hand to mouth consumption to be positive is that aggregate consumption is positive. This is the case if

$$c = \left(Y_\alpha - \beta(1-\tau^D)\Theta_\alpha\right) L + \frac{1-\beta s(1-\delta)}{\psi} > 0\,,$$

which holds true if $\tau^D > \underline{\tau}^D$ because of $\frac{\lambda+\alpha}{1+\alpha} < 1$. From Appendix A.3, we know that there exists a positive employment stead state $L > 0$ satisfying

$$\nu L^\varphi \left(\left(Y_\alpha - \beta(1-\tau^D)\Theta_\alpha\right) L + \frac{1-\beta s(1-\delta)}{\psi}\right) = \frac{\epsilon_w - 1}{\epsilon_w}(1-\alpha)\alpha^{\frac{2\alpha}{1-\alpha}}\,, \tag{18}$$

as the left hand side is strictly increasing in L and takes a zero value for $L = 0$. Additionally, we have the following upper tax bounds ensuring positive steady state consumption of hand-to-mouth households, respectively savers and productivity growth

$$\begin{aligned}
\underline{\tau}^D &= -\frac{\lambda}{\alpha}\,,\\
\overline{\tau}^D &= 1 + \frac{1-\lambda}{\alpha(1-\beta)} + \frac{1-\beta s(1-\delta)}{\psi(1-\beta)\Theta_\alpha L}\,,\\
\tilde{\tau}^{g,D} &= 1 - \frac{\beta^{-1} - s(1-\delta)}{\psi\Theta_\alpha L}\,.
\end{aligned}$$

Notice that $\tilde{\tau}^{g,D} < \overline{\tau}^D$ applies because of

$$\begin{aligned}
&\frac{1}{\psi(1-\beta)\Theta_\alpha L}\left((1-\beta)\left(s(1-\delta) - \beta^{-1}\right) + \beta s(1-\delta) - 1\right) < \frac{1-\lambda}{\alpha(1-\beta)}\\
\Leftrightarrow\ &\frac{1}{\psi(1-\beta)\Theta_\alpha L}\left(-\beta^{-1} + s(1-\delta)\right) < \frac{1-\lambda}{\alpha(1-\beta)}\,,
\end{aligned}$$

which holds true because of $\beta^{-1} > s(1-\delta)$. As a result, positive consumption of saver and hand-to-mouth households as well as positive steady state growth is guaranteed by $\underline{\tau}^D < \tilde{\tau}^{g,D}$, which can be rearranged to

$$L > \frac{\beta^{-1} - s(1-\delta)}{\psi\alpha^{-1}(\alpha+\lambda)\Theta_\alpha} > 0\,, \tag{19}$$

which is precisely the condition stated in the Proposition. As the left hand side of (18) strictly increases in ν one can write $L(\nu)$ with $L'(\nu) < 0$. As $L(0) = \infty$ and $L(\infty) = 0$, there exists by continuity an unique ν^* such that (19) is satisfied for all $\nu < \nu^*$. Notice that the wage Phillips curve also admits a labor steady state with $L = 0$ that leads however to negative steady state growth $g^A < 1$ and zero hand-to-mouth consumption and is therefore ruled out, i.e. the strictly positive employment steady state is the unique steady state consistent with strictly positive consumption of both households and strictly positive productivity growth. The terminal statement of Proposition 1 follows by rewriting the aggregate resource constraint

$$y = \lambda y^H + (1-\lambda)y^S = \lambda c^H + (1-\lambda)c^S + \frac{g^A - 1}{\psi} \, ,$$

which can be rearranged because of $c^H = y^H$ to

$$\begin{aligned}
& y^S = c^S + \frac{1}{1-\lambda}\frac{g^A-1}{\psi} \\
\Leftrightarrow\ & \Gamma_y = \frac{c^S}{y^H} + \frac{1}{1-\lambda}\frac{g^A-1}{\psi}\frac{1}{y^H} = \Gamma + \frac{1}{1-\lambda}\frac{g^A-1}{\psi y}\frac{y}{y^H} = \Gamma + \frac{1-s_c}{1-\lambda}\frac{y}{y^H} = \Gamma + \frac{1-s_c}{1-\lambda}\left(\lambda + (1-\lambda)\Gamma_y\right) \\
\Leftrightarrow\ & \Gamma_y = \frac{\Gamma}{s_c} + \frac{\lambda}{1-\lambda}\frac{1-s_c}{s_c} \, .
\end{aligned}$$

As a result, income inequality Γ_y is strictly larger than consumption inequality Γ if $s_c < 1$, which concludes the proof. □

B.2 Cyclical Fluctuations in Consumption and Income Inequality

B.2.1 Proof Lemma 1

Proof. The cyclicality of hand-to-mouth consumption is given by

$$c_t^H = w_t L_t^H + \frac{\tau_t^D}{\lambda}\Theta_\alpha Z_t L_t = (1-\alpha)\alpha^{\frac{2\alpha}{1-\alpha}}\left(Z_t L_t^H + \alpha\frac{\tau_t^D}{\lambda}Z_t L_t\right) \, .$$

Log-linearization provides us with

$$\begin{aligned}
& \hat{c}_t^H = \frac{1}{c^H}\left((1-\alpha)\alpha^{\frac{2\alpha}{1-\alpha}}L\left[1+\alpha\frac{\tau^D}{\lambda}\right]\hat{z}_t + (1-\alpha)\alpha^{\frac{2\alpha}{1-\alpha}}L\left(\mu+\alpha\frac{\tau^D}{\lambda}\right)\hat{L}_t + (1-\alpha)\alpha^{\frac{2\alpha}{1-\alpha}}L\alpha\frac{\tau^D}{\lambda}\hat{\tau}_t^D\right) \\
\Leftrightarrow\ & \hat{c}_t^H = \hat{z}_t + \hat{L}_t + (\mu-1)\frac{(1-\alpha)\alpha^{\frac{2\alpha}{1-\alpha}}L}{(1-\alpha)\alpha^{\frac{2\alpha}{1-\alpha}}L\left[1+\alpha\frac{\tau^D}{\lambda}\right]}\hat{L}_t + \frac{(1-\alpha)\alpha^{\frac{2\alpha}{1-\alpha}}L\alpha\frac{\tau^D}{\lambda}}{(1-\alpha)\alpha^{\frac{2\alpha}{1-\alpha}}L\left[1+\alpha\frac{\tau^D}{\lambda}\right]}\hat{\tau}_t^D \\
\Leftrightarrow\ & \hat{c}_t^H = \left(1+(\mu-1)\frac{\lambda}{\lambda+\alpha\tau^D}\right)(\hat{z}_t+\hat{L}_t) - (\mu-1)\frac{\lambda}{\lambda+\alpha\tau^D}\hat{z}_t + \frac{\alpha\tau^D}{\lambda+\alpha\tau^D}\hat{\tau}_t^D \\
\Leftrightarrow\ & \hat{c}_t^H = \chi\hat{y}_t + (1-\chi)\hat{z}_t + \chi_\tau\hat{\tau}_t^D \, ,
\end{aligned}$$

where we have defined the auxiliary parameters (χ, χ_τ) as

$$\chi \equiv 1 + (\mu - 1)\frac{\lambda}{\lambda + \alpha\tau^D} , \quad \chi_\tau \equiv \frac{\alpha\tau^D}{\lambda + \alpha\tau^D} .$$

It is straightforward to see that χ strictly increases in μ as $\frac{\lambda}{\lambda+\alpha\tau^D} > 0$ due to $\tau^D > \underline{\tau}^D$. Its minimal value is obtained by $\chi|_{\mu=0} = \frac{\alpha\tau^D}{\lambda+\alpha\tau^D}$ which is positive if $\tau^D > 0$. This concludes the proof. □

B.2.2 Cyclical Income Inequality The identity for total income is given by

$$\lambda y_t^H + (1-\lambda) y_t^S = y_t ,$$

which gives in log-linearized terms

$$\lambda y^H + (1-\lambda)y^S + \lambda y^H \hat{y}_t^H + (1-\lambda)y^S \hat{y}_t^S = y + y\hat{y}_t \quad \Leftrightarrow \quad \hat{y}_t^S = \frac{y\hat{y}_t - \lambda y^H \hat{y}_t^H}{(1-\lambda)y^S} .$$

Expanding terms results in

$$\hat{y}_t^S - \hat{y}_t^H = \frac{y\hat{y}_t - (\lambda y^H + (1-\lambda)y^S)\hat{y}_t^H}{(1-\lambda)y^S} = \frac{1}{1-\lambda}\frac{y}{y^S}\left(\hat{y}_t - \hat{y}_t^H\right)$$

such that we obtain after substituting $\hat{c}_t^H = \hat{y}_t^H = \chi\hat{y}_t + (1-\chi)\hat{z}_t + \chi_\tau \hat{\tau}_t^D$ from Lemma 1

$$\hat{y}_t^S - \hat{y}_t^H = \frac{1}{1-\lambda}\frac{y}{y^S}\left((1-\chi)(\hat{y}_t - \hat{z}_t) - \chi_\tau \hat{\tau}_t^D\right) ,$$

which corresponds to the expression stated in the main text. Conditional on tax shocks, fluctuations in income are hence solely driven by fluctuations in labor income.

B.3 Four Equations Representation

B.3.1 Proof Proposition 2

Proof. To derive the aggregate IS equation, we proceed in two steps: First, we compute cyclical saver consumption as a function of aggregate output $\hat{y}_t$, endogenous productivity $\hat{g}_{t+1}^A$ and exogenous technology $\hat{z}_t$. Second, we substitute cyclical hand-to-mouth and saver consumption into the Euler equation. To begin with, we know from Appendix A.4 that

$$\hat{c}_t^S = \frac{c\hat{c}_t - \lambda c^H \hat{c}_t^H}{(1-\lambda)c^S} = \left(1 + \frac{\lambda}{1-\lambda}\frac{1}{\Gamma}\right)\hat{c}_t - \frac{\lambda}{1-\lambda}\frac{1}{\Gamma}\hat{c}_t^H .$$

From the aggregate resource constraint, we have $\hat{y}_t = s_c\hat{c}_t + (1-s_c)\frac{g^A}{g^A-1}\hat{g}_{t+1}^A$. Substituting in for

$\hat{c}_t$ and $\hat{c}_t^H$ from Lemma 1, we thus obtain

$$\begin{aligned}
\hat{c}_t^S &= \left(1+\frac{\lambda}{1-\lambda}\frac{1}{\Gamma}\right)\left(\frac{\hat{y}_t}{s_c}-\frac{1-s_c}{s_c}\frac{g^A}{g^A-1}\hat{g}_{t+1}^A\right)-\frac{\lambda}{1-\lambda}\frac{1}{\Gamma}\left(\chi\hat{y}_t+(1-\chi)\hat{z}_t+\chi_\tau\hat{\tau}_t^D\right) \\
&= \frac{(1-\lambda)\Gamma+\lambda-\lambda s_c\chi}{(1-\lambda)\Gamma s_c}\hat{y}_t-\frac{(1-\lambda)\Gamma+\lambda}{(1-\lambda)\Gamma}\frac{1-s_c}{s_c}\frac{g^A}{g^A-1}\hat{g}_{t+1}^A-\frac{\lambda(1-\chi)}{(1-\lambda)\Gamma}\hat{z}_t-\frac{\lambda\chi_\tau}{(1-\lambda)\Gamma}\hat{\tau}_t^D \\
&= \mathcal{E}_y\hat{y}_t-\mathcal{E}_g\hat{g}_{t+1}^A-\mathcal{E}_z\hat{z}_t-\mathcal{E}_\tau\hat{\tau}_t^D\ ,
\end{aligned}$$

where $\mathcal{E}_y, \mathcal{E}_g, \mathcal{E}_z, \mathcal{E}_\tau$ denote partial equilibrium elasticities of saver consumption with respect to aggregate income, respectively endogenous productivity and exogenous technology, i.e.

$$\mathcal{E}_y \equiv \frac{(1-\lambda)\Gamma+\lambda-\lambda s_c\chi}{(1-\lambda)\Gamma s_c}\ ,\quad \mathcal{E}_g \equiv \frac{(1-\lambda)\Gamma+\lambda}{(1-\lambda)\Gamma}\frac{1-s_c}{s_c}\frac{g^A}{g^A-1}\ ,\quad \mathcal{E}_z \equiv \frac{\lambda(1-\chi)}{(1-\lambda)\Gamma}\ ,\quad \mathcal{E}_\tau \equiv \frac{\lambda\chi_\tau}{(1-\lambda)\Gamma}\ .$$

From Appendix A.4 the saver Euler equation is given by

$$\hat{c}_t^S = \tilde{s}\mathbb{E}_t\left[\hat{c}_{t+1}^S\right]+(1-\tilde{s})\mathbb{E}_t\left[\hat{c}_{t+1}^H\right]-(\hat{i}_t-\mathbb{E}_t\left[\hat{\pi}_{t+1}\right])+\hat{g}_{t+1}^A\ .$$

Substituting in for the consumption of saver households results in

$$\begin{aligned}
\mathcal{E}_y\hat{y}_t-\mathcal{E}_g\hat{g}_{t+1}^A-\mathcal{E}_z\hat{z}_t-\mathcal{E}_\tau\hat{\tau}_t^D &= \tilde{s}\mathbb{E}_t\left[\mathcal{E}_y\hat{y}_{t+1}-\mathcal{E}_g\hat{g}_{t+2}^A-\mathcal{E}_z\hat{z}_{t+1}-\mathcal{E}_\tau\hat{\tau}_{t+1}^D\right] \\
&\quad +(1-\tilde{s})\mathbb{E}_t\left[\chi\hat{y}_{t+1}+(1-\chi)\hat{z}_{t+1}+\chi_\tau\hat{\tau}_{t+1}^D\right]-(\hat{i}_t-\mathbb{E}_t\left[\hat{\pi}_{t+1}\right])+\hat{g}_{t+1}^A\ ,
\end{aligned}$$

which can be rearranged to

$$\begin{aligned}
\hat{y}_t &= \left(\tilde{s}+(1-\tilde{s})\frac{\chi}{\mathcal{E}_y}\right)\mathbb{E}_t\left[\hat{y}_{t+1}\right]-\frac{1}{\mathcal{E}_y}(\hat{i}_t-\mathbb{E}_t\left[\hat{\pi}_{t+1}\right])+\frac{1+\mathcal{E}_g}{\mathcal{E}_y}\hat{g}_{t+1}^A-\tilde{s}\frac{\mathcal{E}_g}{\mathcal{E}_y}\mathbb{E}_t\left[\hat{g}_{t+2}^A\right]+ \\
&\quad +\frac{\mathcal{E}_z}{\mathcal{E}_y}\hat{z}_t+\frac{1}{\mathcal{E}_y}\left((1-\tilde{s})(1-\chi)-\tilde{s}\mathcal{E}_z\right)\mathbb{E}_t\left[\hat{z}_{t+1}\right]+\frac{\mathcal{E}_\tau}{\mathcal{E}_y}\hat{\tau}_t^D-\frac{\mathcal{E}_\tau}{\mathcal{E}_y}\left(\tilde{s}-(1-\tilde{s})\frac{\chi_\tau}{\mathcal{E}_\tau}\right)\mathbb{E}_t\left[\hat{\tau}_{t+1}^D\right]\ .
\end{aligned}$$

The former equation can be restated as

$$\hat{y}_t = \zeta_f\mathbb{E}_t\left[\hat{y}_{t+1}\right]-\zeta_r(\hat{i}_t-\mathbb{E}_t\left[\hat{\pi}_{t+1}\right])+\zeta_g\hat{g}_{t+1}^A-\zeta_{g'}\mathbb{E}_t\left[\hat{g}_{t+2}^A\right]+\zeta_z\hat{z}_t+\zeta_{z'}\mathbb{E}_t\left[\hat{z}_{t+1}\right]+\zeta_\tau\hat{\tau}_t^D-\zeta_{\tau'}\mathbb{E}_t\left[\hat{\tau}_{t+1}^D\right]\ ,$$

where the compounding parameter is defined as follows

$$\begin{aligned}
\zeta_f \equiv \tilde{s}+(1-\tilde{s})\frac{\chi}{\mathcal{E}_y} &= 1+(1-\tilde{s})\left(\frac{\chi}{\mathcal{E}_y}-1\right) = 1+(1-\tilde{s})\left(\frac{(1-\lambda)\Gamma s_c\chi-((1-\lambda)\Gamma+\lambda-\lambda s_c\chi)}{(1-\lambda)\Gamma+\lambda-\lambda s_c\chi}\right) \\
&= 1+(1-\tilde{s})\left(\frac{(1-\lambda)\Gamma+\lambda}{(1-\lambda)\Gamma+\lambda-\lambda s_c\chi}\right)(s_c\chi-1) = 1+(1-\tilde{s})\frac{s_c\chi-1}{1-\lambda\chi\frac{y^H}{y}}\ ,
\end{aligned}$$

where the last equality makes use of the identity $s_c = ((1-\lambda)\Gamma + \lambda)\,\frac{y^H}{y}$. Additionally, we have

$$
\begin{aligned}
\zeta_r &\equiv \frac{1}{\mathcal{E}_y} = \frac{(1-\lambda)\Gamma s_c}{(1-\lambda)\Gamma + \lambda - \lambda s_c \chi}\,, \\
\zeta_g &\equiv \frac{1+\mathcal{E}_g}{\mathcal{E}_y} = \frac{\frac{(1-\lambda)\Gamma s_c (g^A-1)}{(1-\lambda)\Gamma s_c (g^A-1)} + \frac{(1-\lambda)\Gamma+\lambda}{(1-\lambda)\Gamma}\frac{1-s_c}{s_c}\frac{g^A}{g^A-1}}{\frac{(1-\lambda)\Gamma+\lambda-\lambda s_c\chi}{(1-\lambda)\Gamma s_c}} \\
&= \frac{1}{g^A-1}\frac{(1-\lambda)\Gamma s_c (g^A-1) + ((1-\lambda)\Gamma+\lambda)\,(1-s_c) g^A}{(1-\lambda)\Gamma + \lambda - \lambda s_c\chi} = \frac{\frac{(1-\lambda)\Gamma s_c}{(1-\lambda)\Gamma+\lambda} + (1-s_c)\frac{g^A}{g^A-1}}{1-\lambda\chi\frac{y^H}{y}} \\
&= \frac{(1-\lambda)\frac{c^S}{y} + \frac{g^A}{\psi y}}{1-\lambda\chi\frac{y^H}{y}}\,,
\end{aligned}
$$

where the last equality follows from $1-s_c = \frac{g^A-1}{\psi y}$. Moreover,

$$
\begin{aligned}
\zeta_{g'} &\equiv \tilde{s}\frac{\mathcal{E}_g}{\mathcal{E}_y} = \tilde{s}\frac{\frac{(1-\lambda)\Gamma+\lambda}{(1-\lambda)\Gamma}\frac{1-s_c}{s_c}\frac{g^A}{g^A-1}}{\frac{(1-\lambda)\Gamma+\lambda-\lambda s_c\chi}{(1-\lambda)\Gamma s_c}} = \tilde{s}\frac{g^A}{g^A-1}\frac{((1-\lambda)\Gamma+\lambda)\,(1-s_c)}{(1-\lambda)\Gamma+\lambda-\lambda s_c\chi} = \tilde{s}\frac{g^A}{\psi y}\frac{1}{1-\lambda\chi\frac{y^H}{y}}\,, \\
\zeta_z &\equiv \frac{\mathcal{E}_z}{\mathcal{E}_y} = \frac{\frac{\lambda(1-\chi)}{(1-\lambda)\Gamma}}{\frac{(1-\lambda)\Gamma+\lambda-\lambda s_c\chi}{(1-\lambda)\Gamma s_c}} = \frac{\lambda s_c(1-\chi)}{(1-\lambda)\Gamma+\lambda-\lambda s_c\chi}\,, \\
\zeta_{z'} &= \frac{(1-\tilde{s})(1-\chi) - \tilde{s}\mathcal{E}_z}{\mathcal{E}_y} = \frac{(1-\tilde{s})(1-\lambda)\Gamma s_c(1-\chi) - \tilde{s}\lambda s_c(1-\chi)}{(1-\lambda)\Gamma+\lambda-\lambda s_c\chi} \\
&= \frac{(1-\tilde{s})\Gamma(1-\lambda) - \tilde{s}\lambda}{(1-\lambda)\Gamma+\lambda-\lambda s_c\chi} s_c(1-\chi)\,, \\
\zeta_\tau &= \frac{\mathcal{E}_\tau}{\mathcal{E}_y} = \frac{\lambda s_c \chi_\tau}{(1-\lambda)\Gamma+\lambda-\lambda s_c\chi}\,, \\
\zeta_{\tau'} &= \frac{\mathcal{E}_\tau}{\mathcal{E}_y}\left(\tilde{s} - (1-\tilde{s})\frac{\chi_\tau}{\mathcal{E}_\tau}\right) = \frac{\lambda s_c\chi_\tau}{(1-\lambda)\Gamma+\lambda-\lambda s_c\chi}\left(\tilde{s} - (1-\tilde{s})\frac{\chi_\tau}{\mathcal{E}_\tau}\right)\,.
\end{aligned}
$$

which concludes the proof of Proposition 2. □

B.3.2 Proof Corollary 1

Proof. We show each of the three statements $(a)-(c)$ separately.

Statement (a): Recall from Proposition 2 that the elasticity of aggregate demand with respect to real interest changes is given by

$$
\zeta_r = \frac{1}{\mathcal{E}_y} = \frac{(1-\lambda)\Gamma s_c}{(1-\lambda)\Gamma+\lambda-\lambda s_c\chi}\,.
$$

The numerator is evidently positive such that the overall sign is determined by the sign of the denominator. Thus, ζ_r is positive if and only if $(1-\lambda)\Gamma + \lambda - \lambda s_c \chi > 0$, which is equivalent to

$$\chi < \overline{\chi} \equiv \frac{(1-\lambda)\Gamma + \lambda}{\lambda s_c} ,$$

which is equivalent to

$$1 + (\overline{\mu} - 1)\frac{\lambda}{\lambda + \alpha\tau^D} < \frac{(1-\lambda)\Gamma + \lambda}{\lambda s_c}$$
$$\overline{\mu} < 1 + \frac{(1-\lambda)\Gamma + \lambda(1-s_c)}{\lambda s_c}\frac{\lambda + \alpha\tau^D}{\lambda} ,$$

which corresponds to the threshold stated in the main text. Finally, notice that $\overline{\mu} > 1$ as $\Gamma > 0, s_c < 1, \lambda > 0$ and $\tau^D > \underline{\tau}^D$.

Statement (b): Recall from Proposition 2 that the compounding coefficient of the aggregate IS equation is given by

$$\zeta_f = \tilde{s} + (1-\tilde{s})\frac{\chi}{\mathcal{E}_y} = 1 + (1-\tilde{s})\left(\frac{(1-\lambda)\Gamma + \lambda}{(1-\lambda)\Gamma + \lambda - \lambda s_c \chi}\right)(s_c\chi - 1) .$$

Given the upper bound $\overline{\mu}$ from statement (a) it is – under $\tilde{s} \in [0,1)\,, \Gamma > 0$–straightforward to see that $\zeta_f > 1$ if and only if $\chi s_c > 1$, which can be rewritten as

$$1 + (\overline{\mu} - 1)\frac{\lambda}{\lambda + \alpha\tau^D} > \frac{1}{s_c} \quad \Leftrightarrow \quad \overline{\mu} > \equiv \underline{\mu} = 1 + \frac{1-s_c}{s_c}\frac{\lambda + \alpha\tau^D}{\lambda}$$

which corresponds to the lower bound stated in the main text. As a result, the IS equation under HANK-GS features compounding and admits a negative elasticity of aggregate demand with respect tot real interest rates if $1 < \underline{\mu} < \mu < \overline{\mu}$. Notice that the former interval is non-empty, i.e. $\underline{\mu} < \overline{\mu}$, as long as $\frac{1-\lambda}{\lambda s_c}\Gamma > 0$, i.e. there is a strictly positive mass of saver households. Notice that we recover the lower and upper bounds on χ by Bilbiie (2020, 2021) under $s_c = 1, \Gamma = 1$.

Statement (c): The compounding coefficient can be written as

$$\zeta_f = \tilde{s} + (1-\tilde{s})\frac{\chi}{\mathcal{E}_y} = \frac{s + (1-s)\Gamma\frac{\chi}{\mathcal{E}_y}}{s + (1-s)\Gamma} .$$

With a slide abuse of notation, let us define $\Gamma' \equiv \frac{\partial\Gamma}{\partial\tau^D}$, $\chi' \equiv \frac{\partial\chi}{\partial\tau^D}$, and $\mathcal{E}_y' \equiv \frac{\partial\mathcal{E}_y}{\partial\tau^D}$. As a result, we obtain the following comparative static

$$\frac{\partial \zeta_f}{\partial \tau^D} = \frac{(1-s)\left[\Gamma' \frac{\chi}{\mathcal{E}_y} + \Gamma \frac{\chi' \mathcal{E}_y - \mathcal{E}_y' \chi}{(\mathcal{E}_y)^2}\right](s+(1-s)\Gamma) - (1-s)\Gamma'\left(s + (1-s)\Gamma \frac{\chi}{\mathcal{E}_y}\right)}{(s+(1-s)\Gamma)^2} \,.$$

The sign of the previous derivative is determined by

$$\begin{aligned}
\operatorname{sgn}\left(\frac{\partial \zeta_f}{\partial \tau^D}\right) &= (1-s)\left[s+(1-s)\Gamma\right]\Gamma' \frac{\chi}{\mathcal{E}_y} + (1-s)\left[s+(1-s)\Gamma\right]\Gamma \frac{\chi}{\mathcal{E}_y}\left(\frac{\chi'}{\chi} - \frac{\mathcal{E}_y'}{\mathcal{E}_y}\right) - s(1-s)\Gamma' - (1-s)^2\Gamma'\Gamma\frac{\chi}{\mathcal{E}_y} \\
&= \left[s+(1-s)\Gamma\right]\Gamma' \frac{\chi}{\mathcal{E}_y} + \left[s+(1-s)\Gamma\right]\Gamma \frac{\chi}{\mathcal{E}_y}\left(\frac{\chi'}{\chi} - \frac{\mathcal{E}_y'}{\mathcal{E}_y}\right) - s\Gamma' - (1-s)\Gamma'\Gamma\frac{\chi}{\mathcal{E}_y} \\
&= s\Gamma'\left(\frac{\chi}{\mathcal{E}_y} - 1\right) + \left[s+(1-s)\Gamma\right]\Gamma \frac{\chi}{\mathcal{E}_y}\left(\frac{\chi'}{\chi} - \frac{\mathcal{E}_y'}{\mathcal{E}_y}\right) \\
&= s\frac{\Gamma'}{\Gamma}\left(\frac{\chi}{\mathcal{E}_y} - 1\right) + \left[s+(1-s)\Gamma\right]\frac{\chi}{\mathcal{E}_y}\left(\frac{\chi'}{\chi} - \frac{\mathcal{E}_y'}{\mathcal{E}_y}\right) \\
&= \left[s+(1-s)\Gamma\right]\frac{\chi}{\mathcal{E}_y}\left[\tilde{s}\frac{\Gamma'}{\Gamma}\left(1 - \frac{\mathcal{E}_y}{\chi}\right) + \left(\frac{\chi'}{\chi} - \frac{\mathcal{E}_y'}{\mathcal{E}_y}\right)\right] \,,
\end{aligned}$$

where the second equality divides through $1-s$, the third equality collects terms, the fourth equality divides through Γ, while the terminal equality simplifies terms. As a result, we obtain

$$\operatorname{sgn}\left(\frac{\partial \zeta_f}{\partial \tau^D}\right) = \left[s+(1-s)\Gamma\right]\frac{\chi}{\mathcal{E}_y}\left[\tilde{s}\frac{\Gamma'}{\Gamma}\left(1 - \frac{\mathcal{E}_y}{\chi}\right) + \left(\frac{\chi'}{\chi} - \frac{\mathcal{E}_y'}{\mathcal{E}_y}\right)\right] \,. \tag{20}$$

Additionally, if $s_c = 1$ we have from above $\mathcal{E}_y = \frac{(1-\lambda)\Gamma + \lambda - \lambda\chi}{(1-\lambda)\Gamma}$ such that

$$\begin{aligned}
\frac{\partial \mathcal{E}_y}{\partial \tau^D} &= \frac{\left[(1-\lambda)\Gamma' - \lambda\chi'\right](1-\lambda)\Gamma - (1-\lambda)\Gamma'\left[(1-\lambda)\Gamma + \lambda - \lambda\chi\right]}{((1-\lambda)\Gamma)^2} \\
&= \frac{-\lambda(1-\lambda)\left[\Gamma\chi' + \Gamma'(1-\chi)\right]}{((1-\lambda)\Gamma)^2} \\
&= -\frac{\lambda}{1-\lambda}\frac{1}{\Gamma}\left(\chi' + \frac{\Gamma'}{\Gamma}(1-\chi)\right) \,.
\end{aligned}$$

Hence we get

$$\frac{\mathcal{E}_y'}{\mathcal{E}_y} = -\frac{\lambda\left(\chi' + \frac{\Gamma'}{\Gamma}(1-\chi)\right)}{(1-\lambda)\Gamma + \lambda - \lambda\chi} \,.$$

To further determine the sign of equation (20) we use Result 3.

Result 3 (SIGN $\mathcal{E}_y'$)**.** *If $\mu > 1$ applies, i.e. hand-to-mouth households are over-proportionally exposed to aggregate labor fluctuations, then* $\frac{\partial \mathcal{E}_y}{\partial \tau^D} < 0 \; \forall \tau^D \in \left(\underline{\tau}^D, \overline{\tau}^D\right)$.

Proof. Notice that steady state consumption inequality is in the absence of endogenous growth under $\psi = 0$ and $g^A = 1$ given by

$$\Gamma = \frac{c^S}{c^H} = \frac{1 + \alpha \frac{1-\tau^D}{1-\lambda}}{1 + \alpha \frac{\tau^D}{\lambda}} , \quad \chi = 1 + (\mu - 1) \frac{\lambda}{\lambda + \alpha \tau^D} .$$

It can be seen that hand-to-mouth consumption c^H is strictly positive if $\tau^D > \underline{\tau}^D \equiv -\frac{\lambda}{\alpha}$, whereas saver consumption c^S is strictly positive if $\tau^D < \overline{\tau}^D \equiv 1 + \frac{1-\lambda}{\alpha}$. Obviously it holds that $\Gamma > 0$ for all $\tau^D \in \left(\underline{\tau}^D, \overline{\tau}^D\right)$. As a result, the sign of $\frac{\partial \mathcal{E}_y}{\partial \tau^D}$ is determined by the sign of $\chi' + \frac{\Gamma'}{\Gamma}(1-\chi)$. Using the following derivatives

$$\frac{\partial \chi}{\partial \tau^D} = -\alpha(\mu - 1) \frac{\lambda}{(\lambda + \alpha \tau^D)^2} ,$$

$$\frac{\partial \Gamma}{\partial \tau^D} = \frac{-\frac{\alpha}{1-\lambda}\left(1 + \alpha \frac{\tau^D}{\lambda}\right) - \frac{\alpha}{\lambda}\left(1 + \alpha \frac{1-\tau^D}{1-\lambda}\right)}{\left(1 + \alpha \frac{\tau^D}{\lambda}\right)^2} = \frac{\frac{-\alpha\lambda - \alpha(1-\lambda) - \alpha^2}{\lambda(1-\lambda)}}{\left(1 + \alpha \frac{\tau^D}{\lambda}\right)^2} = -\frac{\alpha(1+\alpha)}{\lambda(1-\lambda)} \frac{1}{\left(1 + \alpha \frac{\tau^D}{\lambda}\right)^2} ,$$

$$\frac{\frac{\partial \Gamma}{\partial \tau^D}}{\Gamma} = -\frac{\alpha(1+\alpha)}{\lambda(1-\lambda)} \frac{1}{\left(1 + \alpha \frac{1-\tau^D}{1-\lambda}\right)\left(1 + \alpha \frac{\tau^D}{\lambda}\right)} ,$$

we finally obtain

$$\frac{\partial \chi}{\partial \tau^D} + \frac{\frac{\partial \Gamma}{\partial \tau^D}}{\Gamma}(1-\chi) = -\alpha(\mu-1) \frac{\lambda}{(\lambda + \alpha\tau^D)^2} + \frac{\alpha(1+\alpha)}{\lambda(1-\lambda)} \frac{1}{\left(1 + \alpha \frac{1-\tau^D}{1-\lambda}\right)\left(1 + \alpha \frac{\tau^D}{\lambda}\right)} \frac{\mu - 1}{1 + \alpha \frac{\tau^D}{\lambda}} .$$

(a) <u>CASE 1:</u> Positive excess labor incidence elasticity.
If $\mu > 1$ applies, the sign of $\mathcal{E}_y'$ is determined by

$$\text{sgn}\left(\frac{\partial \mathcal{E}_y}{\partial \tau^D}\right) = -\text{sgn}\left(-1 + \frac{1+\alpha}{(1-\lambda)\left(1 + \alpha \frac{1-\tau^D}{1-\lambda}\right)}\right) = -\text{sgn}\left(\frac{\lambda + \alpha\tau^D}{1 - \lambda + \alpha(1-\tau^D)}\right) < 0 ,$$

where the strictly negative sign holds true on the interval $\tau^D \in \left(\underline{\tau}^D, \overline{\tau}^D\right)$.

(b) <u>CASE 2:</u> Zero excess labor incidence elasticity.
If $\mu = 1$ applies, it is straightforward to see that $\text{sgn}\left(\frac{\partial \mathcal{E}_y}{\partial \tau^D}\right) = 0$ such that $\mathcal{E}_y = \zeta_f = 1$ for all $\tau^D \in \left(\underline{\tau}^D, \overline{\tau}^D\right)$.

(c) <u>CASE 3:</u> Negative excess labor incidence elasticity.
If $\mu < 1$ applies, then the reverse sign relative to the case $\mu > 1$ holds true, i.e. $\text{sgn}\left(\frac{\partial \mathcal{E}_y}{\partial \tau^D}\right) > 0$ for all $\tau^D \in \left(\underline{\tau}^D, \overline{\tau}^D\right)$.

□

Result 4 (COMPOUNDING REDISTRIBUTION DISCONTINUITY). *If* $1 < \mu < 1 + \frac{1+\alpha}{\lambda}$ *applies, then there exists* $\underline{\tau}^D < \tau^D_{disc} < \overline{\tau}^D$ *such that* $\mathcal{E}_y > 0$ *for* $\tau^D < \tau^D_{disc}$ *and* $\mathcal{E}_y < 0$ *for* $\tau^D > \tau^D_{disc}$.

Proof. The auxiliary parameter $\mathcal{E}_y$ is given by

$$\mathcal{E}_y = \frac{(1-\lambda)\Gamma + \lambda - \lambda\chi}{(1-\lambda)\Gamma} .$$

For $\lambda \in (0,1)$ and $\tau^D \in \left(\underline{\tau}^D, \overline{\tau}^D\right)$ the denominator is strictly positive. Hence the sign of $\mathcal{E}_y$ is determined by the numerator. The latter is given by

$$\begin{aligned}(1-\lambda)\Gamma + \lambda(1-\chi) &= (1-\lambda)\frac{1+\alpha\frac{1-\tau^D}{1-\lambda}}{1+\alpha\frac{\tau^D}{\lambda}} - \lambda\frac{\mu-1}{1+\alpha\frac{\tau^D}{\lambda}} \\ &= \underbrace{\frac{1}{1+\alpha\frac{\tau^D}{\lambda}}}_{>0}\left(1-\lambda+\alpha(1-\tau^D) - \lambda(\mu-1)\right) .\end{aligned}$$

As a result $\mathcal{E}_y(\tau^D_{disc}) = 0$, where

$$\tau^D_{disc} = 1 + \frac{1-\lambda\mu}{\alpha} .$$

The proof concludes by recognizing that $\underline{\tau}^D < \tau^D_{disc} < \overline{\tau}^D$ if and only if $1 < \mu < 1 + (1+\alpha)/\lambda$. As a result, $\tau^D > \tau^D_{disc}$ implies $\mathcal{E}_y < 0$ and *vice versa* $\tau^D < \tau^D_{disc}$ implies $\mathcal{E}_y > 0$. □

Result 5 (COMPARATIVE STATICS COMPOUNDING). *In the case of* $1 < \mu < 1 + \frac{1+\alpha}{\lambda}$, *it follows for* $\tau^D > \tau^D_{disc}$ *that* $\frac{\partial \zeta_f}{\partial \tau^D} > 0$, *i.e. the compounding coefficient increases in redistribution, with asymptotic limit* $\lim\limits_{\tau^D \to \overline{\tau}^D} \zeta_f = 1$.

Proof. As before, $\mu > 1$ ensures that $\tau^D_{disc} < \overline{\tau}^D$, while $\mu < 1 + \frac{1+\alpha}{\lambda}$ guarantees that $\tau^D_{disc} > \underline{\tau}^D$. The first statement of Result 5 follows from equation (20): $\tau^D_{disc} < \tau^D < \overline{\tau}^D$ implies that $\mathcal{E}_y < 0$ such that $\frac{\mathcal{E}_y'}{\mathcal{E}_y} > 0$. As $\Gamma > 0, \Gamma' < 0, \chi > 0, \chi' < 0$ holds as well on $\left(\underline{\tau}^D, \overline{\tau}^D\right)$, it is straightforward to see that $\operatorname{sgn}\left(\frac{\partial \zeta_f}{\partial \tau^D}\right) > 0$. To prove the second statement, notice the following limits:

$$\lim_{\tau^D \to \overline{\tau}^D} \Gamma(\tau^D) = 0 , \quad \lim_{\tau^D \to \overline{\tau}^D} \chi(\tau^D) = 1 + (\mu - 1)\frac{\lambda}{1+\alpha} > 0 , \quad \text{and} \quad \lim_{\tau^D \to \overline{\tau}^D} \mathcal{E}_y(\tau^D) = -\infty .$$

As a result, using the previous limits, we obtain

$$\lim_{\tau^D \to \overline{\tau}^D} \zeta_f = \frac{s - (1-s) \times 0 \times \frac{\chi(\overline{\tau}^D)}{\infty}}{s + (1-s) \times 0} = 1 ,$$

which completes the proof. $\square$

Result 6 (Compounding Redistribution Shape). *If $1 < \mu < 1 + \frac{1+\alpha}{\lambda}$ applies, there exist $\underline{\tau}^D < \tau^D_{eq} < \tau^D_{zero} < \tau^D_{disc} < \overline{\tau}^D$ such that the compounding coefficient decreases in τ^D on $\left(\underline{\tau}^D, \tau^D_{zero}\right)$, while it increases in τ^D on $\left(\tau^D_{zero}, \tau^D_{disc}\right)$. It also holds that $\zeta_f(\tau^D_{zero}) > 1$.*

Proof. On $\underline{\tau}^D < \tau^D < \tau^D_{disc}$, substituting into $\operatorname{sgn}\left(\frac{\partial \zeta_f}{\partial \tau^D}\right)$ the expression for $\frac{\mathcal{E}'_y}{\mathcal{E}_y}$ yields

$$\operatorname{sgn}\left(\frac{\partial \zeta_f}{\partial \tau^D}\right) = \frac{\chi}{\mathcal{E}_y}\left[\tilde{s}\frac{\Gamma'}{\Gamma}\left(1 - \frac{\mathcal{E}_y}{\chi}\right) + \left(\frac{\chi'}{\chi} - \frac{\mathcal{E}'_y}{\mathcal{E}_y}\right)\right] = \underbrace{s\frac{\Gamma'}{\Gamma}\left(\frac{\chi}{\mathcal{E}_y} - 1\right)}_{①} + \underbrace{(s + (1-s)\Gamma)}_{②}\underbrace{\frac{\chi}{\mathcal{E}_y}\left(\frac{\chi'}{\chi} - \frac{\mathcal{E}'_y}{\mathcal{E}_y}\right)}_{③} .$$

There are several sub-components that needs to be explored. To begin with,

$$\mathcal{E}_y = \frac{(1-\lambda)\Gamma + \lambda - \lambda\chi}{(1-\lambda)\Gamma} = \frac{(1-\lambda)\frac{1+\alpha\frac{1-\tau^D}{1-\lambda}}{1+\alpha\frac{\tau^D}{\lambda}} - (\mu-1)\frac{\lambda}{1+\alpha\frac{\tau^D}{\lambda}}}{(1-\lambda)\frac{1+\alpha\frac{1-\tau^D}{1-\lambda}}{1+\alpha\frac{\tau^D}{\lambda}}} = \frac{1-\lambda+\alpha(1-\tau^D) - \lambda(\mu-1)}{1-\lambda+\alpha(1-\tau^D)} > 0\,,$$

which is strictly positive because of $\tau^D < \min\{\tau^D_{disc}, \overline{\tau}^D\}$. Additionally, we have

$$\begin{aligned}
\mathcal{E}'_y &= -\frac{\lambda}{(1-\lambda)\Gamma}\left(\chi' + (1-\chi)\frac{\Gamma'}{\Gamma}\right) \\
&= -\frac{\lambda}{(1-\lambda)}\frac{1+\alpha\frac{\tau^D}{\lambda}}{1+\alpha\frac{1-\tau^D}{1-\lambda}}\left(-\frac{\alpha(\mu-1)}{\lambda(1+\alpha\frac{\tau^D}{\lambda})^2} + \frac{\alpha(1+\alpha)}{\lambda(1-\lambda)}\frac{1}{\left(1+\alpha\frac{1-\tau^D}{1-\lambda}\right)\left(1+\alpha\frac{\tau^D}{\lambda}\right)}\frac{\mu-1}{1+\alpha\frac{\tau^D}{\lambda}}\right) \\
&= -\frac{\alpha(\mu-1)}{1-\lambda+\alpha(1-\tau^D)}\left(-\frac{1}{1+\alpha\frac{\tau^D}{\lambda}} + \frac{1+\alpha}{1-\lambda+\alpha(1-\tau^D)}\frac{1}{1+\alpha\frac{\tau^D}{\lambda}}\right) \\
&= -\frac{\alpha(\mu-1)}{1-\lambda+\alpha(1-\tau^D)}\frac{\lambda}{\lambda+\alpha\tau^D}\frac{\lambda+\alpha\tau^D}{1-\lambda+\alpha(1-\tau^D)} \\
&= -\frac{\alpha(\mu-1)\lambda}{(1-\lambda+\alpha(1-\tau^D))^2}\,.
\end{aligned}$$

Thus, it follows

$$\frac{\mathcal{E}'_y}{\mathcal{E}_y} = -\frac{\alpha(\mu-1)\lambda}{[1-\lambda+\alpha(1-\tau^D)]\,[1-\lambda+\alpha(1-\tau^D)-\lambda(\mu-1))]}\,.$$

Moreover,

$$\frac{\chi'}{\chi} = -\frac{\alpha(\mu-1)\frac{\lambda}{(\lambda+\alpha\tau^D)^2}}{\frac{\lambda+\alpha\tau^D+(\mu-1)\lambda}{\lambda+\alpha\tau^D}} = -\frac{\alpha(\mu-1)}{\left(1+\alpha\frac{\tau^D}{\lambda}\right)(\alpha\tau^D+\lambda\mu)}\,.$$

We also have

$$\frac{\chi}{\mathcal{E}_y} = \frac{\frac{\lambda+\alpha\tau^D+(\mu-1)\lambda}{\lambda+\alpha\tau^D}}{\frac{1-\lambda+\alpha(1-\tau^D)-\lambda(\mu-1)}{1-\lambda+\alpha(1-\tau^D)}} = \frac{\alpha\tau^D+\lambda\mu}{\lambda+\alpha\tau^D}\frac{1-\lambda+\alpha(1-\tau^D)}{1+\alpha-\alpha\tau^D-\lambda\mu}$$

such that

$$\begin{aligned}\frac{\chi}{\mathcal{E}_y}-1 &= \frac{\left[\alpha\tau^D+\lambda\mu\right]\left[1-\lambda+\alpha(1-\tau^D)\right]-\left[\lambda+\alpha\tau^D\right]\left[1+\alpha-\alpha\tau^D-\lambda\mu\right]}{\left[\lambda+\alpha\tau^D\right]\left[1+\alpha-\alpha\tau^D-\lambda\mu\right]} \\ &= \frac{\lambda(1+\alpha)(\mu-1)}{\left[\lambda+\alpha\tau^D\right]\left[1+\alpha-\alpha\tau^D-\lambda\mu\right]}\ .\end{aligned}$$

As a result, the first two terms determining sgn $\left(\frac{\partial\zeta_f}{\partial\tau^D}\right)$ are given by

$$\begin{aligned}① &= s\frac{\Gamma'}{\Gamma}\left(\frac{\chi}{\mathcal{E}_y}-1\right) = -\frac{s\alpha(1+\alpha)}{\left(1-\lambda+\alpha(1-\tau^D)\right)\left(\lambda+\alpha\tau^D\right)}\frac{\lambda(1+\alpha)(\mu-1)}{\left[\lambda+\alpha\tau^D\right]\left[1+\alpha-\alpha\tau^D-\lambda\mu\right]}\ , \\ ② &= s+(1-s)\frac{1+\alpha\frac{1-\tau^D}{1-\lambda}}{1+\alpha\frac{\tau^D}{\lambda}}\ ,\end{aligned}$$

and the third one by

$$\begin{aligned}③ &= \frac{\chi}{\mathcal{E}_y}\left(\frac{\chi'}{\chi}-\frac{\mathcal{E}_y'}{\mathcal{E}_y}\right) \\ &= \frac{\left[\alpha\tau^D+\lambda\mu\right]\left[1-\lambda+\alpha(1-\tau^D)\right]\alpha(\mu-1)\lambda}{\left[\lambda+\alpha\tau^D\right]\left[1+\alpha-\alpha\tau^D-\lambda\mu\right]}\times \\ &\left(-\frac{1}{\left(\lambda+\alpha\tau^D\right)\left(\alpha\tau^D+\lambda\mu\right)}+\frac{1}{\left[1-\lambda+\alpha(1-\tau^D)\right]\left[1-\lambda+\alpha(1-\tau^D)-\lambda(\mu-1)\right]}\right)\ .\end{aligned}$$

As a result, we obtain on $\tau^D < \tau^D_{disc}$

$$\begin{aligned}\operatorname{sgn}\left(\frac{\partial\zeta_f}{\partial\tau^D}\right) &= ① + ② \times ③ \\ &= \frac{-s(1+\alpha)^2}{\left(1-\lambda+\alpha(1-\tau^D)\right)\left(\lambda+\alpha\tau^D\right)} + ② \times \left(-\frac{1-\lambda+\alpha(1-\tau^D)}{\lambda+\alpha\tau^D}+\frac{\alpha\tau^D+\lambda\mu}{1+\alpha(1-\tau^D)-\lambda\mu}\right)\ ,\end{aligned}$$

where the last component can be simplified to

$$-\frac{1-\lambda+\alpha(1-\tau^D)}{\lambda+\alpha\tau^D}+\frac{\alpha\tau^D+\lambda\mu}{1+\alpha-\alpha\tau^D-\lambda\mu} = -\frac{(1+\alpha)\left[1+\alpha-\lambda(1+\mu)-2\alpha\tau^D\right]}{\left[\lambda+\alpha\tau^D\right]\left[1+\alpha-\alpha\tau^D-\lambda\mu\right]}\ .$$

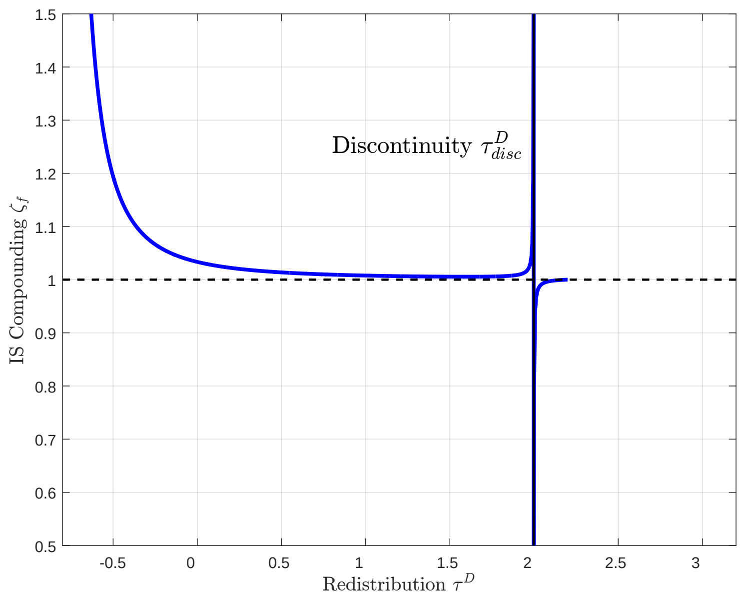
Multiplying through by $\lambda+\alpha\tau^D > 0$, we finally obtain

$$\operatorname{sgn}\left(\frac{\partial\zeta_f}{\partial\tau^D}\right) = -\frac{s(1+\alpha)^2}{1-\lambda+\alpha(1-\tau^D)} - \left(s+(1-s)\frac{1+\alpha\frac{1-\tau^D}{1-\lambda}}{1+\alpha\frac{\tau^D}{\lambda}}\right)\frac{(1+\alpha)\left[1+\alpha-\lambda(1+\mu)-2\alpha\tau^D\right]}{1+\alpha-\alpha\tau^D-\lambda\mu}\ .$$

There are four things worth mentioning: (i) the first component is always negative for $\tau^D < \overline{\tau}^D$; (ii) the second pre-multiplying component is strictly positive for all $\tau^D \in \left(\underline{\tau}^D, \overline{\tau}^D\right)$; (iii) the numerator of the third term is positive if $\tau^D < \frac{1+\alpha-\lambda-\lambda\mu}{2\alpha}$ which is larger than $\underline{\tau}^D$ and smaller than τ^D_{disc} if $\mu < 1 + \frac{1+\alpha}{\lambda}$ which holds by assumption; (iv) the denominator of the third term is positive if $\tau^D < \tau^D_{disc}$ and negative if $\tau^D > \tau^D_{disc}$. As a result, $\tau^D < \frac{1+\alpha-\lambda-\lambda\mu}{2\alpha} < \tau^D_{disc}$ is a sufficient condition for $\text{sgn}\left(\frac{\partial \zeta_f}{\partial \tau^D}\right) < 0$. Additionally, on the interval $(\frac{1+\alpha-\lambda-\lambda\mu}{2\alpha}, \tau^D_{disc})$ there exists a threshold τ^D_{zero} above which $\text{sgn}\left(\frac{\partial \zeta_f}{\partial \tau^D}\right) > 0$. The proof of statement (c) follows by combining Results 3 - 6. □

In Figure 10, we illustrate the previously discussed properties. To do so, we use the following parameters, i.e., $\alpha = 0.50, \beta = 0.99, \mu = 1.25, \lambda = 0.40, s = 0.95$. For these values, ζ_f decreases on $\tau^D \in [0, 1]$ such that the U-shape is only for excessive steady-state tax redistribution apparent.

Figure 10. Comparative statics of IS compounding regarding redistribution under tractable HANK model.



□

B.3.3 Proof Proposition 3

Proof. From Appendix A.4 the endogenous productivity equation is given by

$$\hat{g}^A_{t+1} = \hat{c}^S_t - \mathbb{E}_t\left[\hat{c}^S_{t+1}\right] + \mathscr{M}\mathbb{E}_t\left[\hat{z}_{t+1} + \hat{L}_{t+1}\right] - \mathscr{M}_\tau \mathbb{E}_t\left[\hat{\tau}^D_{t+1}\right] .$$

Substituting in for cyclical saver consumption, i.e. $\hat{c}_t^S = \mathcal{E}_y \hat{y}_t - \mathcal{E}_g \hat{g}_{t+1}^A - \mathcal{E}_z \hat{z}_t - \mathcal{E}_\tau \hat{\tau}_t^D$, we obtain

$$\hat{g}_{t+1}^A = \mathcal{E}_y \hat{y}_t - \mathcal{E}_g \hat{g}_{t+1}^A - \mathcal{E}_z \hat{z}_t - \mathcal{E}_\tau \hat{\tau}_t^D - \mathbb{E}_t \left[\mathcal{E}_y \hat{y}_{t+1} - \mathcal{E}_g \hat{g}_{t+2}^A - \mathcal{E}_z \hat{z}_{t+1} - \mathcal{E}_\tau \hat{\tau}_{t+1}^D \right]$$
$$+ \mathscr{M} \mathbb{E}_t \left[\hat{y}_{t+1} \right] - \mathscr{M}_\tau \mathbb{E}_t \left[\hat{\tau}_{t+1}^D \right] \ ,$$

where we have used $\mathbb{E}_t \left[\hat{y}_{t+1} \right] = \mathbb{E}_t \left[\hat{z}_{t+1} + \hat{L}_{t+1} \right]$. Collecting terms gives us finally

$$\hat{g}_{t+1}^A = \xi_y \hat{y}_t + \xi_{y'} \mathbb{E}_t \left[\hat{y}_{t+1} \right] + \xi_{g'} \mathbb{E}_t \left[\hat{g}_{t+2} \right] - \xi_z z_t + \xi_{z'} \left[z_{t+1} \right] - \xi_\tau \hat{\tau}_t^D + \xi_{\tau'} \mathbb{E}_t \left[\hat{\tau}_{t+1}^D \right] \ ,$$

where we have the following auxiliary variables

$$\xi_y \equiv \tfrac{\mathcal{E}_y}{1+\mathcal{E}_g} \ , \quad \xi_{y'} \equiv \tfrac{\mathscr{M} - \mathcal{E}_y}{1+\mathcal{E}_g} \ , \quad \xi_{g'} \equiv \tfrac{\mathcal{E}_g}{1+\mathcal{E}_g} \ , \quad \xi_z = \xi_{z'} \equiv \tfrac{\mathcal{E}_z}{1+\mathcal{E}_g} \ , \quad \xi_\tau \equiv \tfrac{\mathcal{E}_\tau}{1+\mathcal{E}_g} \ , \quad \xi_{\tau'} \equiv \tfrac{\mathcal{E}_\tau - \mathscr{M}_\tau}{1+\mathcal{E}_g}$$

where $\mathcal{E}_y, \mathcal{E}_g, \mathcal{E}_z, \mathcal{E}_\tau, \mathscr{M}$, and $\mathscr{M}_\tau$ have been defined above. □

B.3.4 Proof Proposition 4

Proof. From Appendix A.4, we know that the log-linear static wage Phillips curve is given by

$$\hat{\pi}_t^w = \kappa \left(\varphi \hat{L}_t + \hat{c}_t - \hat{z}_t \right) \ .$$

Substituting in for $\hat{L}_t$ and $\hat{c}_t$ results in

$$\hat{\pi}_t^w = \kappa \left(\varphi \left(\hat{y}_t - \hat{z}_t \right) + \frac{\hat{y}_t - (1 - s_c) \frac{g^A}{g^A - 1} \hat{g}_{t+1}^A}{s_c} - \hat{z}_t \right) \ ,$$
$$= \kappa \frac{1 + \varphi s_c}{s_c} \hat{y}_t - \kappa \frac{1 - s_c}{s_c} \frac{g^A}{g^A - 1} \hat{g}_{t+1}^A - \kappa (1 + \varphi) \hat{z}_t \ .$$

Making use of the equation linking price and wage inflation, one finally obtains

$$\hat{\pi}_t = \hat{\pi}_t^w + \hat{z}_{t-1} - \hat{z}_t - \hat{g}_t^A$$
$$= \kappa \frac{1 + \varphi s_c}{s_c} \hat{y}_t - \kappa \frac{1 - s_c}{s_c} \frac{g^A}{g^A - 1} \hat{g}_{t+1}^A - \kappa (1 + \varphi) \hat{z}_t + \hat{z}_{t-1} - \hat{z}_t - \hat{g}_t^A$$
$$= \kappa_y \hat{y}_t - \kappa_g \hat{g}_{t+1}^A - \hat{g}_t^A - \kappa_z \hat{z}_t + \hat{z}_{t-1} \ ,$$

where the auxiliary parameters are defined as follows

$$\kappa_y \equiv \kappa \frac{1 + \varphi s_c}{s_c} \ , \quad \kappa_g \equiv \kappa \frac{1 - s_c}{s_c} \frac{g^A}{g^A - 1} \ , \quad \kappa_z \equiv 1 + \kappa (1 + \varphi) \ .$$

Notice that the former definition also allows to write

$$\hat{\pi}_t^w = \kappa_y \hat{y}_t - \kappa_g \hat{g}_{t+1}^A + (1 - \kappa_z)\hat{z}_t \ .$$

□

B.4 Dissecting the Role of Cyclical Income Inequality

B.4.1 Proof Proposition 5

Proof. In a rational expectations equilibrium to an exogenous monetary policy, technology or tax shock of persistence $\rho \in (0,1)$ the endogenous technology impact multiplier is, using the method of undetermined coefficients and guessing that $\hat{y}_t = \mathcal{M}_y \epsilon_t$ and $\hat{g}_{t+1}^A = \mathcal{M}_g \epsilon_t$, given by

$$\mathcal{M}_g = \frac{\xi_y + \rho \xi_{y'}}{1 - \rho \xi_{g'}} \mathcal{M}_g - \frac{1}{1 - \rho \xi_{g'}} \left(\xi_z (1 - \rho) + \xi_\tau - \rho \xi_{\tau'} \right) = \frac{\mathcal{E}_y (1 - \rho) + \rho \mathscr{M}}{1 + \mathcal{E}_g (1 - \rho)} \mathcal{M}_y - \frac{\Xi_{z,\tau,m}}{1 + \mathcal{E}_g (1 - \rho)} \ ,$$

where $\Xi_{z,\tau,m} = 0$ in case of a monetary policy shock, $\Xi_{z,\tau,m} = (1-\rho)\mathcal{E}_z$ in case of a technology shock, and $\Xi_{z,\tau,m} = (1-\rho)\mathcal{E}_\tau + \mathscr{M}_\tau$ in case of a tax shock. Taking the derivative w.r.t. to the degree of countercyclical inequality χ results in

$$\frac{\partial \mathcal{M}_g}{\partial \chi} = \frac{1}{1 + \mathcal{E}_g (1 - \rho)} \left((1 - \rho) \mathcal{M}_y \frac{\partial \mathcal{E}_y}{\partial \chi} + \left(\mathcal{E}_y (1 - \rho) + \rho \mathscr{M} \right) \frac{\partial \mathcal{M}_y}{\partial \chi} - \frac{\partial \Xi_{z,\tau,m}}{\partial \chi} \right)$$

as $\mathcal{E}_g$ and $\mathscr{M}$ are independent of χ. We consider an exogenous shock which is recessionary, i.e. $\mathcal{M}_y < 0$. The scars irrelevance frontier of countercyclical inequality at level $\overline{g}$ is thus given by

$$(1 - \rho) \mathcal{M}_y \frac{\partial \mathcal{E}_y}{\partial \chi} + \left(\mathcal{E}_y (1 - \rho) + \rho \mathscr{M} \right) \frac{\partial \mathcal{M}_y}{\partial \chi} - \frac{\partial \Xi_{z,\tau,m}}{\partial \chi} \leq \left(1 + \mathcal{E}_g (1 - \rho) \right) \overline{g} \ ,$$

which can be rearranged to

$$\mathcal{M}_y \frac{\partial \mathcal{E}_y}{\partial \chi} \frac{\chi}{\mathcal{E}_y} \leq \frac{\chi}{(1 - \rho) \mathcal{E}_y} \left(\left(1 + \mathcal{E}_g (1 - \rho) \right) \overline{g} + \frac{\partial \Xi_{z,\tau,m}}{\partial \chi} \right) - \left(1 + \frac{\rho}{1 - \rho} \frac{\mathscr{M}}{\mathcal{E}_y} \right) \frac{\partial \mathcal{M}_y}{\partial \chi} \chi \ .$$

Division by $\mathcal{M}_y$ finally results in

$$\frac{\partial \mathcal{E}_y}{\partial \chi} \frac{\chi}{\mathcal{E}_y} \geq \frac{\chi}{(1 - \rho) \mathcal{E}_y \mathcal{M}_y} \left(\left(1 + \mathcal{E}_g (1 - \rho) \right) \overline{g} + \frac{\partial \Xi_{z,\tau,m}}{\partial \chi} \right) - \left(1 + \frac{\rho}{1 - \rho} \frac{\mathscr{M}}{\mathcal{E}_y} \right) \frac{\partial \mathcal{M}_y}{\partial \chi} \frac{\chi}{\mathcal{M}_y} \ ,$$

which completes the derivation. □

B.5 A Detailed Look at the Propagation of Monetary Policy Shocks

B.5.1 Proof Proposition 6

Proof. The relevant system of equations is given by

$$\begin{aligned}
\hat{y}_t &= \zeta_f \mathbb{E}_t\left[\hat{y}_{t+1}\right] - \zeta_r\left(\hat{i}_t - \mathbb{E}_t\left[\hat{\pi}_{t+1}\right]\right) + \zeta_g \hat{g}^A_{t+1} - \zeta_{g'}\mathbb{E}_t\left[\hat{g}^A_{t+2}\right] \,, \\
\hat{g}^A_{t+1} &= \xi_y \hat{y}_t + \xi_{y'}\mathbb{E}_t\left[\hat{y}_{t+1}\right] + \xi_{g'}\mathbb{E}_t\left[\hat{g}^A_{t+2}\right] \,, \\
\hat{\pi}^w_t &= \kappa_y \hat{y}_t - \kappa_g \hat{g}^A_{t+1} \,, \\
\hat{\pi}_t &= \hat{\pi}^w_t - \hat{g}^A_t \,, \\
\hat{i}_t &= \phi_\pi \hat{\pi}^w_t + \epsilon_{mt} \,.
\end{aligned}$$

As stated in the main body of the text, we assume that ϵ_{mt} follows and AR(1) process with persistence ρ_m. Using the method of undetermined coefficients, we guess a solution of the form

$$\hat{y}_t = \mathcal{M}_y \epsilon_{mt} \,, \quad \hat{g}^A_{t+1} = \mathcal{M}_g \epsilon_{mt} \,, \quad \hat{\pi}_t = \mathcal{M}_\pi \epsilon_{mt} \,, \quad \hat{\pi}^w_t = \mathcal{M}_{\pi^w}\epsilon_{mt}.$$

Endogenous Growth Multiplier. To begin with, we obtain from the second equation

$$\mathcal{M}_g = \frac{\xi_y + \rho_m \xi_{y'}}{1-\rho_m \xi_{g'}}\mathcal{M}_y = \frac{\frac{\mathcal{E}_y}{1+\mathcal{E}_g} + \rho_m \frac{\mathscr{M}-\mathcal{E}_y}{1+\mathcal{E}_g}}{1-\rho_m\frac{\mathcal{E}_g}{1+\mathcal{E}_g}}\mathcal{M}_y = \frac{\mathcal{E}_y(1-\rho_m)+\rho_m\mathscr{M}}{1+\mathcal{E}_g(1-\rho_m)}\mathcal{M}_y \equiv \Omega\mathcal{M}_y \,,$$

where the auxiliary parameter is defined as

$$\Omega = \frac{\mathcal{E}_y(1-\rho_m)+\rho_m\mathscr{M}}{1+\mathcal{E}_g(1-\rho_m)} \,.$$

Notice that using the definition of $\mathscr{F} \equiv \mathcal{E}_y/(1+\mathcal{E}_g)$, that describes a pure contemporaneous cost of funds effect, one can rewrite Ω as weighted sum, i.e.

$$\Omega = \omega\mathscr{F} + (1-\omega)\mathscr{M} \,,$$

where ω is the solution to

$$\omega\left(\frac{\mathcal{E}_y}{1+\mathcal{E}_g} - \mathscr{M}\right) + \mathscr{M} = \frac{\mathcal{E}_y(1-\rho_m)+\rho_m\mathscr{M}}{1+\mathcal{E}_g(1-\rho_m)} \,,$$

which can be rearranged to

$$\omega = \frac{1+\mathcal{E}_g}{\mathcal{E}_y - \left(1+\mathcal{E}_g\right)\mathscr{M}}\frac{\mathcal{E}_y(1-\rho_m) - (1-\rho_m)\left(1+\mathcal{E}_g\right)\mathscr{M}}{1+\mathcal{E}_g(1-\rho_m)} = (1-\rho_m)\frac{1+\mathcal{E}_g}{1+\mathcal{E}_g(1-\rho_m)} \,.$$

There are two things worth noting: First, $\mathcal{M}_g$ has the same sign as $\mathcal{M}_y$, i.e. $\Omega > 0$. Second, we have $\Omega|_{\rho_m=0} = \mathscr{F} = \mathcal{E}_y/(1+\mathcal{E}_g)$. Moreover, $\lim_{\rho_m \to 1} \Omega|_{\rho_m} = \mathscr{M}$ which is a pure market size effect.

Output Multiplier. From the aggregate IS equation, we have

$$\mathcal{M}_y = \rho_m \zeta_f \mathcal{M}_y - \zeta_r \Big(\phi_\pi (\kappa_y \mathcal{M}_y - \kappa_g \mathcal{M}_g) - \rho_m (\kappa_y \mathcal{M}_y - \kappa_g \mathcal{M}_g) + \mathcal{M}_g \Big) + \zeta_g \mathcal{M}_g - \rho_m \zeta_{g'} \mathcal{M}_g - \zeta_r \ .$$

Rearranging results in

$$\Big(1 + \phi_\pi \zeta_r \kappa_y - \rho_m(\zeta_f + \zeta_r \kappa_y)\Big)\mathcal{M}_y = \Big(\phi_\pi \zeta_r \kappa_g + \zeta_g - \zeta_r - \rho_m(\zeta_r \kappa_g + \zeta_{g'})\Big)\mathcal{M}_g - \zeta_r$$

such that the output multiplier to a positive monetary innovation becomes

$$\mathcal{M}_y = -\frac{\zeta_r}{1 + \phi_\pi \zeta_r \kappa_y - \rho_m(\zeta_f + \zeta_r \kappa_y) - \big(\phi_\pi \zeta_r \kappa_g + \zeta_g - \zeta_r - \rho_m(\zeta_r \kappa_g + \zeta_{g'})\big)\Omega} \ .$$

The previous expression can be rearranged to

$$\begin{aligned} \mathcal{M}_y &= -\frac{1}{\zeta_r^{-1}(1-\rho_m \zeta_f) + \kappa_y(\phi_\pi - \rho_m) - \big(\kappa_g(\phi_\pi - \rho_m) - 1 + \zeta_r^{-1}(\zeta_g - \rho_m \zeta_{g'})\big)\Omega} \ . \\ &= -\frac{1}{\zeta_r^{-1}(1-\rho_m \zeta_f) + (\kappa_y - \Omega\kappa_g)(\phi_\pi - \rho_m) - (1-\rho_m \tilde{s})\,\mathcal{E}_g \Omega} \ , \end{aligned}$$

where the last line follows as

$$\zeta_r^{-1}(\zeta_g - \zeta_r - \rho_m \zeta_{g'}) = \zeta_r^{-1}\left(\frac{\mathcal{E}_g}{\mathcal{E}_y} - \rho_m \tilde{s} \frac{\mathcal{E}_g}{\mathcal{E}_y}\right) = (1-\rho_m \tilde{s})\,\mathcal{E}_g \ .$$

Inflation Multiplier. From the contemporaneous wage inflation Phillips curve, we obtain

$$\mathcal{M}_{\pi^w} = \kappa_y \mathcal{M}_y - \kappa_g \mathcal{M}_g = (\kappa_y - \Omega \kappa_g)\mathcal{M}_y \ ,$$

which equals at impact also $\mathcal{M}_\pi$ as $\hat{g}_t^A = 0$. This concludes the proof of the Proposition. □

B.5.2 Proof Corollary 2

Proof. Recall that the output response of a monetary policy shock is given by

$$\frac{1}{|\mathcal{M}_y|} = \mathcal{E}_y(1-\rho_m \zeta_f) + (\kappa_y - \Omega\kappa_g)(\phi_\pi - \rho_m) - (1-\rho_m \tilde{s})\,\mathcal{E}_g \Omega$$

Recall the definition of $\mathcal{E}_y$ and κ_y and let us define $\tilde{\mathcal{E}}_y$, respectively $\tilde{\kappa}_y$:

$$\begin{aligned}
\mathcal{E}_y &= \frac{(1-\lambda)\Gamma + \lambda - \lambda s_c \chi}{(1-\lambda)\Gamma s_c} \,, \quad \tilde{\mathcal{E}}_y = \frac{(1-\lambda)\Gamma + \lambda - \lambda \chi}{(1-\lambda)\Gamma} \,, \\
\kappa_y &= \kappa \frac{1+\varphi s_c}{s_c} \,, \quad \tilde{\kappa}_y = \kappa(1+\varphi) \,.
\end{aligned}$$

Having this at hands, we obtain

$$\begin{aligned}
\frac{1}{|\mathcal{M}_y|} &= (1-\rho_m)\mathcal{E}_y - \rho_m(1-\tilde{s})\left(\chi - \mathcal{E}_y\right) + \kappa_y(\phi_\pi - \rho_m) - \Omega\kappa_g(\phi_\pi - \rho_m) - (1-\rho_m\tilde{s})\,\mathcal{E}_g\Omega \\
&= (1-\rho_m) + \kappa_y(\phi_\pi - \rho_m) + (1-\rho_m)\left(\mathcal{E}_y - 1\right) - \rho_m(1-\tilde{s})\left(\chi - \mathcal{E}_y\right) \\
&\quad - \Omega\kappa_g(\phi_\pi - \rho_m) - (1-\rho_m\tilde{s})\,\mathcal{E}_g\Omega \\
&= (1-\rho_m) + \tilde{\kappa}_y(\phi_\pi - \rho_m) + (1-\rho_m)\left(\tilde{\mathcal{E}}_y - 1\right) + \rho_m(1-\tilde{s})\left(\tilde{\mathcal{E}}_y - \chi\right) \\
&\quad + \left(\kappa(\phi_\pi - \rho_m) + (1-\rho_m\tilde{s})\,\frac{(1-\lambda)\Gamma+\lambda}{(1-\lambda)\Gamma}\right)\frac{1-s_c}{s_c} - \Omega\kappa_g(\phi_\pi - \rho_m) - (1-\rho_m\tilde{s})\,\mathcal{E}_g\Omega \,,
\end{aligned}$$

where the first line follows from using $\zeta_f = 1 + (1-\tilde{s})\left(\frac{\chi}{\mathcal{E}_y} - 1\right)$, the second line by adding an intelligent zero, and the third one by replacing $\mathcal{E}_y, \kappa_y$ with their counterfactual ones from a model without endogenous growth, i.e. $\tilde{\mathcal{E}}_y, \tilde{\kappa}_y$. Notice that $\tilde{\mathcal{E}}_y - 1 < 0$ if $\chi > 1$, and also $\tilde{\mathcal{E}}_y - \chi < 0$ if $\chi > 1$. This former expression can finally be rearranged to the one stated in the main text by recognizing that

$$-\Omega\kappa_g(\phi_\pi - \rho_m) = -\Omega^{RA}\kappa_g(\phi_\pi - \rho_m) + (\Omega^{RA} - \Omega)\kappa_g(\phi_\pi - \rho_m)$$

and additionally that

$$-\left(1-\rho_m\tilde{s}\right)\mathcal{E}_g\Omega = -(1-\rho_m)\tilde{s}\mathcal{E}_g\Omega - (1-\tilde{s})\mathcal{E}_g\Omega \,.$$

This concludes the derivation of the decomposition. □

B.5.3 Proof Proposition 7

Proof. The proof proceeds in two steps. In the first one, we derive $\frac{\partial \mathcal{M}_y}{\partial \chi}$ and show that its sign is characterized by a second order polynomial in ρ_m. In the second step, we conduct a case distinction to determine the sign of this polynomial.

Step 1: Derivation of Second Order Polynomial

To begin with, let us denote $\mathcal{M}_y = -\mathcal{M}_y^+$, where $\mathcal{M}_y^+$ denotes the positive part of the output multiplier. Notice that the denominator of $\mathcal{M}_y^+$ can be rewritten as

$$\mathcal{E}_y(1-\rho_m\zeta_f) + (\kappa_y - \Omega\kappa_g)(\phi_\pi - \rho_m) - (1-\rho_m\tilde{s})\,\mathcal{E}_g\Omega \,,$$

such that we obtain

$$\frac{\partial \mathcal{M}_y}{\partial \chi} = \left(\mathcal{M}_y^+\right)^2 \left((1-\rho_m\zeta_f)\frac{\partial \mathcal{E}_y}{\partial \chi} - \rho_m \mathcal{E}_y \frac{\partial \zeta_f}{\partial \chi} - \left[\kappa_g(\phi_\pi - \rho_m) + (1-\rho_m\tilde{s})\,\mathcal{E}_g\right] \frac{\partial \Omega}{\partial \chi} \right) \,.$$

Because of

$$\begin{aligned} \frac{\partial \Omega}{\partial \chi} &= \frac{1-\rho_m}{1+\mathcal{E}_g(1-\rho_m)} \frac{\partial \mathcal{E}_y}{\partial \chi} < 0 \,, \\ \frac{\partial \zeta_f}{\partial \chi} &= -(1-\tilde{s})\frac{\chi}{(\mathcal{E}_y)^2}\frac{\partial \mathcal{E}_y}{\partial \chi} + (1-\tilde{s})\frac{1}{\mathcal{E}_y} > 0 \,, \end{aligned}$$

the previous expression can be rewritten as

$$\begin{aligned} \frac{\partial \mathcal{M}_y}{\partial \chi} =& \left(\mathcal{M}_y^+\right)^2 \left(\left[1-\rho_m\zeta_f + \rho_m\,(1-\tilde{s})\,\frac{\chi}{\mathcal{E}_y}\right] \frac{\partial \mathcal{E}_y}{\partial \chi} - \rho_m\,(1-\tilde{s}) \right) \\ & - \left(\mathcal{M}_y^+\right)^2 \left[\kappa_g(\phi_\pi - \rho_m) + (1-\rho_m\tilde{s})\mathcal{E}_g\right] \frac{1-\rho_m}{1+\mathcal{E}_g(1-\rho_m)} \frac{\partial \mathcal{E}_y}{\partial \chi} \,. \end{aligned}$$

Making use of

$$1-\rho_m\zeta_f + \rho_m\,(1-\tilde{s})\,\frac{\chi}{\mathcal{E}_y} = 1-\rho_m\left(\tilde{s} + (1-\tilde{s})\,\frac{\chi}{\mathcal{E}_y}\right) + \rho_m\,(1-\tilde{s})\,\frac{\chi}{\mathcal{E}_y} = 1-\rho_m\tilde{s} \,,$$

we obtain

$$\frac{\partial \mathcal{M}_y}{\partial \chi} = \left(\mathcal{M}_y^+\right)^2 \left(\frac{1}{1+\mathcal{E}_g(1-\rho_m)} \left[1-\rho_m\tilde{s} - \kappa_g(\phi_\pi - \rho_m)(1-\rho_m)\right] \frac{\partial \mathcal{E}_y}{\partial \chi} - \rho_m\,(1-\tilde{s}) \right) \,.$$

Notice that $\frac{\partial \mathcal{M}_y}{\partial \chi}$ is continuously differentiable on $\rho_m \in (0,1)$. Its upper limit is given by

$$\lim_{\rho_m \to 1} \frac{\partial \mathcal{M}_y}{\partial \chi} = \left(\mathcal{M}_y^+|_{\rho_m=1}\right)^2 (1-\tilde{s}) \left(\frac{\partial \mathcal{E}_y}{\partial \chi} - 1\right) < 0 \,,$$

where

$$\mathcal{M}_y^+|_{\rho_m=1} = \frac{1}{\mathcal{E}_y(1-\zeta_f) + (\kappa_y - \mathscr{M}\kappa_g)(\phi_\pi - 1) - (1-\tilde{s})\,\mathcal{E}_g\mathscr{M}} \neq 0 \,.$$

The sign of the penultimate inequality follows as $\frac{\partial \mathcal{E}_y}{\partial \chi} < 0$. Similarly, the lower limit is given by

$$\lim_{\rho_m \to 0} \frac{\partial \mathcal{M}_y}{\partial \chi} = \left(\mathcal{M}_y^+|_{\rho_m=0}\right)^2 \frac{1}{1+\mathcal{E}_g} \left[1-\kappa_g\phi_\pi\right] \frac{\partial \mathcal{E}_y}{\partial \chi} \,,$$

where

$$\mathcal{M}_y^+|_{\rho_m=0} = \frac{1}{\frac{\mathcal{E}_y}{1+\mathcal{E}_g}(1-\kappa_g\phi_\pi)+\kappa_y\phi_\pi} \neq 0 \; .$$

There arise three cases:

1. If $\phi_\pi > \kappa_g^{-1}$, then $\lim\limits_{\rho_m \to 0} \frac{\partial \mathcal{M}_y}{\partial \chi} > 0$.

2. If $\phi_\pi = \kappa_g^{-1}$, then $\lim\limits_{\rho_m \to 0} \frac{\partial \mathcal{M}_y}{\partial \chi} = 0$.

3. If $\phi_\pi < \kappa_g^{-1}$, then $\lim\limits_{\rho_m \to 0} \frac{\partial \mathcal{M}_y}{\partial \chi} < 0$.

To understand how the sign of $\frac{\partial \mathcal{M}_y}{\partial \chi}$ depends on ρ_m based on this case distinction, we solve for its roots by rewriting

$$\begin{aligned} \frac{1}{1+\mathcal{E}_g(1-\rho_m)} \left[(1-\rho_m\tilde{s}) - \kappa_g(\phi_\pi-\rho_m)(1-\rho_m)\right] \frac{\partial \mathcal{E}_y}{\partial \chi} - \rho_m(1-\tilde{s}) &= 0 \\ \Leftrightarrow \left[(1-\rho_m\tilde{s}) - \kappa_g(\phi_\pi-\rho_m)(1-\rho_m)\right] \frac{\partial \mathcal{E}_y}{\partial \chi} - (1+\mathcal{E}_g(1-\rho_m))\rho_m(1-\tilde{s}) &= 0 \\ \Leftrightarrow \left[1-\rho_m\tilde{s} - \kappa_g\left(\phi_\pi - (1+\phi_\pi)\rho_m + \rho_m^2\right)\right] \frac{\partial \mathcal{E}_y}{\partial \chi} - \left((1+\mathcal{E}_g)\rho_m - \mathcal{E}_g\rho_m^2\right)(1-\tilde{s}) &= 0 \; . \end{aligned}$$

As a result, one obtains the quadratic polynomial

$$f(\rho_m) \equiv a\rho_m^2 + b\rho_m + c = 0 \; ,$$

where

$$\begin{aligned} a &= -\kappa_g \frac{\partial \mathcal{E}_y}{\partial \chi} + \mathcal{E}_g\left(1-\tilde{s}\right) \; , \\ b &= \left[-\tilde{s} + \kappa_g(1+\phi_\pi)\right] \frac{\partial \mathcal{E}_y}{\partial \chi} - \left(1+\mathcal{E}_g\right)\left(1-\tilde{s}\right) \; , \\ c &= (1-\kappa_g\phi_\pi)\frac{\partial \mathcal{E}_y}{\partial \chi} \; . \end{aligned}$$

Notice that $a > 0$ due to $\frac{\partial \mathcal{E}_y}{\partial \chi} < 0$ and $\mathcal{E}_g > 0$ such that $f(\rho_m)$ is strictly convex, and attains its minimum at $\rho_m^{min} = -\frac{b}{2a}$ with $f(\rho_m^{min}) = -\frac{1}{4}\frac{b^2}{a} + c$. By standard calculus, the roots of $f(\rho_m)$ are characterized by

$$\rho_m^{1,2} = \frac{-b \pm \sqrt{b^2-4ac}}{2a} \; .$$

STEP 2: SIGN OF SECOND ORDER POLYNOMIAL

To make progress on the previous cases, we use the following auxiliary result.

Result 7 (Roots Quadratic Polynomial). *Let us assume that the roots of $f(\rho_m)$ are real-valued, i.e. $b^2 - 4ac > 0$. Under our particular values for a, b and c, it follows that $\max\{\rho_m^1, \rho_m^2\} > 1$ and $\min\{\rho_m^1, \rho_m^2\} < 1$.*

Proof. Because of $a > 0$ and $b^2 - 4ac > 0$, the largest root ρ_m^l is given by

$$\rho_m^l = \max\{\rho_m^1, \rho_m^2\} = \frac{-b + \sqrt{b^2 - 4ac}}{2a} .$$

Consequently, $\rho_m^l > 1$ applies if

$$\frac{-b + \sqrt{b^2 - 4ac}}{2a} > 1 \quad \Leftrightarrow \quad b^2 - 4ac > 4a^2 + 4ab + b^2 \quad \Leftrightarrow \quad a(a + b + c) < 0 .$$

Notice that the first inequality applies by assumption if $2a + b < 0$, while the second one is necessary to check if $2a + b > 0$. Substituting in for a, b, c, the latter conditions trivially holds as

$$\begin{aligned} a + b + c &= -\kappa_g \frac{\partial \mathcal{E}_y}{\partial \chi} + \mathcal{E}_g (1 - \tilde{s}) + \left[-\tilde{s} + \kappa_g (1 + \phi_\pi)\right] \frac{\partial \mathcal{E}_y}{\partial \chi} \\ &\quad - (1 + \mathcal{E}_g)(1 - \tilde{s}) + (1 - \kappa_g \phi_\pi) \frac{\partial \mathcal{E}_y}{\partial \chi} \\ &= (1 - \tilde{s}) \left(\frac{\partial \mathcal{E}_y}{\partial \chi} - 1 \right) < 0 . \end{aligned}$$

To show the second statement, notice that the smaller root ρ_m^s is consequently given by

$$\rho_m^s = \min\{\rho_m^1, \rho_m^2\} = \frac{-b - \sqrt{b^2 - 4ac}}{2a} .$$

If $2a + b > 0$ applies, then $\rho_m^s < 1$ holds by assumption. If however $2a + b < 0$ applies, then $\rho_m^s < 1$ holds true if

$$\frac{-b - \sqrt{b^2 - 4ac}}{2a} < 1 \quad \Leftrightarrow \quad \sqrt{b^2 - 4ac} > -(2a + b) \quad \Leftrightarrow \quad a(a + b + c) < 0 ,$$

which is satisfied by the above reasoning. This concludes the proof. □

Coming back to the case distinction from the first part, we obtain:

1. If $\phi_\pi > \kappa_g^{-1}$, then $b < 0$ and $c > 0$ such that $\rho_m^{min} > 0$. Because of $\lim_{\rho_m \to 0} \frac{\partial \mathcal{M}_y}{\partial \chi} > 0$ and $\lim_{\rho_m \to 1} \frac{\partial \mathcal{M}_y}{\partial \chi} < 0$, it follows by the intermediary value theorem for continuous functions and the fact that $f(\rho_m)$ is a quadratic polynomial that there exists a unique $\rho_m^{SR} \in (0, 1)$ such that $\frac{\partial \mathcal{M}_y}{\partial \chi} > 0$ on $\rho_m \in (0, \rho_m^{SR})$, while $\frac{\partial \mathcal{M}_y}{\partial \chi} < 0$ on $\rho_m \in (\rho_m^{SR}, 1)$. Consequently, $f(\rho_m)$ has two real-valued roots, as a second order polynomial cannot jointly have real and complex roots, and hence

$b^2 - 4ac > 0$ follows. Finally, using the above auxiliary result, the threshold is

$$\rho_m^{SR} = \frac{-b - \sqrt{b^2 - 4ac}}{2a} \, .$$

2. If $\phi_\pi = \kappa_g^{-1}$, then $b < 0$ and $c = 0$ such that $\rho_m^{min} > 0$. As a result, both roots are given by

   $$\rho_m^s = 0 \, , \quad \rho_m^l = -\frac{b}{a} > 0 \, .$$

   Because of

   $$-\frac{b}{a} = -\frac{\left[1 - \tilde{s} + \kappa_g\right] \frac{\partial \mathcal{E}_y}{\partial \chi} - \left(1 + \mathcal{E}_g\right)\left(1 - \tilde{s}\right)}{-\kappa_g \frac{\partial \mathcal{E}_y}{\partial \chi} + \mathcal{E}_g \left(1 - \tilde{s}\right)} > 1 \, ,$$

   it then follows that $\frac{\partial \mathcal{M}_y}{\partial \chi} < 0$ on $\rho_m \in (0,1)$, where $\lim_{\rho_m \to 0} \frac{\partial \mathcal{M}_y}{\partial \chi} = 0$ describes the smaller root.

3. If $\phi_\pi < \kappa_g^{-1}$, then $c < 0$ while the sign of b is ambiguous. Additionally, $b^2 - 4ac > 0$ such that both roots are real-valued. As $b^2 - 4ac > b^2$ applies, it thus follows – independently of the sign of b – that $\rho_m^s < 0$ and $\rho_m^l > 0$. Because of $\lim_{\rho_m \to 0} \frac{\partial \mathcal{M}_y}{\partial \chi} < 0$ and $\lim_{\rho_m \to 1} \frac{\partial \mathcal{M}_y}{\partial \chi} < 0$, we must have that $\rho_m^l > 1$ such that $\frac{\partial \mathcal{M}_y}{\partial \chi} < 0$ on $\rho_m \in (0,1)$.

Combining cases 2. $-$ 3. gives the first statement in Proposition 7, whereas first case corresponds to the second statement. This concludes the proof. □

B.5.4 Proof Proposition 8

Proof. The proof proceeds analogously to the one for Proposition 7. In the first step, we compute $\frac{\partial \mathcal{M}_g}{\partial \chi}$ and show that it constitutes a quadratic polynomial in ρ_m. In the second step we show that there exists a unique threshold $\rho_m^{LR} \in (0,1)$ that determines the sign of $\frac{\partial \mathcal{M}_g}{\partial \chi}$.

To begin with, we have $\mathcal{M}_g = \Omega \mathcal{M}_y$ such that

$$\frac{\partial \mathcal{M}_g}{\partial \chi} = -\left(\frac{\partial \Omega}{\partial \chi} \mathcal{M}_y^+ + \Omega \frac{\partial \mathcal{M}_y^+}{\partial \chi}\right) \, .$$

From the proof of Proposition 7, we have

$$\frac{\partial \mathcal{M}_y^+}{\partial \chi} = -\left(\mathcal{M}_y^+\right)^2 \left(\frac{1}{1 + \mathcal{E}_g(1 - \rho_m)} \left[1 - \rho_m \tilde{s} - \kappa_g(\phi_\pi - \rho_m)(1 - \rho_m)\right] \frac{\partial \mathcal{E}_y}{\partial \chi} - \rho_m \left(1 - \tilde{s}\right)\right) \, ,$$
$$\frac{\partial \Omega}{\partial \chi} = \frac{1 - \rho_m}{1 + \mathcal{E}_g(1 - \rho_m)} \frac{\partial \mathcal{E}_y}{\partial \chi} \, ,$$

such that we get

$$\begin{aligned}\frac{\partial \mathcal{M}_g}{\partial \chi} =& -\mathcal{M}_y^+ \frac{1-\rho_m}{1+\mathcal{E}_g(1-\rho_m)} \frac{\partial \mathcal{E}_y}{\partial \chi} \\ &+ \left(\mathcal{M}_y^+\right)^2 \frac{1}{1+\mathcal{E}_g(1-\rho_m)} \left[1-\rho_m \tilde{s} - \kappa_g(\phi_\pi - \rho_m)(1-\rho_m)\right] \frac{\partial \mathcal{E}_y}{\partial \chi} \Omega \,, \\ &- \left(\mathcal{M}_y^+\right)^2 \rho_m \left(1-\tilde{s}\right) \Omega \,,\end{aligned}$$

where we additionally have

$$\Omega = \frac{\mathcal{E}_y(1-\rho_m) + \rho_m \mathscr{M}}{1+\mathcal{E}_g(1-\rho_m)} \,.$$

As a result, the derivative takes the following limits

$$\lim_{\rho_m \to 1} \frac{\partial \mathcal{M}_g}{\partial \chi} = \left(\mathcal{M}_y^+|_{\rho_m=1}\right)^2 \mathscr{M}(1-\tilde{s}) \left(\frac{\partial \mathcal{E}_y}{\partial \chi} - 1\right) < 0 \,,$$

where the sign follows from $\frac{\partial \mathcal{E}_y}{\partial \chi} < 0$. Additionally, we have

$$\begin{aligned}\lim_{\rho_m \to 0} \frac{\partial \mathcal{M}_g}{\partial \chi} &= -\frac{\mathcal{M}_y^+|_{\rho_m=0}}{1+\mathcal{E}_g} \frac{\partial \mathcal{E}_y}{\partial \chi} + \frac{\left(\mathcal{M}_y^+|_{\rho_m=0}\right)^2}{1+\mathcal{E}_g} \left(1-\kappa_g \phi_\pi\right) \frac{\partial \mathcal{E}_y}{\partial \chi} \frac{\mathcal{E}_y}{1+\mathcal{E}_g} \\ &= -\frac{\mathcal{M}_y^+|_{\rho_m=0}}{1+\mathcal{E}_g} \frac{\partial \mathcal{E}_y}{\partial \chi} \left(1 - \frac{\mathcal{E}_y}{1+\mathcal{E}_g} \left(1-\kappa_g \phi_\pi\right) \mathcal{M}_y^+|_{\rho_m=0}\right) \\ &= -\frac{\mathcal{M}_y^+|_{\rho_m=0}}{1+\mathcal{E}_g} \frac{\partial \mathcal{E}_y}{\partial \chi} \left(1 - \frac{\frac{\mathcal{E}_y}{1+\mathcal{E}_g}\left(1-\kappa_g \phi_\pi\right)}{\frac{\mathcal{E}_y}{1+\mathcal{E}_g}\left(1-\kappa_g \phi_\pi\right) + \kappa_y \phi_\pi}\right) \\ &= -\frac{\mathcal{M}_y^+|_{\rho_m=0}}{1+\mathcal{E}_g} \frac{\kappa_y \phi_\pi}{\frac{\mathcal{E}_y}{1+\mathcal{E}_g}\left(1-\kappa_g \phi_\pi\right) + \kappa_y \phi_\pi} \frac{\partial \mathcal{E}_y}{\partial \chi} \\ &= -\kappa_y \phi_\pi \frac{\left(\mathcal{M}_y^+|_{\rho_m=0}\right)^2}{1+\mathcal{E}_g} \frac{\partial \mathcal{E}_y}{\partial \chi} > 0 \,,\end{aligned}$$

where the last inequality follows in turn from $\frac{\partial \mathcal{E}_y}{\partial \chi} < 0$. Additionally, to determine the sign of $\frac{\partial \mathcal{M}_g}{\partial \chi}$, one can simplify

$$\begin{aligned}\frac{\partial \mathcal{M}_g}{\partial \chi} =& -\frac{\left(\mathcal{M}_y^+\right)^2}{1+\mathcal{E}_g(1-\rho_m)} \left(\frac{1-\rho_m}{\mathcal{M}_y^+} - \Omega\left[1-\rho_m \tilde{s} - \kappa_g(\phi_\pi - \rho_m)(1-\rho_m)\right]\right) \frac{\partial \mathcal{E}_y}{\partial \chi} \\ &- \frac{\left(\mathcal{M}_y^+\right)^2}{1+\mathcal{E}_g(1-\rho_m)} \Big((1+\mathcal{E}_g(1-\rho_m))\rho_m \left(1-\tilde{s}\right) \Omega \Big) \,,\end{aligned}$$

such that the sign of $\frac{\partial \mathcal{M}_g}{\partial \chi}$ is determined by

$$\text{sgn}\frac{\partial \mathcal{M}_g}{\partial \chi} = -\left(\left[\frac{1-\rho_m}{\mathcal{M}_y^+} - \Omega\big(1-\rho_m\tilde{s} - \kappa_g(\phi_\pi - \rho_m)(1-\rho_m)\big)\right]\frac{\partial \mathcal{E}_y}{\partial \chi} + \big((1+\mathcal{E}_g(1-\rho_m))\rho_m\,(1-\tilde{s})\,\Omega\big)\right) \;.$$

Substituting in for $\mathcal{M}_y^+$, one obtains

$$\begin{aligned}
\text{sgn}\frac{\partial \mathcal{M}_g}{\partial \chi} = & -\big[(1-\rho_m)\left(\mathcal{E}_y(1-\rho_m\zeta_f) + (\kappa_y - \Omega\kappa_g)(\phi_\pi - \rho_m) - (1-\rho_m\tilde{s})\,\mathcal{E}_g\Omega\right)\big]\frac{\partial \mathcal{E}_y}{\partial \chi} \\
& + \Omega\big((1-\rho_m\tilde{s}) - \kappa_g(\phi_\pi-\rho_m)(1-\rho_m)\big)\frac{\partial \mathcal{E}_y}{\partial \chi} \\
& - \big(1+\mathcal{E}_g(1-\rho_m)\big)\rho_m(1-\tilde{s})\Omega \\
= & -\left[(1-\rho_m)\left(\mathcal{E}_y(1-\rho_m\zeta_f) + \kappa_y(\phi_\pi-\rho_m) - (1-\rho_m\tilde{s})\,\frac{1+(1-\rho_m)\mathcal{E}_g}{1-\rho_m}\Omega\right)\right]\frac{\partial \mathcal{E}_y}{\partial \chi} \\
& - \big(1+\mathcal{E}_g(1-\rho_m)\big)\rho_m(1-\tilde{s})\Omega \;.
\end{aligned}$$

Finally, substituting in for Ω delivers

$$\begin{aligned}
\text{sgn}\frac{\partial \mathcal{M}_g}{\partial \chi} = & -\left[(1-\rho_m)\Big(\mathcal{E}_y(1-\rho_m\zeta_f) + \kappa_y(\phi_\pi-\rho_m)\Big) - (1-\rho_m\tilde{s})\,\Big(\mathcal{E}_y(1-\rho_m) + \rho_m\mathscr{M}\Big)\right]\frac{\partial \mathcal{E}_y}{\partial \chi} \\
& - \rho_m(1-\tilde{s})\Big(\mathcal{E}_y(1-\rho_m) + \rho_m\mathscr{M}\Big) \;.
\end{aligned}$$

The previous expression is a quadratic polynomial in ρ_m whose roots are the solution to

$$\tilde{f}(\rho_m) = \tilde{a}\rho_m^2 + \tilde{b}\rho_m + \tilde{c} = 0 \;,$$

where

$$\begin{aligned}
\tilde{a} &= -\big(\mathcal{E}_y\zeta_f + \kappa_y - \tilde{s}(\mathcal{E}_y - \mathscr{M})\big)\frac{\partial \mathcal{E}_y}{\partial \chi} + (1-\tilde{s})(\mathcal{E}_y - \mathscr{M}) \;, \\
\tilde{b} &= -\big(-\mathcal{E}_y\zeta_f - \kappa_y(1+\phi_\pi) + \tilde{s}\mathcal{E}_y - \mathscr{M}\big)\frac{\partial \mathcal{E}_y}{\partial \chi} - (1-\tilde{s})\mathcal{E}_y \;, \\
\tilde{c} &= -\kappa_y\phi_\pi\frac{\partial \mathcal{E}_y}{\partial \chi} \;.
\end{aligned}$$

Result 8. *A sufficient condition for $\tilde{a}$ to be positive is given by*

$$s \geq \frac{(1-\lambda)\Gamma^2}{\lambda + (1-\lambda)\Gamma^2} \;,$$

which holds in most calibrations. Additionally, $\tilde{b} < 0$ and $\tilde{c} > 0$ applies.

Proof. To show the first claim notice that one can rewrite

$$\begin{aligned}\tilde{a} &= -\left(\mathcal{E}_y\zeta_f + \kappa_y - \tilde{s}(\mathcal{E}_y - \mathscr{M})\right)\frac{\partial \mathcal{E}_y}{\partial \chi} + (1-\tilde{s})(\mathcal{E}_y - \mathscr{M}) \\ &= -\left(\mathcal{E}_y\left(\zeta_f - \tilde{s}\right) + \kappa_y + \tilde{s}\mathscr{M}\right)\frac{\partial \mathcal{E}_y}{\partial \chi} + (1-\tilde{s})(\mathcal{E}_y - \mathscr{M}) \\ &= -\left((1-\tilde{s})\chi + \kappa_y\right)\frac{\partial \mathcal{E}_y}{\partial \chi} + (1-\tilde{s})\mathcal{E}_y + \frac{\tilde{s}}{s}\mathscr{M}\left(-s\frac{\partial \mathcal{E}_y}{\partial \chi} - (1-s)\Gamma\right) .\end{aligned}$$

Evidently, the first two terms are strictly positive if $\chi \geq 0$ such that a sufficient condition for $\tilde{a}$ to be positive is given by

$$-s\frac{\partial \mathcal{E}_y}{\partial \chi} - (1-s)\Gamma \geq 0 \quad \Leftrightarrow \quad \frac{s\lambda}{(1-\lambda)\Gamma} - (1-s)\Gamma \geq 0 \quad \Leftrightarrow \quad s \geq \frac{(1-\lambda)\Gamma^2}{\lambda + (1-\lambda)\Gamma^2} .$$

To show the second statement, notice that we have

$$\begin{aligned}\tilde{b} &= -\left(-\mathcal{E}_y\zeta_f - \kappa_y(1+\phi_\pi) + \tilde{s}\mathcal{E}_y - \mathscr{M}\right)\frac{\partial \mathcal{E}_y}{\partial \chi} - (1-\tilde{s})\mathscr{E}_y \\ &= -\left(-\mathcal{E}_y\left(\zeta_f - \tilde{s}\right) - \kappa_y(1+\phi_\pi) - \mathscr{M}\right)\frac{\partial \mathcal{E}_y}{\partial \chi} - (1-\tilde{s})\mathcal{E}_y \\ &= -\left(-(1-\tilde{s})\chi - \kappa_y(1+\phi_\pi) - \mathscr{M}\right)\frac{\partial \mathcal{E}_y}{\partial \chi} - (1-\tilde{s})\mathcal{E}_y < 0 ,\end{aligned}$$

where the negative sign follows because of $\chi \geq 0$. Finally, the sign of $\tilde{c}$ is obvious. □

There arise two cases:

1. If the conditions of the previous auxiliary result apply, i.e. $\tilde{a} > 0$, $\tilde{b} < 0$, and $\tilde{c} > 0$, then $\tilde{f}(\rho_m)$ is strictly convex, and attains its minimum at $\rho_m^{min} = -\frac{\tilde{b}}{2\tilde{a}} > 0$. As a result, we have in this case two real-valued roots, with $\rho_m^s < 1$ and $\rho_m^l > 1$, where

$$\rho_m^{LR} = \rho_m^s = \frac{-\tilde{b} - \sqrt{\tilde{b}^2 - 4\tilde{a}\tilde{c}}}{2\tilde{a}} .$$

2. If on the contrary $\tilde{a} < 0$, $\tilde{b} < 0$, and $\tilde{c} > 0$ applies, then $\tilde{f}(\rho_m)$ is strictly concave, and attains its maximum at $\rho_m^{max} = -\frac{\tilde{b}}{2\tilde{a}} < 0$. As a result, we have in this case two real-valued roots, with $\rho_m^s < 0$ and $\rho_m^l < 1$, where

$$\rho_m^{LR} = \rho_m^l = \frac{-\tilde{b} + \sqrt{\tilde{b}^2 - 4\tilde{a}\tilde{c}}}{2\tilde{a}} .$$

As a result, in both cases it holds that: $\frac{\partial \mathcal{M}_g}{\partial \chi} > 0$ on $\rho_m \in (0, \rho_m^{LR})$, $\frac{\partial \mathcal{M}_g}{\partial \chi} = 0$ if $\rho_m = \rho_m^{LR}$, and $\frac{\partial \mathcal{M}_g}{\partial \chi} < 0$ on $\rho_m \in (\rho_m^{LR}, 1)$.

Comparison of Persistence Thresholds

In order to compare the thresholds ρ_m^{LR}, ρ_m^{SR} regarding output and productivity response, recall that

$$\frac{\partial \mathcal{M}_g}{\partial \chi} = -\left(\frac{\partial \Omega}{\partial \chi}\mathcal{M}_y^+ + \Omega \frac{\partial \mathcal{M}_y^+}{\partial \chi}\right) = \frac{\partial \Omega}{\partial \chi}\mathcal{M}_y + \Omega \frac{\partial \mathcal{M}_y}{\partial \chi} .$$

Let us denote by ρ_m^{LR} the threshold value such that $\frac{\partial \mathcal{M}_g}{\partial \chi}|_{\rho_m = \rho_m^{LR}} = 0$, and by $\rho_m^{SR} \in (0,1)$ the threshold value such that $\frac{\partial \mathcal{M}_y}{\partial \chi}|_{\rho_m = \rho_m^{SR}} = 0$. Because $\frac{\partial \Omega}{\partial \chi}\mathcal{M}_y > 0 \ \forall \rho_m \in (0,1)$, we must have $\Omega \frac{\partial \mathcal{M}_y}{\partial \chi} < 0$ evaluated at ρ_m^{LR}. From Proposition 7, we know that $\frac{\partial \mathcal{M}_y}{\partial \chi} < 0$ on $\rho_m \in (0,1)$ if $\phi_\pi \leq \kappa_g^{-1}$ such that no threshold ordering is possible. On the contrary, if $\phi_\pi > \kappa_g^{-1}$, we know that $\frac{\partial \mathcal{M}_y}{\partial \chi} > 0$ on $\rho_m \in (0, \rho_m^{SR})$ and $\frac{\partial \mathcal{M}_y}{\partial \chi} < 0$ on $\rho_m \in (\rho_m^{SR}, 1)$. As a result, it follows that $\rho_m^{LR} > \rho_m^{SR}$ in this case. □

B.6 Effects from Progressive Redistribution Policies

B.6.1 Proof Proposition 9

Proof. We show the exogenous and endogenous growth cases separately.

PART I: PROGRESSIVE REDISTRIBUTION UNDER EXOGENOUS GROWTH

Statement (a) If wages are completely flexible, i.e. $\theta = 0$, the labor supply decisions made by unions is characterized by

$$\nu L_t^{\varphi} c_t = \frac{\epsilon_w - 1}{\epsilon_w}(1-\alpha)\alpha^{\frac{2\alpha}{1-\alpha}} Z_t ,$$

which results in $\hat{z}_t = \varphi \hat{L}_t + \hat{c}_t$ in log-linear terms. Under $\hat{z}_t = 0$, one recovers $\hat{y}_t = \hat{L}_t = \hat{c}_t$ such that the labor supply equation implies $\hat{y}_t = 0$. The aggregate IS equation then provides us with the associated real interest rate. On the contrary, if wages are sticky but the monetary authority replicates the real interest absent nominal rigidity, the equilibrium is characterized by

$$\begin{aligned} \hat{y}_t &= \zeta_f \mathbb{E}_t\left[\hat{y}_{t+1}\right] - \zeta_r\left(\hat{i}_t - \mathbb{E}_t\left[\hat{\pi}_{t+1}\right]\right) + \zeta_\tau \hat{\tau}_t^D - \zeta_{\tau'}\mathbb{E}_t\left[\hat{\tau}_{t+1}^D\right] , \\ \hat{\pi}_t &= \kappa\left(\varphi \hat{L}_t + \hat{c}_t\right) . \end{aligned}$$

As $\hat{i}_t - \mathbb{E}_t\left[\hat{\pi}_{t+1}\right] = \hat{r}_t^{real}|_{flex}$, for a given series of shocks $\{\hat{\tau}_t^D, \mathbb{E}_t\left[\hat{\tau}_{t+1}^D\right]\}$, the output series $\{\hat{y}_t, \mathbb{E}_t\left[\hat{y}_{t+1}\right]\}$ must be – due to the IS equation – equivalent to the one under flexible wages as well.

Statement (b) In the case of nominal wage rigidity, the equilibrium is summarized by

$$\begin{aligned} \hat{y}_t &= \zeta_f \mathbb{E}_t\left[\hat{y}_{t+1}\right] - \zeta_r\left(\hat{i}_t - \mathbb{E}_t\left[\hat{\pi}_{t+1}\right]\right) + \zeta_\tau \hat{\tau}_t^D - \zeta_{\tau'}\mathbb{E}_t\left[\hat{\tau}_{t+1}^D\right] , \\ \hat{\pi}_t &= \kappa\left(\varphi \hat{L}_t + \hat{c}_t\right) , \\ \hat{i}_t &= \phi_\pi \hat{\pi}_t , \end{aligned}$$

where we have directly used that $\hat{\pi}_t = \hat{\pi}_t^w$ applies. Notice that $(\zeta_f, \zeta_r, \zeta_\tau, \zeta_{\tau'})$ are defined under $s_c = 1$. Additionally, $\hat{y}_t = \hat{c}_t$ applies. Assuming an AR(1)-process on $\hat{\tau}_t^D$ with persistence $\rho_\tau \in (0,1)$, the output impact multiplier is given by

$$\mathcal{M}_y = \frac{\zeta_\tau - \rho_\tau \zeta_{\tau'}}{1 - \rho_\tau \zeta_f + \zeta_r \kappa_y (\phi_\pi - \rho_\tau)} ,$$

where $\kappa_y \equiv \kappa(1+\varphi)$. The sign of the denominator is positive if $\phi_\pi > \rho_\tau + (\rho_\tau \zeta_f - 1)/(\zeta_r \kappa_y)$, which is ensured due to $\rho_\tau \in (0,1)$ by local determinacy, i.e. $\phi_\pi > 1 + (\zeta_f - 1)/(\zeta_r \kappa_y)$. As a result, the sign of $\mathcal{M}_y$ is determined by the sign of the numerator. We obtain

$$\zeta_\tau - \rho_\tau \zeta_{\tau'} = \frac{\mathcal{E}_\tau}{\mathcal{E}_y}\left(1 - \rho_\tau \left[\tilde{s} - (1-\tilde{s})\frac{\chi_\tau}{\mathcal{E}_\tau}\right]\right) = \frac{\mathcal{E}_\tau}{\mathcal{E}_y}\left(1 - \rho_\tau \left[\tilde{s} - (1-\tilde{s})\frac{(1-\lambda)\Gamma}{\lambda}\right]\right) ,$$

where the second equality follows from substituting in for χ_τ and $\mathcal{E}_\tau$. As a result, we have

$$\zeta_\tau - \rho_\tau \zeta_{\tau'} = \frac{\mathcal{E}_\tau}{\mathcal{E}_y}\left(1 - \rho_\tau \tilde{s} + \rho_\tau (1-\tilde{s})\frac{(1-\lambda)\Gamma}{\lambda}\right) > 0 ,$$

where the positive sign follows as $\mathcal{E}_y > 0$ and $\mathcal{E}_\tau > 0$ if $\tau^D > 0$. Consequently, $\mathcal{M}_y > 0$ holds true.

PART II: PROGRESSIVE REDISTRIBUTION UNDER ENDOGENOUS GROWTH

Statement (c) Under flexible prices the equilibrium is summarized by

$$\begin{aligned}
\hat{y}_t &= \zeta_f \mathbb{E}_t[\hat{y}_{t+1}] - \zeta_r\left(\hat{i}_t - \mathbb{E}_t[\hat{\pi}_{t+1}]\right) + \zeta_g \hat{g}_{t+1}^A - \zeta_{g'}\mathbb{E}_t\left[\hat{g}_{t+2}^A\right] + \zeta_\tau \hat{\tau}_t^D - \zeta_{\tau'}\mathbb{E}_t\left[\hat{\tau}_{t+1}^D\right] , \\
\hat{g}_{t+1}^A &= \frac{\mathcal{E}_y}{1+\mathcal{E}_g}\hat{y}_t + \frac{\mathscr{M} - \mathcal{E}_y}{1+\mathcal{E}_g}\mathbb{E}_t[\hat{y}_{t+1}] + \frac{\mathcal{E}_g}{1+\mathcal{E}_g}\mathbb{E}_t\left[\hat{g}_{t+2}^A\right] - \frac{\mathcal{E}_\tau}{1+\mathcal{E}_g}\hat{\tau}_t^D + \frac{\mathcal{E}_\tau - \mathscr{M}_\tau}{1+\mathcal{E}_g}\mathbb{E}_t\left[\hat{\tau}_{t+1}^D\right] , \\
\hat{y}_t &= s_c \hat{c}_t + (1-s_c)\frac{g^A}{g^A - 1}\hat{g}_{t+1}^A , \\
0 &= \varphi \hat{L}_t + \hat{c}_t .
\end{aligned}$$

Together with $\hat{y}_t = \hat{L}_t$, the output and endogenous technology multiplier are characterized by

$$\begin{aligned}
\mathcal{M}_y &= \frac{1-s_c}{1+\varphi s_c}\frac{g^A}{g^A-1}\mathcal{M}_g , \\
\mathcal{M}_g &= \frac{\mathcal{E}_y(1-\rho_\tau) + \rho_\tau \mathscr{M}}{1+\mathcal{E}_g(1-\rho_\tau)}\mathcal{M}_y - \frac{\mathcal{E}_\tau(1-\rho_\tau) + \rho_\tau \mathscr{M}_\tau}{1+\mathcal{E}_g(1-\rho_\tau)} .
\end{aligned}$$

The first equation implies $\text{sgn}\mathcal{M}_y = \text{sgn}\mathcal{M}_g$. Substituting the first equation into the latter gives

$$\left(1 - \frac{1-s_c}{1+\varphi s_c}\frac{g^A}{g^A-1}\frac{\mathcal{E}_y(1-\rho_\tau) + \rho_\tau \mathscr{M}}{1+\mathcal{E}_g(1-\rho_\tau)}\right)\mathcal{M}_g = -\frac{\mathcal{E}_\tau(1-\rho_\tau) + \rho_\tau \mathscr{M}_\tau}{1+\mathcal{E}_g(1-\rho_\tau)} .$$

As $\tau^D > 0$ holds, the sign of the right hand side is negative such that the sign of $\mathcal{M}_g$ is determined by its pre-multiplying expression. As a result, sgn$\mathcal{M}_g > 0$ if

$$\begin{aligned} (1+\varphi s_c)\left(g^A-1\right)\left(1+\mathcal{E}_g(1-\rho_\tau)\right) &> (1-s_c)g^A\left(\mathcal{E}_y(1-\rho_\tau)+\rho_\tau\mathscr{M}\right) \;, \\ \Leftrightarrow \frac{(1+\varphi s_c)\left(g^A-1\right)}{(1-s_c)g^A} &> \frac{\mathcal{E}_y(1-\rho_\tau)+\rho_\tau\mathscr{M}}{1+\mathcal{E}_g(1-\rho_\tau)} \;. \end{aligned} \tag{21}$$

The derivative of the right hand side with respect to ρ_τ is given by

$$\frac{\partial\left(\frac{\mathcal{E}_y(1-\rho_\tau)+\rho_\tau\mathscr{M}}{1+\mathcal{E}_g(1-\rho_\tau)}\right)}{\partial\rho_\tau} = \frac{\mathscr{M}(1+\mathcal{E}_g)-\mathcal{E}_y}{\left(1+\mathcal{E}_g(1-\rho_\tau)\right)^2} \;.$$

There are two subcases. First, in the case $\mathscr{M}(1+\mathcal{E}_g)-\mathcal{E}_y \geq 0$, equation (21) holds on $\rho_\tau \in (0,1)$ if

$$\frac{(1+\varphi s_c)\left(g^A-1\right)}{(1-s_c)g^A} > \mathscr{M} \quad \Leftrightarrow \quad \frac{1+\varphi s_c}{1-s_c}\left(g^A-1\right) > g^A - \beta s(1-\delta) \;.$$

The previous equality can be rearranged to

$$g^A > 1 + \frac{1-s_c}{s_c}\frac{1-\beta s(1-\delta)}{1+\varphi} \;,$$

which corresponds to the condition stated in the main text. Second, in the case $\mathscr{M}(1+\mathcal{E}_g)-\mathcal{E}_y <$ 0, equation (21) holds on $\rho_\tau \in (0,1)$ if

$$\frac{(1+\varphi s_c)\left(g^A-1\right)}{(1-s_c)g^A} > \frac{\mathcal{E}_y}{1+\mathcal{E}_g} = \frac{(1-\lambda)\Gamma+\lambda-\lambda s_c\chi}{(1-\lambda)\Gamma\left(1-\frac{s_c}{g^A}\right)+\lambda(1-s_c)}\frac{g^A-1}{g^A} \;.$$

After some tedious algebra, the condition can be rewritten as

$$(1-\lambda)\Gamma s_c\left(1-\frac{1}{g^A}\right)+(1-\lambda)\Gamma\left(1-\frac{s_c}{g^A}\right)\varphi s_c+\lambda s_c(1-s_c)\varphi > -\lambda s_c(1-s_c)\chi \;,$$

which applies under the sufficient condition $\chi \geq 0$.

Statement (d) Under the full HANK-GS economy, the equilibrium is summarized by

$$\begin{aligned} \hat{y}_t &= \zeta_f\mathbb{E}_t\left[\hat{y}_{t+1}\right]-\zeta_r\left(\hat{i}_t-\mathbb{E}_t\left[\hat{\pi}_{t+1}\right]\right)+\zeta_g\hat{g}^A_{t+1}-\zeta_{g'}\mathbb{E}_t\left[\hat{g}^A_{t+2}\right]+\zeta_\tau\hat{\tau}^D_t-\zeta_{\tau'}\mathbb{E}_t\left[\hat{\tau}^D_{t+1}\right] \;, \\ \hat{g}^A_{t+1} &= \frac{\mathcal{E}_y}{1+\mathcal{E}_g}\hat{y}_t+\frac{\mathscr{M}-\mathcal{E}_y}{1+\mathcal{E}_g}\mathbb{E}_t\left[\hat{y}_{t+1}\right]+\frac{\mathcal{E}_g}{1+\mathcal{E}_g}\mathbb{E}_t\left[\hat{g}^A_{t+2}\right]-\frac{\mathcal{E}_\tau}{1+\mathcal{E}_g}\mathbb{E}_t\left[\hat{\tau}^D_t\right]+\frac{\mathcal{E}_\tau-\mathscr{M}_\tau}{1+\mathcal{E}_g}\mathbb{E}_t\left[\hat{\tau}^D_{t+1}\right] \;, \\ \hat{\pi}^w_t &= \kappa_y\hat{y}_t-\kappa_g\hat{g}^A_{t+1} \;, \\ \hat{i}_t &= \phi_\pi\hat{\pi}^w_t \;, \end{aligned}$$

where price and wage inflation relate according to $\hat{\pi}_t = \hat{\pi}_t^w - \hat{g}_t^A$. As a result, the impact multiplier are written as

$$\left(1+\zeta_r\phi_\pi\kappa_y - \rho_\tau\left(\zeta_f+\zeta_r\kappa_y\right)\right)\mathcal{M}_y = \left(\zeta_r\phi_\pi\kappa_g+\zeta_g-\zeta_r-\rho_\tau\left(\zeta_r\kappa_g+\zeta_{g'}\right)\right)\mathcal{M}_g+\zeta_\tau-\rho_\tau\zeta_{\tau'} \;,$$
$$\mathcal{M}_g = \Omega_y\mathcal{M}_y - \Omega_\tau \;,$$

where we have defined the auxiliary parameters

$$\Omega_y \equiv \frac{\mathcal{E}_y(1-\rho_\tau)+\rho_\tau\mathscr{M}}{1+\mathcal{E}_g(1-\rho_\tau)} > 0\,, \qquad \Omega_\tau \equiv \frac{\mathcal{E}_\tau(1-\rho_\tau)+\rho_\tau\mathscr{M}_\tau}{1+\mathcal{E}_g(1-\rho_\tau)} > 0\,.$$

As a result, the output and technology multiplier become

$$\mathcal{M}_y = \frac{-\left(\zeta_r\phi_\pi\kappa_g+\zeta_g-\zeta_r-\rho_\tau\left(\zeta_r\kappa_g+\zeta_{g'}\right)\right)\Omega_\tau+\zeta_\tau-\rho_\tau\zeta_{\tau'}}{1+\zeta_r\phi_\pi\kappa_y-\rho_\tau\left(\zeta_f+\zeta_r\kappa_y\right)-\left(\zeta_r\phi_\pi\kappa_g+\zeta_g-\zeta_r-\rho_\tau\left(\zeta_r\kappa_g+\zeta_{g'}\right)\right)\Omega_y}\,,$$
$$\mathcal{M}_g = \frac{\Omega_y(\zeta_\tau-\rho_\tau\zeta_{\tau'})-\Omega_\tau\left(1+\zeta_r\phi_\pi\kappa_y-\rho_\tau(\zeta_f+\zeta_r\kappa_y)\right)}{1+\zeta_r\phi_\pi\kappa_y-\rho_\tau\left(\zeta_f+\zeta_r\kappa_y\right)-\left(\zeta_r\phi_\pi\kappa_g+\zeta_g-\zeta_r-\rho_\tau\left(\zeta_r\kappa_g+\zeta_{g'}\right)\right)\Omega_y}\,.$$

The denominator of $\mathcal{M}_y$ and $\mathcal{M}_g$ is strictly positive if ϕ_π is sufficiently large, ensured for instance through the Taylor principle. To see this, notice that

$$\zeta_r\phi_\pi\left(\kappa_y-\kappa_g\Omega_y\right) = \frac{\zeta_r\phi_\pi}{1+\mathcal{E}_g(1-\rho_\tau)}\left((1+\mathcal{E}_g(1-\rho_\tau))\kappa_y-(\mathcal{E}_y(1-\rho_\tau)+\rho_\tau\mathscr{M})\kappa_g\right)\,.$$

As a result, the sign of $\zeta_r\phi_\pi\left(\kappa_y-\kappa_g\Omega_y\right)$ is a linear function in ρ_τ. Because of $\lim_{\rho_\tau\to 0}\left(\kappa_y-\kappa_g\Omega_y\right) = \kappa_y-\frac{\mathcal{E}_y}{1+\mathcal{E}_g}\kappa_g > 0$ and $\lim_{\rho_\tau\to 1}\left(\kappa_y-\kappa_g\Omega_y\right) = \kappa_y-\mathscr{M}\kappa_g > 0$ such that $\kappa_y-\kappa_g\Omega_y > 0\ \forall\rho_\tau\in(0,1)$. The second positive sign follows as we consider an upward sloping AS regime, while the first one follows from

$$\kappa_y-\frac{\mathcal{E}_y}{1+\mathcal{E}_g}\kappa_g = \frac{\kappa}{s_c}\left(1+\varphi s_c-(1-s_c)\frac{g^A}{g^A-1}\frac{\mathcal{E}_y}{1+\mathcal{E}_g}\right)$$
$$= \frac{\kappa}{s_c}\left(1+\varphi s_c-(1-s_c)\frac{(1-\lambda)\Gamma+\lambda-\lambda s_c\chi}{(1-\lambda)\Gamma\left(1-\frac{s_c}{g^A}\right)+\lambda(1-s_c)}\right)\,,$$

where the term inside the brackets can be rewritten as

$$\varphi s_c+\frac{(1-\lambda)\Gamma\left(1-\frac{s_c}{g^A}\right)+\lambda(1-s_c)-(1-s_c)(1-\lambda)\Gamma-(1-s_c)\lambda+(1-s_c)\lambda s_c\chi}{(1-\lambda)\Gamma\left(1-\frac{s_c}{g^A}\right)+\lambda(1-s_c)}$$
$$= \varphi s_c+\frac{(1-\lambda)s_c\Gamma\left(1-\frac{1}{g^A}\right)+(1-s_c)\lambda s_c\chi}{(1-\lambda)\Gamma\left(1-\frac{s_c}{g^A}\right)+\lambda(1-s_c)} > 0\,.$$

As a result, a sufficiently high ϕ_π ensures that the denominators of $\mathcal{M}_y$ and $\mathcal{M}_g$ are positive. Consequently, the overall signs are determined by the sign of their numerators. Accordingly, the sign of $\mathcal{M}_y$ is positive if

$$\begin{aligned}\zeta_\tau - \rho_\tau \zeta_{\tau'} &> \left(\zeta_r \phi_\pi \kappa_g + \zeta_g - \zeta_r - \rho_\tau\left(\zeta_r \kappa_g + \zeta_{g'}\right)\right)\Omega_\tau \\ &= \frac{\mathcal{E}_\tau(1-\rho_\tau) + \rho_\tau \mathscr{M}_\tau}{1+\mathcal{E}_g(1-\rho_\tau)}\left(\frac{\kappa_g}{\mathcal{E}_y}(\phi_\pi - \rho_\tau) + (1-\rho_\tau \tilde{s})\frac{\mathcal{E}_g}{\mathcal{E}_y}\right) \ ,\end{aligned}$$

while the sign of $\mathcal{M}_g$ is positive if

$$\begin{aligned}\zeta_\tau - \rho_\tau \zeta_{\tau'} &> \left(1 + \zeta_r \phi_\pi \kappa_y - \rho_\tau(\zeta_f + \zeta_r \kappa_y)\right)\frac{\Omega_\tau}{\Omega_y} \\ &= \frac{\mathcal{E}_\tau(1-\rho_\tau) + \rho_\tau \mathscr{M}_\tau}{\mathcal{E}_y(1-\rho_\tau) + \rho_\tau \mathscr{M}}\left(1 + \zeta_r \phi_\pi \kappa_y - \rho_\tau(\zeta_f + \zeta_r \kappa_y)\right) \ .\end{aligned}$$

Case (d.1): No Persistence. In the case of $\rho_\tau = 0$, the condition on output reduces to

$$\frac{\mathcal{E}_\tau}{\mathcal{E}_y} > \frac{\mathcal{E}_\tau}{1+\mathcal{E}_g}\left(\frac{\phi_\pi \kappa_g + \mathcal{E}_g}{\mathcal{E}_y}\right) \quad \Leftrightarrow \quad \phi_\pi < \frac{1}{\kappa_g} \ ,$$

where the last inequality follows from $\mathcal{E}_\tau > 0$ under $\tau^D > 0$. Analogously, the condition on the sign of the technology multiplier reduces to

$$\frac{\mathcal{E}_\tau}{\mathcal{E}_y} > \frac{\mathcal{E}_\tau}{\mathcal{E}_y}\left(1 + \zeta_r \phi_\pi \kappa_y\right) \ ,$$

which is never satisfied for $\mathcal{E}_\tau > 0$. We thus have $\mathcal{M}_y > 0$ and $\mathcal{M}_g < 0$. □

Case (d.2): Arbitrary Persistence. Let us denote the threshold values of the previous conditions by $\mathcal{D}_y$, respectively $\mathcal{D}_g$. Additionally, notice that $\mathcal{D}_g > \mathcal{D}_y$ applies if

$$\left(1 + \zeta_r \phi_\pi \kappa_y - \rho_\tau(\zeta_f + \zeta_r \kappa_y)\right)\frac{1}{\Omega_y} > \left(\zeta_r \phi_\pi \kappa_g + \zeta_g - \zeta_r - \rho_\tau\left(\zeta_r \kappa_g + \zeta_{g'}\right)\right) \ ,$$

which holds as the denominators of $(\mathcal{M}_y, \mathcal{M}_g)$ are positive under the HANG-GS Taylor principle. Consider now the case in which $\mathcal{D}_g > \zeta_\tau - \rho_\tau \zeta_{\tau'} > \mathcal{D}_y$. In this case, we obtain $\mathcal{M}_y > 0$ and $\mathcal{M}_g < 0$. Using results from Appendix B.7, one can show that the difference in the level of output in comparison to a counterfactual unshocked economy can be written as

$$\ln Y_{t+T} - \ln Y_{t+T}^{NS} = \sum_{i=0}^{T-1} \rho_\tau^i \mathcal{M}_g + \rho_\tau^T \mathcal{M}_y \ .$$

Using the relation that $\sum_{i=0}^{T-1} \rho_\tau^i = \frac{1-\rho_\tau^T}{1-\rho_\tau}$, the threshold time span T^* is thus implicitly defined by

$$\frac{1-\rho_\tau^{T^*}}{1-\rho_\tau}\mathcal{M}_g + \rho_\tau^{T^*}\mathcal{M}_y = 0 \quad \Leftrightarrow \quad \rho_\tau^{T^*}\left((1-\rho_\tau)\mathcal{M}_y - \mathcal{M}_g\right) = -\mathcal{M}_g \,,$$

which can finally be rearranged to

$$\rho_\tau^{T^*} = \frac{1}{1-(1-\rho_\tau)\frac{\mathcal{M}_y}{\mathcal{M}_g}} \quad \Leftrightarrow \quad T^* = -\frac{\ln\left(1-(1-\rho_\tau)\frac{\mathcal{M}_y}{\mathcal{M}_g}\right)}{\ln(\rho_\tau)} > 0 \,.$$

B.7 Definition 1: Scars from Inequality

In Definition 1 we stated that scars from inequality can be simply written in terms of the difference of the productivity multiplier under HANK-GS relative to one obtained from a counterfactual RANK-GS economy. We formally derive this result below.

Case I: Transitory Shock. Let us denote the impact output multiplier by $\ln\left(\frac{y_t}{y}\right) = \mathcal{M}_y$ and the impact endogenous productivity multiplier by $\ln\left(\frac{g_{t+1}^A}{g^A}\right) = \mathcal{M}_g$. Consequently, we obtain

$$\begin{aligned}
\ln y_t &= \ln y + \mathcal{M}_y \,, \\
\ln Y_t &= \ln A_t + \ln y + \mathcal{M}_y \,, \\
\ln g_{t+1}^A &= \ln g^A + \mathcal{M}_g \,, \\
\ln A_{t+1} &= \ln A_t + \ln g^A + \mathcal{M}_g \,.
\end{aligned}$$

As a result, we obtain for a purely transitory shock occurring in period t

$$\ln Y_{t+1} = \ln A_{t+1} + \ln y = \ln A_t + \ln y + \ln g^A + \mathcal{M}_g = \ln Y_{t+1}^{NS} + \mathcal{M}_g \,,$$

where $\ln Y_{t+1}^{NS} = \ln A_{t+1} + \ln y = \ln A_t + \ln y + \ln g^A$ denotes the counterfactual output in period $t+1$ in the absence of a shock in period t. Thus, permanent output scars are given by

$$\ln Y_{t+1} - \ln Y_{t+1}^{NS} = \mathcal{M}_g \,.$$

Case II: Persistent Shock. Similar to above, one can write the output response $\ln\left(\frac{y_{t+T}}{y}\right) = \rho^T\mathcal{M}_y$, respectively the endogenous productivity response $\ln\left(\frac{g^A_{t+T+1}}{g^A}\right) = \rho^T \mathcal{M}_g$. As a result,

$$\begin{aligned}
\ln y_{t+T} &= \ln y + \rho^T \mathcal{M}_y \\
\ln Y_{t+T} &= \ln A_{t+T} + \ln y + \rho^T \mathcal{M}_y \\
\ln g^A_{t+T+1} &= \ln g^A + \rho^T \mathcal{M}_g \\
\ln A_{t+T+1} &= \ln A_{t+T} + \ln g^A + \rho^T \mathcal{M}_g
\end{aligned}$$

Consequently, we can rewrite

$$\begin{aligned}
\ln Y_{t+T} &= \ln A_{t+T} + \ln y + \rho^T \mathcal{M}_y \\
&= \ln A_{t+T-1} + \ln g^A + \rho^{T-1} \mathcal{M}_g + \ln y + \rho^T \mathcal{M}_y \\
&= \ln A_t + \sum_{i=0}^{T-1} \rho^i \mathcal{M}_g + T \ln g^A + \ln y + \rho^T \mathcal{M}_y
\end{aligned}$$

In the absence of shocks, we have $\ln\left(\frac{y^{NS}_{t+T}}{y}\right) = 0$ such that we can state

$$\ln y^{NS}_{t+T} = \ln y \Leftrightarrow \ln Y^{NS}_{t+T} = \ln A^{NS}_{t+T} + \ln y = \ln A_t + T \ln g^A + \ln y \; .$$

Using the former expression, we can rewrite

$$\begin{aligned}
\ln Y_{t+T} &= \ln A_t + \sum_{i=0}^{T-1} \rho^i \mathcal{M}_g + T \ln g^A + \ln y + \rho^T \mathcal{M}_y \\
&= \ln Y^{NS}_{t+T} + \sum_{i=0}^{T-1} \rho^i \mathcal{M}_g + \rho^T \mathcal{M}_y \; ,
\end{aligned}$$

such that the permanent output loss under a persistent shock with $|\rho| < 1$ is up to first order

$$\lim_{T\to\infty} \left(\ln Y_{t+T} - \ln Y^{\mathrm{NS}}_{t+T}\right) = \frac{\mathcal{M}_g}{1-\rho} \; .$$

Consequently, *scars from inequality* are computed by

$$-\frac{\mathcal{M}^{HA}_g - \mathcal{M}^{RA}_g}{1-\rho} \; .$$

B.8 Social Welfare and Optimal Balanced Growth Path

B.8.1 Social Welfare Criterion The *utilitarian* social welfare criterion is written as

$$\begin{aligned}
\mathcal{W} &= \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\lambda \mathcal{U}\left(C_t^H, L_t^H\right) + (1-\lambda)\mathcal{U}\left(C_t^S, L_t^S\right)\right] \\
&= \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\lambda\left(\ln C_t^H - \nu \frac{(L_t^H)^{1+\varphi}}{1+\varphi}\right) + (1-\lambda)\left(\ln C_t^S - \nu \frac{(L_t^S)^{1+\varphi}}{1+\varphi}\right)\right] \\
&= \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\lambda\left(\ln c_t^H - \nu \frac{(L_t^H)^{1+\varphi}}{1+\varphi}\right) + (1-\lambda)\left(\ln c_t^S - \nu \frac{(L_t^S)^{1+\varphi}}{1+\varphi}\right) + \ln A_t\right] .
\end{aligned}$$

Rewriting the discounted sum of endogenous productivity delivers

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left(\ln A_t\right) = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{\beta}{1-\beta} \ln g_{t+1}^A\right) + \frac{1}{1-\beta} \ln A_0 ,$$

such that the welfare criterion is given for some A_0 by

$$\mathcal{W} = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\lambda\left(\ln c_t^H - \nu \frac{(L_t^H)^{1+\varphi}}{1+\varphi}\right) + (1-\lambda)\left(\ln c_t^S - \nu \frac{(L_t^S)^{1+\varphi}}{1+\varphi}\right) + \frac{\beta}{1-\beta} \ln g_{t+1}^A\right] + \frac{1}{1-\beta} \ln A_0 .$$

The life-time welfare criterion $\mathcal{W}$ stated in terms of the per period welfare function $\mathcal{W}_t$ reads

$$\begin{aligned}
\mathcal{W} &= \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \mathcal{W}_t + \frac{1}{1-\beta} \ln A_0 , \\
\mathcal{W}_t &= \underbrace{\lambda\left(\ln c_t^H - \nu \frac{(L_t^H)^{1+\varphi}}{1+\varphi}\right)}_{\text{①: hand-to-mouth term } \mathcal{W}_t^H} + \underbrace{(1-\lambda)\left(\ln c_t^S - \nu \frac{(L_t^S)^{1+\varphi}}{1+\varphi}\right)}_{\text{②: saver term } \mathcal{W}_t^S} + \underbrace{\frac{\beta}{1-\beta} \ln g_{t+1}^A}_{\text{③: endogenous growth term } \mathcal{W}_t^{EG}}
\end{aligned}$$

B.8.2 Optimal Redistribution on the Balanced Growth Path In the absence of shocks, the economy is on the balanced growth path in all periods. As a result, the welfare objective becomes for a given initial condition A_0

$$\max_{\{\tau^D\}} \mathcal{W}(\tau^D) = \lambda \ln c^H + (1-\lambda) \ln c^S - \nu \frac{L^{1+\varphi}}{1+\varphi} + \frac{\beta}{1-\beta} \ln g^A .$$

The first order condition to this problem writes

$$\frac{\partial \mathcal{W}}{\partial \tau^D} = \frac{\lambda}{c^H} \frac{\partial c^H}{\partial \tau^D} + \frac{1-\lambda}{c^S} \frac{\partial c^S}{\partial \tau^D} + \frac{\beta}{1-\beta} \frac{1}{g^A} \frac{\partial g^A}{\partial \tau^D} = 0 ,$$

For what follows, let us define an auxiliary parameter. As a result, we can rewrite the steady state as follows

$$g^A = \beta\left((1-\tau^D)\psi\Theta_\alpha L + s(1-\delta)\right) \ , \quad \frac{\partial g^A}{\partial \tau^D} = -\beta\psi\Theta_\alpha L \ ,$$
$$c^H = \alpha^{-1}\Theta_\alpha L\left(1+\alpha\frac{\tau^D}{\lambda}\right) \ , \quad \frac{\partial c^H}{\partial \tau^D} = \frac{\Theta_\alpha L}{\lambda} \ ,$$
$$c^S = \alpha^{-1}\Theta_\alpha L\left(1+\alpha\frac{1-\tau^D}{1-\lambda}\right) - \frac{g^A-1}{(1-\lambda)\psi} \ , \quad \frac{\partial c^S}{\partial \tau^D} = -\frac{(1-\beta)\Theta_\alpha L}{1-\lambda} \ .$$

Lemma 2 (SOCIAL WELFARE FUNCTION BGP)**.** *The social welfare function is well-defined and strictly concave on the interval* $\tau \in \left(\underline{\tau}^D, \overline{\tau}^D\right)$.

(a) If $\psi = 0$ *such that* $g^A = 1$ *and* $\beta = 0$*, the bounds are given by*

$$\underline{\tau}^D = -\frac{\lambda}{\alpha} \ , \quad \overline{\tau}^D = 1 + \frac{1-\lambda}{\alpha} \ .$$

(b) If $\psi > 0$ *and* $\beta > 0$*, the corresponding bounds are given by*

$$\underline{\tau}^D = -\frac{\lambda}{\alpha} \ , \quad \overline{\tau}^D = 1 + \frac{s(1-\delta)}{\psi\Theta_\alpha L} \ .$$

Proof. Statement (a) follows immediately by ensuring that steady state consumption of both households is strictly positive. Regarding statement (b) notice that the SWF is now well-defined if $c^H > 0, c^S > 0, g^A > 0$ applies. The first condition is again satisfied if $\tau^D_{c^H} > -\frac{\lambda}{\alpha}$. Analogously, the condition on productivity growth is satisfied if $\tau^D_{g^A} < 1 + \frac{s(1-\delta)}{\psi\Theta_\alpha L}$. Finally, the condition on consumption of saver households is now satisfied if

$$c^S = \alpha^{-1}\Theta_\alpha L\left(1+\alpha\frac{1-\tau^D}{1-\lambda}\right) - \frac{g^A-1}{(1-\lambda)\psi} = \alpha^{-1}\Theta_\alpha L + \Theta_\alpha L(1-\beta)\frac{1-\tau^D}{1-\lambda} + \frac{1-\beta s(1-\delta)}{(1-\lambda)\psi} > 0$$
$$\Leftrightarrow \tau^D_{c^S} < 1 + \frac{1-\lambda}{\alpha(1-\beta)} + \frac{1-\beta s(1-\delta)}{(1-\beta)\psi\Theta_\alpha L} \ .$$

To complete the second statement, we show that $\tau^D_{g^A} < \tau^D_{c^S}$ holds. This is indeed the case as

$$\begin{aligned} 1 + \frac{s(1-\delta)}{\psi\Theta_\alpha L} &< 1 + \frac{1-\lambda}{\alpha(1-\beta)} + \frac{1-\beta s(1-\delta)}{(1-\beta)\psi\Theta_\alpha L} \\ s(1-\delta) &< \frac{1-\lambda}{\alpha(1-\beta)}\psi\Theta_\alpha L + \frac{1-\beta s(1-\delta)}{1-\beta} \\ 0 &< \frac{1-\lambda}{\alpha(1-\beta)}\psi\Theta_\alpha L + \frac{1-s+s\delta}{1-\beta} \end{aligned}$$

applies. Consequently, c^S is positive as long as g^A is positive. To conclude, the SWF is strictly concave as

$$\frac{\partial^2 \mathcal{W}}{\partial \tau^D \partial \tau^D} = -\frac{\lambda}{(c^H)^2}\left(\frac{\partial c^H}{\partial \tau^D}\right)^2 - \frac{1-\lambda}{(c^S)^2}\left(\frac{\partial c^S}{\partial \tau^D}\right)^2 - \frac{\beta}{1-\beta}\frac{1}{(g^A)^2}\left(\frac{\partial g^A}{\partial \tau^D}\right)^2 < 0\,,$$

where the previous equation holds as c^H, c^S and g^A are linear functions in τ^D such that their second derivatives become zero. □

Proposition 10 (OPTIMAL BGP REDISTRIBUTION)**.** *The following balanced growth path tax result applies in the absence of exogenous shocks:*

(a) If $\beta = 0$ and $\psi = 0$, then $\tau^{D,*} = \lambda$ such that $\Gamma = 1$.

(b) If $\beta > 0$ and $\psi > 0$, then $\tau^{D,*} \in \left(\underline{\tau}^D, \overline{\tau}^D\right)$ is chosen optimally such that $\Gamma > 1$, i.e. the social planner never establishes an egalitarian system. Moreover, there exists $\tilde{\beta}$ such that for all $\beta > \tilde{\beta} > \underline{\beta}$ endogenous productivity grows on the balanced growth path, i.e. $g^A > 1$.

Proof. The proof proceeds in two parts. In the first part, we show how optimal redistribution and balanced growth path inequality are linked. In the second part, we relate optimal redistribution to the sign of optimal endogenous productivity growth.

PART I: OPTIMAL REDISTRIBUTION AND BGP INEQUALITY.

The first order condition can be rewritten as

$$\frac{\lambda}{c^H}\frac{\partial c^H}{\partial \tau^D} + \frac{1-\lambda}{c^S}\frac{\partial c^S}{\partial \tau^D} + \frac{\beta}{1-\beta}\frac{1}{g^A}\frac{\partial g^A}{\partial \tau^D} = 0 \quad \Leftrightarrow \frac{1}{c^H} = \frac{1-\beta}{c^S} + \beta\frac{\beta}{1-\beta}\frac{\psi}{g^A}\,. \tag{22}$$

To show statement (a), for $\beta = 0$ and $\psi = 0$ the previous FOC implies that $c^H = c^S$. This is the case if $\frac{\tau^D}{\lambda} = \frac{1-\tau^D}{1-\lambda}$ which implies $\tau^D = \lambda$, i.e. the social planner establishes perfect insurance.

To show statement (b), notice that equation (22) characterizes implicitly an unique $\tau^{*,D}$ on $\left(\underline{\tau}^D, \overline{\tau}^D\right)$. This is the case, as the left hand side becomes larger than the right hand side for $\tau \to \underline{\tau}^D$ due to $c^H \to 0$ while c^S and g^A converge to a strictly positive number. Contrary, at the upper boundary $\tau \to \overline{\tau}^D$, the right hand side becomes larger than the left hand side due to $g^A \to 0$ while c^H and c^S converge to a positive number. As the second derivative of the social welfare function is strictly negative, there exists by the intermediate value theorem a unique solution $\tau^{*,D}$ that satisfies the first order condition. To show that an egalitarian society is never optimal, we rewrite the FOC as follows

$$\frac{1}{c^H(\tau^{*,D})} = \frac{1}{c^S(\tau^{*,D})} + \beta\left(-\frac{1}{c^S(\tau^{*,D})} + \frac{\beta}{1-\beta}\frac{\psi}{g^A(\tau^{*,D})}\right)\,.$$

Notice that the left hand side is strictly decreasing in τ^D, whereas the right hand side is strictly increasing in τ^D. Also, under $\psi > 0$ and $\beta > 0$ there exists a tax value $\tau^{EQ,D}$ such that $c^H = c^S$ applies. Thus, the proof for $\Gamma > 1$ is established by showing that the second term on the right hand side is strictly positive for $\tau^{EQ,D}$ which implies $\tau^{*,D} < \tau^{EQ,D}$.

We thus proceed in two steps. First, we derive explicitly $\tau^{EQ,D}$. Second, we verify that

$$-\frac{1}{c^S(\tau^{EQ,D})} + \frac{\beta}{1-\beta}\frac{\psi}{g^A(\tau^{EQ,D})} > 0\,.$$

To begin with, $\tau^{EQ,D}$ is obtained by equating consumption of hand-to-mouth and saver households, i.e.

$$\begin{aligned}
\alpha^{-1}\Theta_\alpha L\left(1+\alpha\frac{\tau^{EQ,D}}{\lambda}\right) &= \alpha^{-1}\Theta_\alpha L\left(1+\alpha(1-\beta)\frac{1-\tau^{EQ,D}}{1-\lambda}\right) + \frac{1-\beta s(1-\delta)}{(1-\lambda)\psi} \\
\Leftrightarrow \tau^{EQ,D}\Theta_\alpha L\left(\frac{1}{\lambda}+\frac{1-\beta}{1-\lambda}\right) &= \Theta_\alpha L\frac{1-\beta}{1-\lambda} + \frac{1-\beta s(1-\delta)}{(1-\lambda)\psi} \\
\Leftrightarrow \tau^{EQ,D}\Theta_\alpha L\frac{1-\beta\lambda}{\lambda(1-\lambda)} &= \Theta_\alpha L\frac{1-\beta}{1-\lambda} + \frac{1-\beta s(1-\delta)}{(1-\lambda)\psi} \\
\Leftrightarrow \tau^{EQ,D} &= \lambda\frac{1-\beta}{1-\beta\lambda} + \lambda\frac{1-\beta s(1-\delta)}{1-\beta\lambda}\frac{1}{\psi\Theta_\alpha L} > 0\,.
\end{aligned}$$

Accordingly, we get

$$1-\tau^{EQ,D} = \frac{1-\lambda}{1-\beta\lambda} - \lambda\frac{1-\beta s(1-\delta)}{1-\beta\lambda}\frac{1}{\psi\Theta_\alpha L}\,.$$

There arise now two cases. In the first one, $\tau^{EQ,D} \geq \overline{\tau}^D$ such that $\tau^{*,D} < \tau^{EQ,D}$ applies by construction. This case is the relevant one if the following inequality holds

$$\begin{aligned}
\lambda\frac{1-\beta}{1-\beta\lambda} + \lambda\frac{1-\beta s(1-\delta)}{1-\beta\lambda}\frac{1}{\psi\Theta_\alpha L} &\geq 1 + \frac{s(1-\delta)}{\psi\Theta_\alpha L} \\
\Leftrightarrow -\frac{1-\lambda}{1-\beta\lambda} + \frac{1}{\psi\Theta_\alpha L(1-\beta\lambda)}\big(\lambda - \beta\lambda s(1-\delta) - (1-\beta\lambda)s(1-\delta)\big) &\geq 0 \\
\Leftrightarrow -\frac{1-\lambda}{1-\beta\lambda} - \frac{s-\delta-\lambda}{\psi\Theta_\alpha L(1-\beta\lambda)} &\geq 0 \\
\Leftrightarrow -\frac{1}{\psi\Theta_\alpha L(1-\beta\lambda)}\big(\psi\Theta_\alpha L(1-\lambda) + s - \delta - \lambda\big) &\geq 0\,,
\end{aligned}$$

which is the case if $\delta \geq \overline{\delta} \equiv s - \lambda + \psi\Theta_\alpha L(1-\lambda)$. In the second case, i.e. $\delta < \overline{\delta}$, we have $\tau^{EQ,D} < \overline{\tau}^D$ such that $c^S(\tau^{EQ,D}) > 0, g^A(\tau^{EQ,D}) > 0$ and thus need to verify the inequality

$$-\frac{1}{c^S(\tau^{EQ,D})} + \frac{\beta}{1-\beta}\frac{\psi}{g^A(\tau^{EQ,D})} > 0 \quad \Leftrightarrow \quad c^S(\tau^{EQ,D}) > \frac{1-\beta}{\beta}\frac{g^A(\tau^{EQ,D})}{\psi}\,.$$

Substituting in for $\tau^{EQ,D}$ we obtain

$$\alpha^{-1}\Theta_\alpha L+\Theta_\alpha L\frac{1-\beta}{1-\lambda}\left[\frac{1-\lambda}{1-\beta\lambda}-\lambda\frac{1-\beta s(1-\delta)}{1-\beta\lambda}\frac{1}{\psi\Theta_\alpha L}\right]+\frac{1-\beta s(1-\delta)}{(1-\lambda)\psi}>$$
$$\frac{1-\beta}{\beta}\frac{\beta}{\psi}\left(\psi\Theta_\alpha L\left[\frac{1-\lambda}{1-\beta\lambda}-\lambda\frac{1-\beta s(1-\delta)}{1-\beta\lambda}\frac{1}{\psi\Theta_\alpha L}\right]+s(1-\delta)\right)$$
$$\Leftrightarrow\alpha^{-1}\Theta_\alpha L+\Theta_\alpha L\frac{1-\beta}{1-\beta\lambda}-\frac{\lambda(1-\beta)}{(1-\lambda)\psi}\frac{1-\beta s(1-\delta)}{1-\beta\lambda}+\frac{1-\beta s(1-\delta)}{(1-\lambda)\psi}>$$
$$\frac{(1-\beta)s(1-\delta)}{\psi}+\Theta_\alpha L\frac{(1-\beta)(1-\lambda)}{1-\beta\lambda}-\frac{\lambda(1-\beta)}{\psi}\frac{1-\beta s(1-\delta)}{1-\beta\lambda}\,.$$

Rearranging terms results in

$$\alpha^{-1}\Theta_\alpha L\left(1+\alpha\lambda\frac{1-\beta}{1-\beta\lambda}\right)-\frac{\lambda}{1-\lambda}\frac{\lambda(1-\beta)}{\psi}\frac{1-\beta s(1-\delta)}{1-\beta\lambda}+\frac{1-s\beta+s\beta\delta-s(1-\beta)(1-\delta)(1-\lambda)}{(1-\lambda)\psi}>0$$
$$\Leftrightarrow\alpha^{-1}\Theta_\alpha L\left(1+\alpha\lambda\frac{1-\beta}{1-\beta\lambda}\right)-\frac{\lambda}{1-\lambda}\frac{\lambda(1-\beta)}{\psi}\frac{1-\beta s(1-\delta)}{1-\beta\lambda}+\frac{s\lambda(1-\beta)(1-\delta)+1-s+s\delta}{(1-\lambda)\psi}>0$$
$$\Leftrightarrow\alpha^{-1}\Theta_\alpha L\left(1+\alpha\lambda\frac{1-\beta}{1-\beta\lambda}\right)-\frac{\lambda}{1-\lambda}\frac{\lambda(1-\beta)}{\psi}\frac{1-\beta s(1-\delta)}{1-\beta\lambda}+\frac{s\delta+1-s}{\psi}+\frac{\lambda(1-\beta s(1-\delta))}{(1-\lambda)\psi}>0$$
$$\Leftrightarrow\alpha^{-1}\Theta_\alpha L\left(1+\alpha\lambda\frac{1-\beta}{1-\beta\lambda}\right)+\frac{s\delta+1-s}{\psi}+\frac{\lambda(1-\beta s(1-\delta))}{(1-\lambda)\psi}\left(1-\frac{\lambda(1-\beta)}{1-\beta\lambda}\right)>0$$
$$\Leftrightarrow\alpha^{-1}\Theta_\alpha L\left(1+\alpha\lambda\frac{1-\beta}{1-\beta\lambda}\right)+\frac{s\delta+1-s}{\psi}+\frac{\lambda}{\psi}\frac{1-\beta s(1-\delta)}{1-\beta\lambda}>0\,,$$

where the last strict inequality obviously holds. As a result, we must have $\tau^{*,D}<\tau^{EQ,D}$, which completes the proof of the first statement.

PART II: SIGN ENDOGENOUS PRODUCTIVITY.

To determine the sign of productivity growth under the optimal redistribution schedule, we proceed in two steps. First, we determine the tax rate $\tau^{NG,D}$ under which there is no productivity growth, i.e. $g^A=1$. Second, we apply a similar argument as under Part I and derive a threshold value on the discount factor such that the right hand side of the FOC (22) is larger than the left hand side, implying that $\tau^{*,D}<\tau^{NG,D}$ and consequently $g^A>1$. To begin with, the tax value under which endogenous technology does not grow on the balanced growth path is defined by

$$g^A=\beta\left((1-\tau^{NG,D})\psi\Theta_a L+s(1-\delta)\right)=1\quad\Leftrightarrow\tau^{NG,D}=1-\frac{\beta^{-1}+s(\delta-1)}{\psi\Theta_a L}$$

Evidently, we have that $\tau^{NG,D}<\overline{\tau}^D$ such that $g^A>0$. Moreover, if $\tau^{*,D}<\tau^{NG,D}$ we have $g^A>1$, and vice versa, if $\tau^{*,D}>\tau^{NG,D}$ we have $g^A<1$. From Part I, we know that $\tau^{*,D}\in\left(\underline{\tau}^D,\overline{\tau}^D\right)$.

Hence, if $\tau^{NG,D} \leq \underline{\tau}^D$, we know that $g^A < 1$, as $\tau^{*,D} > \tau^{NG,D}$. This is indeed the case if

$$1 - \frac{\beta^{-1}+s(\delta-1)}{\psi\Theta_a L} \leq -\frac{\lambda}{\alpha} \;\Leftrightarrow\; \beta^{-1} \geq \psi\Theta_a L + s(1-\delta) + \lambda\psi\alpha^{-1}\Theta_a L \;\Leftrightarrow\; \beta \leq \left(\psi\Theta_a L\frac{\alpha+\lambda}{\alpha} + s(1-\delta)\right)^{-1} .$$

Hence, if $\beta \leq \underline{\beta} \equiv \left(\psi\Theta_a L\frac{\alpha+\lambda}{\alpha} + s(1-\delta)\right)^{-1}$ we have $g^A < 1$. Remember that the first order condition is given by

$$\frac{1}{c^H} = \frac{1-\beta}{c^S} + \beta\frac{\beta}{1-\beta}\frac{\psi}{g^A} .$$

As a result, if the inequality

$$\frac{1}{c^H(\tau^{NG,D})} < \frac{1-\beta}{c^S(\tau^{NG,D})} + \beta\frac{\beta}{1-\beta}\frac{\psi}{g^A(\tau^{NG,D})}$$

applies, then $\tau^{*,D} < \tau^{NG,D}$ such that $g^A > 1$. Substituting in for $\tau^{NG,D}$ results in

$$\frac{1}{\alpha^{-1}\Theta_\alpha L\left(1+\frac{\alpha}{\lambda}\frac{\psi\Theta_\alpha L-\beta^{-1}+s(1-\delta)}{\psi\Theta_\alpha L}\right)} < \frac{1-\beta}{\alpha^{-1}\Theta_\alpha L\left(1+\frac{\alpha}{1-\lambda}\frac{\beta^{-1}+s(\delta-1)}{\psi\Theta_a L}\right)} + \frac{\beta^2\psi}{1-\beta} .$$

The left hand side of the previous equation is strictly positive if $\beta > \underline{\beta}$. Thus, if $\beta \to \underline{\beta}$ the previous inequality does not hold as the left hand side converges to infinity, whereas the right hand side converges to a positive finite number. Contrary, if $\beta \to 1$, then the right hand side converges to infinity wheres the left hand side converges to a positive number. As a result, there exists at least one β threshold and we denote by $\tilde{\beta}$ the largest one. □

C Quantitative Appendix

C.1 Derivation Wage NKPC

To ease comparability with the tractable model, we derive the NKPC by assuming that unions maximize the following representative household utility

$$\sum_{t=0}^{\infty}\left(\prod_{s=0}^{t}\widetilde{\beta}_s\right)\left[\left(u(C_t)-\nu(L_{kt})\right)-\frac{\theta}{2}\left(\frac{W_{kt}}{W_{t-1,k}}-g^A\right)^2\right] ,$$

where the demand for employment tasks is $L_{kt}=\left(\frac{W_{kt}}{W_t}\right)^{-\epsilon_w}L_t$ and $W_t=\left(\int W_{kt}^{1-\epsilon_w}dk\right)^{\frac{1}{1-\epsilon_w}}$ is the price index for aggregate employment services. Under the above specification, the Phillips curve is given by

$$\frac{\epsilon_w}{\theta}L_t\left(\nu'(L_t)-u'(C_t)w_t\frac{(\epsilon_w-1)}{\epsilon_w}\left(1-\frac{\partial\mathcal{T}(Y^d)}{\partial Y^d}\right)\right)=\pi_t^w(\pi_t^w-g^A)-\widetilde{\beta}\left[\pi_{t+1}^w(\pi_{t+1}^w-g^A)\right] .$$

If we further assume that the adjustments costs are given by $\frac{\theta}{2}\left(\frac{W_{kt}}{W_{t-1}}-g^A\right)^2$, we recover a static Phillips curve of the form

$$\frac{\epsilon_w}{\theta}L_t\left(\nu'(L_t)-u'(C_t)w_t\frac{\epsilon_w-1}{\epsilon_w}\left(1-\frac{\partial\mathcal{T}(Y^d)}{\partial Y^d}\right)\right)=\pi_t^w(\pi_t^w-g^A) .$$

In both cases, we the steady-state is characterized by

$$\overline{L}=\nu^{-1}\left(u'(\overline{C})w_t\frac{\epsilon_w-1}{\epsilon_w}\left(1-\frac{\partial\mathcal{T}(Y^d)}{\partial Y^d}\right)\right) .$$

In Online Appendix OA3.1, we also provide alternative NKPC specifications in which we additionally integrate a heterogeneous hours worked and unemployment incidence. The results turn out to be qualitatively similar.

C.2 Computational Algorithm

We detail the algorithm in the following consecutive steps.

1. Initialize a full dimension grid space over liquid asset values b, the productivity level h, the relative stock of innovations a, the discount rate β, and the employment status e.

2. Guess an initial vector of prices, quantities and growth rate $\{r_t, L_t, g_{t+1}^A\}$. Compute the implied wage rate w_t.

3. Given prices, solve the household's consumption-saving problem. To do so, we use a modified

version of the EGM algorithm introduced by Carroll (2006).

Moreover, the optimal innovative investment amount x satisfies the first order condition

$$\begin{aligned}
&\frac{\partial \mathcal{I}(a',b',\mathbf{s}',x)}{\partial x} = 0 \\
\Leftrightarrow \quad &\frac{\partial \mathcal{I}(a',b',\mathbf{s}',x)}{\partial x} = p_x'(x)\left[\mathbb{E}\mathcal{V}\left(a'+\iota, b'-\frac{\psi(x)}{g_{t+1}^A},\mathbf{s}'\right) - \mathbb{E}\mathcal{V}\left(a', b'-\frac{\psi(x)}{g_{t+1}^A},\mathbf{s}'\right)\right] \\
&+\left(-\frac{\partial\psi(x)}{\partial x}\right)\frac{1}{g_{t+1}^A}\left((1-p_x(x))\mathbb{E}\mathcal{V}_b\left(a', b'-\frac{\psi(x)}{g_{t+1}^A},\mathbf{s}'\right)+p_x(x)\mathbb{E}\mathcal{V}_b\left(a'+\iota, b'-\frac{\psi(x)}{g_{t+1}^A},\mathbf{s}'\right)\right)=0\,,
\end{aligned}$$

which we solve by using a standard one-dimensional Newton algorithm that relies on numerical derivatives.

4. Construct the transition matrix $\mathbf{M}$ generated by $\mathbb{P}_h$, $\mathbb{P}_\beta$, $\mathbb{P}_e$, and $a'(\mathbf{s})$, $b'(\mathbf{s})$, $x(\mathbf{s})$. Compute the associated stationary measure of individuals $\mathcal{G}(\mathbf{s})$ by first guessing an initial distribution, and then by iterating on $\mathcal{G}'(\mathbf{s}) = \mathbf{M}\mathcal{G}(\mathbf{s})$ until convergence.

5. Compute new prices, back to step 2 and iterate until convergence on the equilibrium prices is reached.

Transitional Dynamics The algorithm follows the standard MIT shock solution procedure. We first guess a sequence of prices $\{r_t, g_{t+1}^A\}$ and shocks and solve for the policy function backward by starting with a final guess of prices until period $t = 0$. Then, we solve forward for the distribution $\mathcal{G}_t$ starting at period $t = 0$ and using the steady-state stationary distribution as initial condition.

With a relaxation, we update the vector of prices and quantities using the aggregate model-implied conditions and the NKPC. The interest rate r_t^b is adjusted to clear the bond market. The growth rate follows from the law of motion of innovative investments. Along the transition, we also adjust taxes or government debt depending on which variable adjusts to balance the government budget constraint.

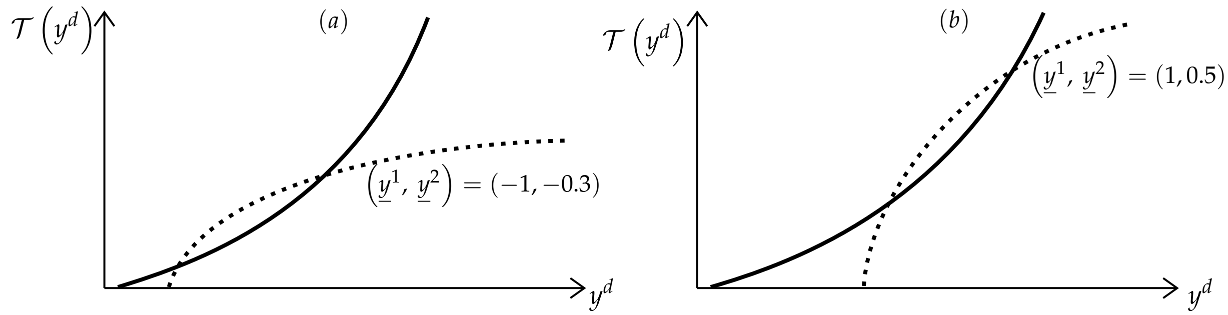
C.2.1 Government Budget Constraint: Debt Financing When the government issues debt to close its budget constraint, we assume that there is a jump in lump-sum transfers T_t in the initial period $t = 1$ of the transition. The transfers then smoothly decay at a rate $\rho_T \in (0, 1)$. Provided the model structure, this jump can be computed using the present value of the government budget constraint, such that

$$T_t = \frac{\frac{\overline{B}}{A}\left(g_{t+z}^A - (1+r_t)\prod_{s=1}^{z-1}\frac{(1+r_{t+s})}{g_{t+s}^A}\right) - \left(\sum_{j=0}^{z-2}\mathcal{P}_{t+j}^{def,T}\prod_{s=j+1}^{z-1}\frac{(1+r_{t+s})}{g_{t+s}^A} + \mathcal{P}_{t+z-1}^{def,T}\right)}{\left(\sum_{j=0}^{z-2}(\rho_T)^j\prod_{s=j+1}^{z-1}\frac{(1+r_{t+s})}{g_{t+s}^A} + \rho_T^{z-1}\right)}\,,$$

where $\mathcal{P}_t^{def,T} = \mathbb{T}_t^y + \mathbb{T}_t^D + \mathbb{T}_t^b - \frac{G_t}{A_t} - \mathbb{S}_t - r_t^b\frac{B_t^G}{A_t}$ is the primary deficit excluding transfers T_t.

C.2.2 Government Budget Constraint: Non-Monotonous Tax Incidence The functional form for the tax incidence used throughout the paper allows to target specific income groups. We illustrate this in Figure 11 for two pairs of values of the scale parameter $\underline{y}^1$ and the constant $\underline{y}^2$. For each $\underline{y}^1$, the parameter $\underline{y}^2$ endogenously adjusts to balance the government budget constraint. It is evident that a higher $\underline{y}^1$ monotonously shifts the tax incidence on richer income deciles.

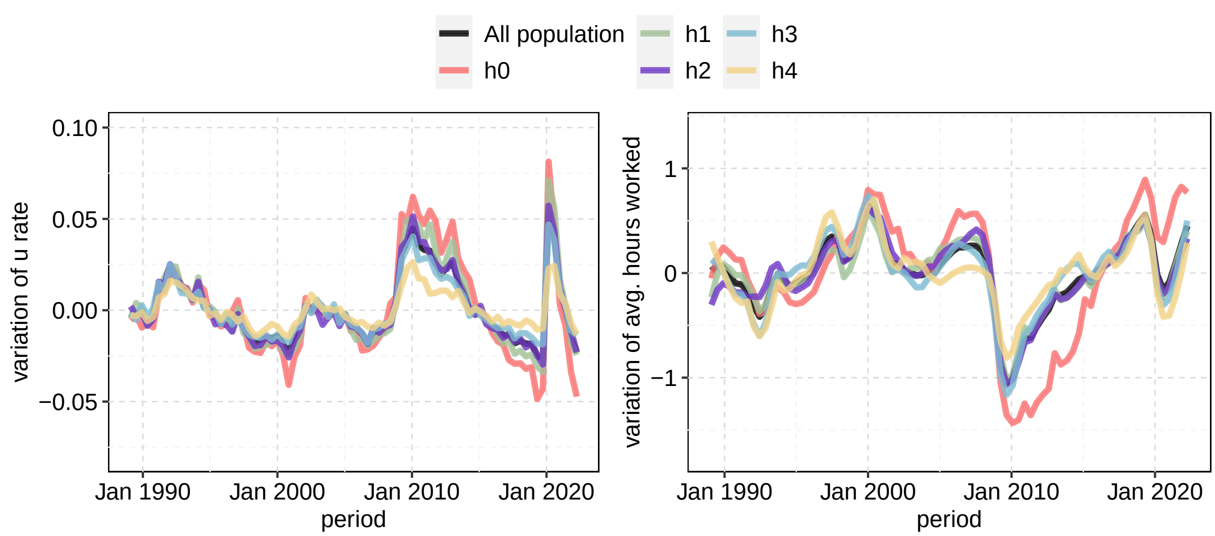
Figure 11. Tax schedule under heterogeneous tax incidence.



Legend: Figure (a) illustrates an incidence on the bottom-middle income groups. Figure (b) illustrates a tax incidence on top-middle income groups.

C.3 CPS Data - Additional Results

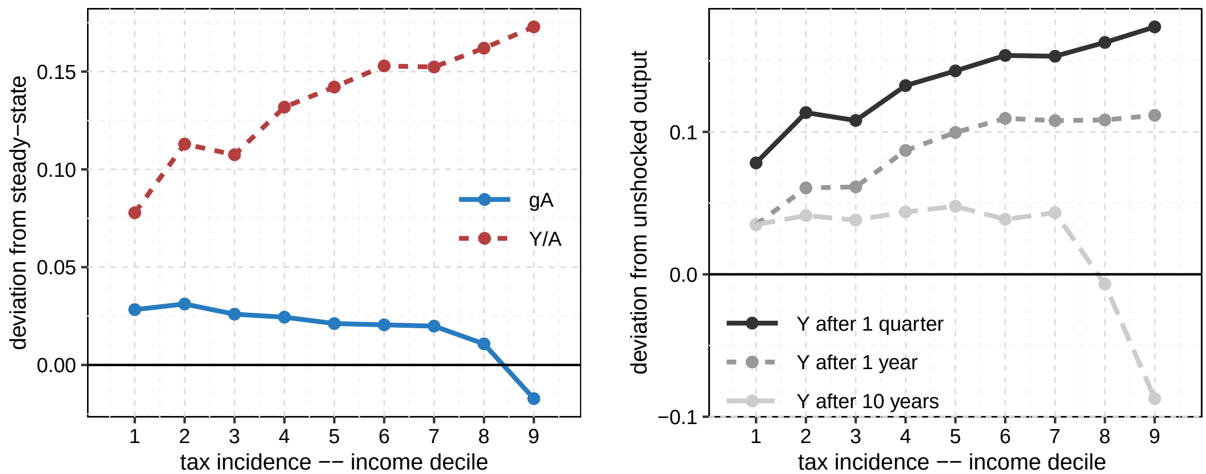
Figure 12. Variation of unemployment rate and hours worked within h-group



C.4 Multiplier as a Function of Income Tax Incidence

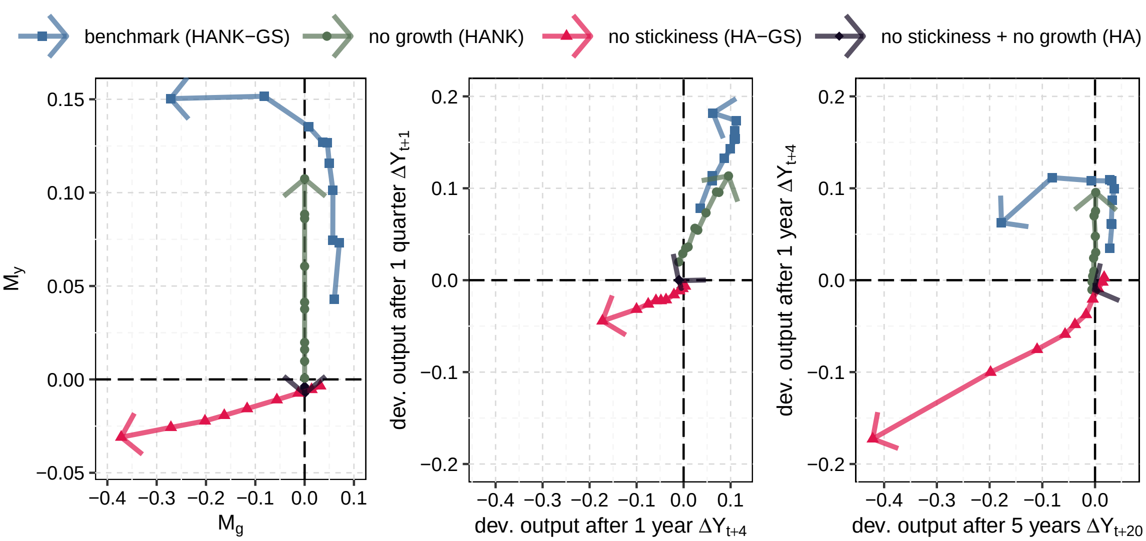
The left panel displays the annualized response of normalized output Y_t/A_t and technology growth g^A_{t+1} after the UI expansion in function of the income tax incidence. The right panel displays the implied output loss relative to the unshocked economy at different time horizon in function of the income tax incidence. The highest output level after ten years is achieved when taxing middle class households, i.e., the fifth income decile.

Figure 13. Multiplier as a function of tax incidence and resulting output deviation at different horizons.



C.5 Effects of Heterogeneous Tax Incidence Including Top 1% Income Tax

Figure 14. Heterogeneous tax incidence and the short- and long-run effects of a temporary UI extension.



Remark: This figure is similar to the one in the main paper. Dots still refer to successive income deciles, whereas the last dot represents now the tax incidence on the top 1% income group.