Welfare Implications of Information Technologies*

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Abstract

In this paper we highlight that information technologies are beneficial to face volatility, but in an economy with competitive and informational efficient markets, are also costly on increasing volatility and idiosyncratic risks. These risks can be shared when markets are complete, but if not, and to obtain more predictable market prices, agents may avoid investing in information intensive technologies, even if free and in spite of their benefits on improving production decisions. Alternatively, they may adopt information technologies and participate less on trading to diversify risk. We show there may be too much information and too little trading in equilibrium, as (i) agents do not internalize that by trading they provide assets that are useful for others to diversify their portfolio and (ii) this externality creates a wedge between individual and social evaluations of information that may lead to coordination failures.

Keywords: Social value of information, Incomplete markets, Risk sharing, Efficient market hypothesis, Safe assets, Data-intensive technologies.

JEL: D52, D53, D8, E21, E23, G12, G14.

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1 Introduction

The digital revolution of the last two decades facilitated enormously the production, processing and access to information. This possibility has pushed firms to increasingly adopt information technologies with the intention of promptly reacting to ongoing changes.¹. At the same time, financial markets became better equipped at predicting the value of firms and assets.² In spite of its ubiquitous availability and low acquisition costs, the adoption of information-intensive technologies is far from becoming universal, with large heterogeneity across firms and across countries. This fact has motivated public efforts to reduce the costs of access and production of data, under the presumption that this is the main hurdle preventing firms to adopting information technologies. It has also lead to strengthen transparency regulations supposing that firms that prefer opacity must be driven by illegal or market power purposes, hence detrimental to capital market allocation.

In this paper, we formalize the alternative idea that, even if beneficial for improving decisions and free to acquire, information may be individually and socially undesirable when financial markets are competitive but incomplete (in the sense of lack of contingent contracts). By producing information that may leak publicly or that leads to actions contingent on public information, agents feed the ability of markets to discriminate business activities. Thus, as financial markets have more elements to process the ex-ante uncertainty about actual valuations of business activities, ex-ante risks about those valuations increase. If markets were complete, such risk could be shared, not entering as a consideration for information production. When markets are incomplete, however, agents face a trade off, between the gains of information for improving decisions and its costs on increasing market risks. How agents weight these forces? Is adopting information technologies socially efficient?

To answer these questions, we design a general equilibrium model aimed at capturing the social and individual value of information in the presence of competitive, but incomplete markets, putting on equal footing the role of information to improve

¹See (Brynjolfsson and McElheran 2016) and (Brynjolfsson, Jin, and McElheran 2021).

²See (Bai, Philippon, and Savov 2016), (Farboodi et al. 2022) and (Davila and Parlatore 2022).

decisions and generate risk. In our model, (ex-ante) identical representative atomistic agents act in four roles: they are consumers and producers, but also buyers and sellers. We assume each agent faces an idiosyncratic shock to the productivity of an individually owned factor of production, which we denote as capital, that can be traded in a centralized competitive market. After trading, the agent can combine available capital with labor, through a standard Cobb-Douglas specification, to produce consumption goods.

Markets are incomplete in that production risks – i.e. the risk of having high or low final production – cannot be shared given the lack of contingent contracts. However, productivity risks can be reduced through two channels. First, trading capital so to diversify risk (market-insurance). Second, adjusting labor to reduce the consumption implications of capital productivity shocks (self-insurance). While market-insurance entails adjustments and trading costs, self-insurance is also costly in terms of labor disutility.

Information about capital productivity can be generated by adopting a technology that acquires, process or interpret data. However, agents decide on the degree of information that such technology can produce, knowing that part of it will unavoidably become public, for example, because they cannot completely hide production choices taken upon it. Hence, by choosing to produce more information for themselves as producers, agents also increase transparency, i.e. information available to others when trading its own capital. On the one side, information allows a more effective self-insurance response. If own capital is not very productive the agent can decide to either compensate by working more or cut back by working less, depending on whether substitution or income effects dominate, respectively. On the other side, as information partly spreads publicly, it comes at the cost of reducing scope for market-insurance. Competition in financial market creates incentives for traders to use all available information, and as a consequence market valuations fluctuate more closely with realized productivity when leakages are large. Hence, the value of an information-intensive technology depends on the extent to which information leaks publicly to financial markets. When public leakage is large, information allows self-insuring through labor choices but not market-insuring, as agents would endogenously trade little. When leakage is small, agents can both self-insure and market-insure.

We characterize information technology adoption and trading choices jointly, and we show that opacity and trading are complements: when agents produce more information they choose to trade less, which encourage them to produce more information. This creates the possibility that individual utility displays multiple local maxima, but still a unique equilibrium. We are however interested on the social consequences of these individual choices. We show there is a wedge between the individual and the social value of information because trading creates positive externalities. By selling capital, agents get rid of their own productivity risk, without transferring the risk to buyers, who in equilibrium fully diversify their portfolio by buying capital from all sellers. Thus, when selling capital agents not only benefit individually by getting rid of idiosyncratic risk, but also benefit the rest of agents by providing inputs for the creation of fully diversified – and so safe – assets. In other words, agents do not internalize that their capital contributes to a diversified portfolio (a safe asset) in fully decentralized markets.

This externality has two effects. First, since individual and social valuations of local maxima differ, there is room for coordination failures in which the equilibrium displays too much information from a social perspective. We derive a simple parametric condition in the case with full leakages that characterizes under what circumstances agents prefer to avoid information technologies when the net benefit of adjusting labor – the ratio of labor share to the Frisch elasticity – is small compared to the net benefit of trading capital – the ratio of capital share to the adjustment cost. We also derive a simple condition under which agents adopt information technologies which are socially undesirable. The second effect of the externality is the underprovision of market insurance, which could be fixed by a government, for instance by subsidizing safe assets trading, or by a financial intermediary, such as a competitive mutual fund owned by agents in shares proportional to their capital contributions. We also provide a simple analytic characterization of the constrained optimal allocation.

In this paper, we highlight that the efficient market hypothesis – traders can-

not make excess profits by investing because prices embed all available information about fundamentals – generate idiosyncratic risks. This is not an issue in a world of complete markets but under incompleteness, this risk has to be factored in when evaluating the welfare of information technologies that improve decision making but also market risk. We do so in a macro-finance general equilibrium model with endogenous information technology adoption and trading choices.

Related Literature: This paper provides a general equilibrium setting in which agents choose an information technology, trading strategies and resource allocations. The setting displays trading in competitive asset markets, but an absence of complete markets, formalizing a trade-off between two opposing faces of information.

The positive face of information is rooted in a long tradition in macroeconomics. In this tradition, the potentially negative social value of information may arise from the interaction of strategic complementarity and disperse information, as in the seminar works of Morris and Shin (2002) and Angeletos and Pavan (2007), but ultimately the socially optimal role of public information, when costless and perfect, is maintained.³ Other papers have found that dispersed information about technological shocks may have perverse welfare effects because of externalities in learning from prices (Amador and Weill (2010), Gaballo (2016)) or because of costly information acquisition (Colombo, Femminis, and Pavan (2014), Llosa and Venkateswaran (2017)). Closer to our multi-faceted analysis, Angeletos, Iovino, and La'O (2016) shows that information about non-distortionary forces, such as technological shocks, cannot be welfare detrimental, whereas information about distortionary forces, such as markups shocks, can indeed be socially inferior. In contrast to these works, in our setting consumption risk cannot be shared with contingent contracts, but can be traded with non-contingent ones, showing that information about technological shocks (non-distortionary forces) that is public (no need to learn from prices), costless (no-information cost) and complete (no dispersed signals) may still be detrimental to welfare by eroding market-insurance.

³The insights from this literature, however, have been shown to be limited in the context of fully micro-founded macro models without consumption risk (Hellwig (2005), Walsh (2007), Baeriswyl and Cornand (2010), Lorenzoni (2010), Roca (2010)).

The negative face of public information has gained more traction in finance, mainly in efforts to understand the relation between information and both the existence as well as the organization of financial intermediaries. Even though at its inception this literature focused on the beneficial role of information for reallocating resources and improving the quality of assets (such as the seminal papers of Leland and Pyle (1977), Bester (1985) and Diamond (1984 and 1991)) more recently it has highlighted information's detrimental effect on the value of liabilities (such as Gorton and Pennacchi (1990) and Dang et al. (2017)). This literature has been motivated by the original insights of Hirshleifer (1971)), who shows that information can prevent the ex-ante use of contingent contracts. Instead we assume contingent contracts cannot be used for reasons other than information, such as commitment or enforcing problems, and show how information is also detrimental in generating uninsurable risks when markets are competitive and informationally efficient.

There are, however, some recent notable attempts to accommodate information trade-offs in welfare analysis. Gottardi and Rahi (2014) combine the negative effect of information on insurance with the positive effect on portfolio optimization in the context of a two-period asset trading model, while Kurlat and Veldkamp (2015) explore the trade-off between risk and return that greater disclosure entails in the context of different types of assets. Our paper is close in spirit to these works, but we frame the trade-off in a general-equilibrium model with production. Eckwert and Zilcha (2001), also consider both production and risk-sharing in a two-period economy with heterogeneous agents (risk-averse consumers and risk-neutral producers); in contrast, we use standard assumptions in macroeconomics and finance, in particular a representative agent who performs all roles (consumer, producer, buyer, and seller). Our approach allows for a tractable characterization of the social value of information and provides a setting that can be easily compared to traditional models in macroeconomics and finance.

Our work highlights the importance of the insurance contracting environment in

⁴These contributions have been used for practical purposes, such as in arguments about the benefits of opacity on promoting liquidity in markets (Andolfatto, Berentsen, and Waller (2014), Chousakos, Gorton, and Ordoñez (2020)) or on improving government interventions (Nosal and Ordoñez (2016), Gorton and Ordoñez (2020a)).

an assessment of the social value of public information. A similar point has been made by Golosov and Iovino (2020), who show that, while full revelation of private information about employment possibilities is always desirable when governments can commit to social insurance, in general it is suboptimal without public commitment. Our setting shows instead that public information about idiosyncratic shocks is socially desirable when private markets are complete, but not necessarily when they are not - particularly when commitment is limited or enforcement is imperfect.

Finally, our work relates to the more recent literature that emphasizes studying origination and trading of assets in a single setting. Vanasco (2017) shows that information acquisition at origination deepens asymmetric information and may lead to a freeze in trading of assets with a collapse of liquidity. In our setting, lack of trading opportunities does not come from asymmetric information in decentralized secondary markets, but rather by common information in centralized secondary markets. Caramp (2017) also studies, but without focusing on information, the negative role of liquidity on the incentives to originate high quality assets. Our work strongly suggests that the positive face of information plays a more prominent role in the origination of assets, while the negative face is more relevant for trading assets.

The next section presents the model. Section 3 characterizes the market equilibrium and allocations. Section 4 computes the social planner's solution: both constrained to respect market compensations, as well as unconstrained to redistribute capital at will. Section 5 concludes.

2 Model

In this section we present a general equilibrium model of production with idiosyncratic risks. Markets are incomplete in that contingent contracts are unfeasible, preventing agents from sharing risks directly. However, imperfect insurance is possible through two channels: agents can *trade* risk by exchanging non-contingent assets in perfectly competitive centralized markets and *reduce* risk by individually adjusting production to anticipated shocks. We study the insurance trade-off that individuals face when choosing the transparency of their technology: while a transparent tech-

nology gives information for better choices to reduce risk, such information may leak to others, narrowing the scope for trading risk.

Preferences and production of consumption goods. There is a single period with a continuum of agents of mass one indexed by $i \in (0,1)$. Agent i has utility function

$$\mathbb{U}(C_i, L_i) \equiv \frac{C_i^{1-\sigma}}{1-\sigma} - \frac{1}{\gamma} L_i^{\gamma},\tag{1}$$

where C_i and L_i are consumption and labor specific to agent i, $\sigma > 0$ is a constant relative risk-aversion parameter and $\gamma > 1$ controls the convexity of labor disutility.

Each agent produces a quantity Y_i of consumption goods combining labor L_i and capital \hat{K}_i according to the following production function:

$$Y_i = L_i^{\alpha} \hat{K}_i^{1-\alpha}, \tag{2}$$

where $\alpha \in (0,1)$ is the labor share in production. In what follows we describe the production of capital that determines \hat{K}_i .

Production of capital The capital \hat{K}_i available to agent i is given by

$$\hat{K}_i = e^{k_i} \tag{3}$$

where k_i denotes a quantity of homogeneous intermediate capital that is obtained by transforming raw capital. Each agent is endowed with one unit of raw capital, and upon endowment, each agent chooses a technology that determines stochastically the idiosyncratic productivity of such unit, $\bar{\theta} + \theta_i \sim N(\bar{\theta}, 1)$, independently distributed across agents, with $\bar{\theta}$ assumed large enough to guarantee productivity is positive almost surely⁵. We describe later how the technological choice translates into the productivity draw and its informational implications.

Agent i can transform her own or others' raw capital into a quantity of interme-

⁵More precisely, such that $\Pr(\bar{\theta} + \theta_i < 0) \approx 0$.

diate capital, y, according to a linear technology:

$$y(\beta_i(j)) = (\overline{\theta} + \theta_j)\beta_i(j) - \frac{\varphi_{ij}}{2}\beta_i^2(j), \tag{4}$$

where $\beta_i(j) \in [0,1]$ is the mass of raw capital from agent $j \in (0,1)$ used by agent i. We define φ_{ih} to be the adjustment cost (an "iceberg cost") in terms of intermediate capital production that agent i incurs for using raw capital from agent j. Further, we assume adjustment costs are symmetric and only exist when agents produce with raw capital of others,

$$\varphi_{ij} = \begin{cases} \varphi > 0 & \text{if } j \neq i, \\ 0 & \text{if } j = i. \end{cases}$$

Every agent chooses the fraction $\beta_i(i)$ of her own raw capital (from here onwards simply β_i) to use in the production of intermediate capital, and sells the rest to other agents; inversely, agent i chooses how much raw capital to buy from other agents, this is $\beta_i(j)$ from all $j \neq i$.

Notice that while we model preferences and the production of consumption goods with standard functional forms (CRRA utility and a Cobb-Douglas technology) the assumption that production of capital follows an exponential function (equation 3) - increasing returns to scale with respect to raw capital - is not. As it will become clear, this assumption is extremely convenient for tractability, yet conservative in that it tends to favor the adoption of information technologies.

Competitive Market for Capital. Agents can exchange raw capital for non-contingent claims on intermediate capital. More specifically, raw capital is traded in a centralized Walrasian market according the following protocol: each agent $h \in H(i) \equiv (0,1)/\{i\}$ can sign a contract with agent i to buy raw capital at a unit price R_i , where the price R_i represents an enforceable claim on agent h's future production of intermediate capital.

An alternative of this contract is that agent h produces with raw capital from agent i in exchange of a repayment promise backed by agent h's subsequent production of intermediate capital. In this case, R_i is the price of raw capital – an asset

price. Another alternative is that agent h just produces with own raw capital and trade claims on the production of intermediate capital at a trading cost of φ . In this case R_i is the price of an intermediate capital firm's share – a stock price.

We will assume throughout that intermediate capital is the only pledgeable asset in the economy, and then these transactions cannot be written in terms of consumption goods. The implication of this assumption is that each agent consumes what produces, $C_i = Y_i$. This is useful to introduce consumption risk in a tractable way but it is not critical for the results as it does not entail any departure from complete markets. Indeed, we show later in section 4.3 that a social planner could implement the unconstrained first-best allocation by operating *contingent transfers* in intermediate capital only.

Technological choices and information implications. Each agent decides the technology that determines the productivity of the unit of endowed raw capital. To focus on the informational implications, we assume this technological choice does not affect the distribution of productivity, just the private and public information about its realization. To focus on the costs of information that arise from insurance considerations, we further assume that the technological choice is free, and does not convey any private cost to the agent. Formally we assume that the process for productivity follows:

$$\theta_i = \sqrt{a_i} \, \eta_{\,\mathrm{pp},i} + \sqrt{1 - a_i} \, \eta_{\,\mathrm{u},i}$$

with

$$\eta_{\,\mathrm{pp},i} = \sqrt{\frac{f(a_i)}{a_i}}\,\eta_{\,\mathrm{c},i} + \sqrt{1 - \frac{f(a_i)}{a_i}}\,\eta_{\,\mathrm{p},i}$$

which effectively leads to

$$\theta_i = \sqrt{f(a_i)} \, \eta_{c,i} + \sqrt{a_i - f(a_i)} \, \eta_{p,i} + \sqrt{1 - a_i} \, \eta_{u,i}$$
 (5)

where $\eta_{c,i}$ is the stochastic component of the productivity for agent i that is commonly known, $\eta_{p,i}$ is the productivity component that is privately known only by agent i, and $\eta_{u,i}$ is the component that is truly unknown. We assume $\eta_{\cdot,i} \sim N(0,1)$

are i.i.d. between them and across agents.

Agent i chooses $a_i \in (0,1)$, which is a direct measure of the fraction of unconditional volatility of θ_i that she is able to anticipate (accounting for $\eta_{c,i}$ and $\eta_{p,i}$). However, a_i also determines $f(a_i)$, which is the fraction of unconditional volatility of θ_i that agents other than i will be able to anticipate (accounting for $\eta_{c,i}$). In words, choosing the transparency a_i also determines the information leakage $f(a_i)$. We assume the following intuitive properties: $a_i \geq f(a_i)$, f(0) = 0 and $f'(a_i) > 0$.

To fix ideas, think about choosing a_i as choosing the fraction of raw capital that comes from a building that is transparent about the raw capital productivity, and the rest from a completely obscure building. A fraction $f(a_i)$ of raw capital that comes from the transparent building is observable to all agents, while the rest is only observable to agent i. The fraction of raw capital that comes from the opaque building is not observable to anybody. Then all raw capital is combined so there is no observation of which came from which building.

An alternative interpretation of the relation between a and f(a) comes from the inference of the market about productivity from actions observed by agents in response to produced information: the more information is produced, the higher the sensitivity of actions to productivity shocks and so the larger the information revealed through actions. The mapping between f(a) and a would be determined by noise in observing those actions, for instance.

Our setting spans the whole space between two extreme benchmarks. In the full-information benchmark all information is infinitely precise about raw capital productivity and public, this is available to all agents in the economy (i.e., $a_i = f(a_i) = 1$ for all i). In the no-information benchmark no one has any information about idiosyncratic productivities until consumption occurs (i.e., $a_i = 0$ for all i).

Two important implications are worth highlighting. First, regardless of the technological choice, θ_i is drawn from N(0,1). This assumption isolates the informational motive of technological choices, but it is possible to generalize it so that an information-intensive technology also affects the expectation and variance of productivity. Second, adopting any transparency level is costless in terms of firm's resources. This assumption can be easily relaxed, but allows us to focus on costs of adopting

information technologies that are external to the firm.

Market incompleteness. In our economy markets are incomplete as we assume restrictions on the ability of agents to write contingent contracts, i.e. enforceable agreements to make transfers contingent on the verifiable realizations of uncertain events. First, we assume enforceability only holds for contracts written after agents are endowed with raw capital, meaning that agents cannot write contracts under the veil of ignorance, i.e. based on assets for which they do not yet hold property rights. Second, we assume that verifiability obtains only with public information, meaning that contracts cannot condition on realizations of $\eta_{p,i}$ or $\eta_{u,i}$ for which there will not be common knowledge.

Timing and Equilibrium. The timing is a sequence of two stages:

- 1. in the ex-ante stage: agents choose technology $\{a_i\}_{i\in(0,1)}$ and the fraction of own raw capital to sell $\{1-\beta_i\}_{i\in(0,1)}$;
- 2. in the ex-post stage: productivity shocks realize $\Theta \equiv \{\eta_{c,i}, \eta_{p,i}, \eta_{u,i}\}_{i \in (0,1)}$, agents set their demand for raw capital $\{\{\beta_i(h)\}_{(h\neq i)}\}_{i\in(0,1)}$ and choose labor supply $\{L_i\}_{i\in(0,1)}$;

Finally, raw capital is exchanged, production of intermediate capital takes place, intermediate capital payments are made, production of the consumption good takes places, and agents consume their output. Given this sequence of events, a market equilibrium is defined as follows:

Definition 1 (Market Equilibrium). For given productivity realizations Θ , a market equilibrium is the cross-sectional allocation $\{\hat{K}_i(\Theta), C_i(\Theta)\}_{i \in (0,1)}$ determined by:

- $\{a_i\}_{i\in(0,1)}$ and $\{\beta_i\}_{i\in(0,1)}$ maximizing $E[\mathbb{U}(C_i,L_i)], \forall\, i;$
- $\{L_i\}_{i\in(0,1)}$ and $\{\{\beta_i(h)\}_{(h\neq i)}\}_{i\in(0,1)}$ maximizing $E_i[\mathbb{U}(C_i,L_i)], \forall i$.

where $E_i[\cdot] \equiv E[\cdot|\eta_{p,i}, \{\eta_{c,j}\}_{j\in(0,1)}]$ is expectation operator conditional to the information set of agent i.

Before moving on, it is useful to highlight two key features of our model. First, this is a general equilibrium setting: any agent in the economy is at the same time a buyer, a seller, a producer and a consumer. Agents only differ in the productivity of the endowed raw capital - they are otherwise ex-ante identical.

Second, our specification of market incompleteness allows a tractable model in which consumption risk cannot be insured away with contingent contracts, but can be managed with labor choices and trading choices. While the first do not induce externalities, the second are subject to general equilibrium forces, which will induce a failure to internalize the effects of trading decisions on consumption risk.

3 Market Equilibrium

In this section, we characterize the equilibrium working backward. First, we solve for an agent's ex-post optimal individual labor supply and raw capital demand, for a given informational technology and raw capital supply. Then, we work out the optimal ex-ante individual informational technology and supply choices.

3.1 Ex-post stage

Optimal Individual Labor Choice

The next Lemma shows the amount of labor that agent i chooses given her expected (conditional on available information) distribution of intermediate capital,

Lemma 1 (Labor Supply). Agent i supplies labor optimally according to

$$L_i = E_i [K_i]^{\frac{\phi}{\gamma}}. \tag{6}$$

with $K_i \equiv \alpha e^{k_i(1-\alpha)(1-\sigma)}$ and

$$\phi \equiv \frac{1}{1 - \frac{\alpha}{\gamma}(1 - \sigma)} \tag{7}$$

Proof. This results follows from maximizing expected equation (1) subject to (2). \Box

Note that labor is increasing in intermediate capital k_i when $\sigma < 1$, and decreasing when $\sigma > 1$. These comparative statics come from standard trade-offs between income and substitution effects. When $\sigma < 1$ a substitution effect dominates: as capital becomes abundant, labor is more productive and agents work more - the additional variance of consumption is not punished as heavily because risk aversion is relatively low. In contrast, when $\sigma > 1$ the income effect dominates: as capital becomes abundant agents work less because they are comparatively more sensitive to variance. When $\sigma = 1$, these two forces exactly offset each other and labor supply does not depend on the amount of intermediate capital.

The role of information on optimal labor choices is captured through the expectation operator. Without information agents can only choose labor based on expected capital, not on each possible realization, as could be done with full-information. This conveys a *positive face* of information: information allows for labor choices to better react to idiosyncratic shocks to capital.

Optimal Individual Demand of Raw Capital

The equilibrium per unit price of agent *i*'s raw capital, which we denote by R_i , is determined competitively from market clearing by equalizing the total demand from agents $h \neq i$ with the supply from agent *i*: $\int_{H(i)} \beta_h(i) dh = 1 - \beta_i$.

After selling a fraction β_i of her own raw capital at a price R_i , buying $\beta_i(h)$ raw capital from agents $h \in H(i)$ at prices R_h and covering adjustment costs $\frac{\varphi}{2}\beta_i^2(h)$, the amount of intermediate capital available to agent i to produce consumption goods is

$$k_i = (\bar{\theta} + \theta_i) \beta_i + (1 - \beta_i) R_i + \int_{H(i)} \Pi_i(h) dh, \tag{8}$$

where $\Pi_i(h)$ is agent i's profit from buying agent h's raw capital and given by

$$\Pi_i(h) = (\bar{\theta} + \theta_h) \beta_i(h) - \frac{\varphi}{2} \beta_i^2(h) - R_h \beta_i(h), \tag{9}$$

for any $h \in H(i)$. In words, an agent will operate with intermediate capital that

comes from three sources: that proceeding from i) transforming a fraction β_i of her own raw capital into intermediate capital with productivity θ_i , ii) selling a fraction $(1 - \beta_i)$ of own her raw capital to other agents in exchange for $(1 - \beta_i)R_i$ units of intermediate capital, and iii) buying raw capital $\beta_i(h)$ from other agents and obtaining a profit $\Pi_i(h)$, in terms of intermediate capital, after repayment. The objects R_i and $\Pi_i(h)$ are endogenous and depend on the availability of information through the formation of expectations as follows.

Lemma 2 (Demand of raw capital). Agent i's utility-maximizing demand of agent h's raw capital is

$$\beta_i^*(h) = \frac{\bar{\theta} + E_i[\theta_h] - R_h}{\varphi},\tag{10}$$

which also maximizes profits (9). In a symmetric equilibrium (this is $\beta_i = \beta$ for all i), market clearing implies that the price of agent h's raw capital satisfies

$$R_h = \bar{\theta} + E_i[\theta_h] - \varphi(1 - \beta), \tag{11}$$

Proof. Postponed to Appendix A.1.

This lemma shows that the optimal individual demand for raw capital equates expected marginal return, $\bar{\theta} + E_i[\theta_h]$, and marginal cost, $R_h + \varphi \beta_i(h)$, of operating with others' capital. It boils down to a linear schedule, decreasing in price and marginal adjustment costs and increasing in expected productivity. It is instructive to notice that demand only depends on the expected productivity of raw capital not on its conditional variance. The reason is that traders simultaneously demand a continuum of capital goods, each with an i.i.d. productivity shock, which allows them to achieve perfect diversification.

Because of market forces, perfect diversification is indeed the only possible equilibrium outcome when there is trade of raw capital (even if $\sigma < 1$ and agents like volatility of intermediate capital). Four features combine in our setting to obtain this convenient result: i) perfect competition among traders, ii) capital-specific, rather than portfolio-specific, adjustment costs φ , iii) CRRA utility in consumption (from

(2)), but constant absolute risk aversion (CARA) in portfolio returns (from (2) and (3) jointly), and iv) quadratic adjustment costs which allow asset investments to be "self-financed." The last two features ensure that h's demand of i's raw capital is independent from the expected returns of h's raw capital, i.e. from the only potential source of individual heterogeneity⁶; combined with capital-specific adjustment costs, this implies that agents have common asset valuations irrespectively of their differences as buyers. Perfect competition requires that the marginal benefit be equalized across all buyers for each type of raw capital, so that, in equilibrium, each buyer must absorb an equal (infinitesimal) amount of raw capital supply. Finally, since per unit adjustment costs are homogeneous across all raw capital types, the distribution of optimal individual demand across types within a portfolio must also be symmetric.

In short, as long as the supply of raw capital is uniform across sellers (which we prove in the next section), buyers perfectly diversify their portfolio as a result of perfect competition. Thus, portfolio profits are deterministic and known by agents under any information configuration, leading to the following result,

Corollary 1. Suppose supply of raw capital is uniform across agents other than i, i.e. $(1 - \beta_h) = (1 - \beta) \in (0, 1)$ for any $h \in H(i)$. Agent i's portfolio profits are deterministic,

$$\int_{H(i)} \Pi_i(h) dh = \frac{\varphi}{2} (1 - \beta)^2. \tag{12}$$

and agent i's quantity of intermediate capital available for production, from (8), is

$$k_i = \bar{\theta} + \beta_i \theta_i + (1 - \beta_i) E_h[\theta_i] - \varphi (1 - \beta_i)^2 + \frac{\varphi}{2} (1 - \beta)^2,$$
 (13)

which depends on both own agent i's supply $(1-\beta_i)$ and other agents' supply $(1-\beta)$.

Proof. Postponed to Appendix A.1.
$$\Box$$

The corollary leads to two important insights. First, even though information

⁶In particular, CARA with self-financing ensure respectively that the marginal utility of portfolio returns and the spending in others' capital does depend on one's own wealth, which in our model is determined by the price at which the agent can sell her own raw capital in the market and, ultimately, on the expected productivity of said capital.

affects the selling price of an agent's raw capital, it does not affect the portfolio profits from buying others' raw capital. By the law of large numbers, the sum of profits obtained from buying a basket of raw capital from other agents is deterministic, so its ex-ante and ex-post evaluations coincide. Thus, information availability is irrelevant to agents in their role as buyers. As we will see next, it is however relevant in their role as sellers. Second, the choices of an agent as a seller do not affect the profits that the agent obtains as a buyer. The reason is that buying others' capital is expected to be self-financed by the production of that purchase, and not constrained by own proceedings from selling. Expected profits are positive (this is, $E_i[\Pi_i(h)] = \frac{\varphi}{2}(1-\beta)^2 > 0$) because the purchase unit price is $\varphi(1-\beta)$ lower than expected productivity, while average unit adjustment cost is only $\frac{\varphi}{2}(1-\beta)$.

3.2 Ex-ante stage

Characterization of ex-ante individual utility.

For a given individual information set let $V(E_i[k_i])$ and $V_i(k_i)$ be the unconditional variance of the expected conditional mean and the conditional variance of k_i . By using the previous optimal decisions in the ex-post stage we can characterize the ex-ante individual utility as follows.

Lemma 3 (Ex-ante Individual Utility). Using (1),(2) and (6), given $\{a_i,\beta_i\}_{i\in(0,1)}$ and $f(\cdot)$, ex-ante utility is:

$$E[\mathbb{U}(E_i[K_i])] \equiv E\left[\frac{K_i E_i[K_i]^{\frac{\phi}{\gamma}\alpha(1-\sigma)}}{\alpha(1-\sigma)} - \frac{1}{\gamma} E_i[K_i]^{\phi}\right]$$
$$= \Phi e^{\phi(1-\alpha)^2(1-\sigma)^2\left(\frac{\phi}{2}V(E_i[k_i]) + \frac{1}{2}V_i(k_i)\right)} \bar{K}$$
(14)

with $\bar{K} \equiv \alpha e^{\phi(1-\alpha)(1-\sigma)E[k_i]}$ and

$$\Phi \equiv \frac{\gamma - \alpha(1 - \sigma)}{\gamma \alpha(1 - \sigma)},$$

which is negative for $\sigma > 1$. Using (5) and (13),

$$E[k_i] = \bar{\theta} - \varphi (1 - \beta_i)^2 + \frac{\varphi}{2} (1 - \beta)^2$$

$$V(E_i[k_i]) = \beta_i^2 (a_i - f(a_i)) + f(a_i),$$

$$V_i(k_i) = \beta_i^2 (1 - a_i),$$

Finally, by the law of total variance, the unconditional variance of k_i is

$$V(k_i) = V(E_i[k_i]) + V_i(k_i) = \beta_i^2 (1 - f(a_i)) + f(a_i).$$

Proof. We postpone the details of the derivation in Appendix A.2.

Our characterization allows for a more complete interpretation of the key parameter ϕ , from equation (7), which not only affects labor reactions but also ex-ante utilities through trading results. To provide a clear intuition, let's focus on the two extreme information benchmarks of no-information and full-information.

With no-information, this is when agents choose only an opaque technology $(a_i = 0 \text{ for all } i)$, $V(E_i[k_i]) = 0$. From equation (14) this implies $E[\mathbb{U}(E_i[K_i])] = \Phi E[K_i]^{\phi}$. In this case, ϕ measures the elasticity of ex-ante utility to expected capital $E[K_i]$. When the substitution effect dominates, i.e. $\sigma < 1$, ϕ is larger than one, meaning that an increase in expected capital is amplified in terms of ex-ante utility because agents react by working more. When the income effect dominates instead, i.e. $\sigma > 1$, ϕ is positive but smaller than one, meaning that an increase in expected capital productivity is weakened in terms of ex-ante utility because agents react by working less. Note that the elasticity ϕ not only depends on σ but also on α/γ – the labor elasticity of production relative to the labor elasticity of utility: the higher this ratio, the cheaper it is for agents to adjust labor, which magnifies these amplification/weakening effects of labor choices on ex-ante utility.

With full-information, when agents choose a fully transparent technology $(a_i = 1 \text{ for all } i)$, $V(k_i) = 0$, and $E[K_i] = K_i$. From equation (14) this implies $E[U(E_i[K_i])] = \Phi E[K_i^{\phi}]$. In this case, ϕ measures the elasticity of ex-post utility to capital K_i , i.e. same economic insights carry over, but instead of holding on average (in ex-ante

terms), they hold for each realization of capital (in ex-post terms).

If the unconditional variance of intermediate capital $V(k_i)$ were fixed and exogenous, information would always improve ex-ante utility by increasing $V(E_i[k_i])$, as it shrinks $V_i(k_i)$. In our framework, however, the individual information choice does affect the unconditional variance of $V(k_i)$. The individual technological choice a_i affects traders' information as it also determines $f(a_i)$. Hence information also increases price volatility, and so the unconditional volatility of capital to the extent the agent supplies raw capital. On the one hand, when supplying everything $(\beta_i = 0)$ all the volatility in intermediate capital comes from the volatility of the selling price, which depends on traders' information. On the other hand, if traders have perfect information $(f(a_i) = 1)$, the unconditional volatility of intermediate capital is the same as the one of productivity, independent of the supply choice. Therefore, the individual assessment of the benefits of information has to be reconsidered in light of the possibly different ex-ante distributions of k_i , which depend both on information and supply choices, and on leakage properties.

Individual information and supply of raw capital choices.

When agent i chooses how much raw capital to sell, she understands that part of the information she chooses to acquire will feed into the selling price. Thus, in contrast to the problem of agents as buyers, the optimal supply of raw capital does depend on the amount of information traders choose to have.

We now derive the first order conditions of two joint choices, supply of raw capital, $(1 - \beta_i)$, and technology, a_i . In terms of the supply of raw capital, it is optimal to increase β_i (this is, reduce the supply of raw capital) as long as

$$\phi(1-\alpha)^2(1-\sigma)^2\left(\phi\beta_i(a_i-f(a_i)) + \beta_i(1-a_i) + \frac{2\varphi(1-\beta_i)}{(1-\alpha)(1-\sigma)}\right)E[\mathbb{U}(E_i[K_i])] > 0.$$
 (15)

When $\sigma < 1$ (that is $\phi > 1$) this expression is positive and agents chose the corner solution, $\beta_i = 1$. When $\sigma > 1$, however, Φ switches sign and there is a non-trivial trade-off between decreasing the variance of intermediate capital (by selling more on

⁷This is evident when $\phi > 1$ (i.e. $\sigma \in (0,1)$); when instead $\phi \in (0,1)$ (i.e. $\sigma > 1$) note that $\Phi < 0$, so information generates a "less negative" expected utility.

the market) and decreasing average productivity (due to facing adjustment costs). In terms of technology, it is optimal to increase a_i as long as

$$\phi(1-\alpha)^2(1-\sigma)^2\frac{1}{2}\left(f'(a)\phi(1-\beta_i^2) + \beta_i^2(\phi-1)\right)E[\mathbb{U}(E_i[K_i])] > 0.$$
 (16)

Again, with $\sigma < 1$ (that is $\phi > 1$), the expression is always positive and then the solution is the corner $a_i = 1$. When $\sigma > 1$, however, Φ switches sign and there is a non-trivial trade-off between decreasing the variance of intermediate capital from trading (by choosing a more opaque technology) and from adjusting labor (by choosing a more transparent technology).

These two conditions yield the following characterization.

Lemma 4 (Technology and Supply of Raw Capital). The optimal $\beta_i^*(a_i)$ and $a_i^*(\beta_i)$ are given by

$$\beta_i^*(a_i) = \max \left\{ \min \left\{ 1, \frac{A}{A + 1 - a_i(1 - \phi) - \phi f(a_i)} \right\}, 0 \right\}$$
 (17)

$$a_i^*(\beta_i) = \max\left\{\min\left\{1, f_{-1}'\left(\frac{\beta_i^2 - \phi\beta_i^2}{\phi - \phi\beta_i^2}\right)\right\}, 0\right\}$$
(18)

where $A \equiv \frac{2\varphi}{(1-\alpha)(\sigma-1)}$ and $f'_{-1}(\cdot)$ being such that $f'_{-1}(f'(x)) = x$ for any $x \in \mathbb{R}$. For $\sigma > 1$ we have

$$\frac{E[\partial \mathbb{U}(E_i[K_i])]}{\partial \beta_i} > 0 \quad if \quad \beta_i < \beta_i^* \tag{19}$$

$$\frac{\partial E[\mathbb{U}(E_i[K_i])]}{\partial a_i} > 0 \quad if \quad \beta_i > \tilde{\beta}_i \equiv \left(\frac{f'(a_i)\phi}{1 - \phi(1 - f'(a_i))}\right)^{\frac{1}{2}}.$$
 (20)

Proof. It just comes directly from the first order conditions above, by restricting the possible range of both β_i and a_i to [0,1].

Now we have all the elements to characterize the equilibrium choices, both ex-post (labor and raw capital demand) and ex-ante (technology and raw capital supply).

Proposition 1 (Market Equilibrium). Labor supply is given by equation (6) in Lemma 1. Demand of raw capital is given by (10) in Lemma 2. Technology and

supply of raw capital are jointly determined by equations (17) and (18) in Lemma 4

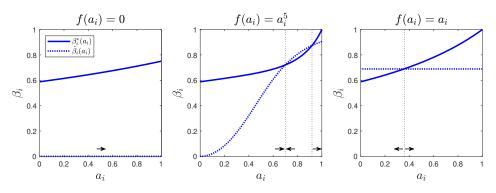
When $\sigma \leq 1$, $\phi \geq 1$ and equations (15) and (16) are strictly positive, which leads to the following Corollary.

Corollary 2. When $\sigma \leq 1$ the unique equilibrium is given by full transparency, $a_i^* = 1$, and no trading, $\beta_i^* = 1$

When $\sigma \leq 1$, agents are not so risk averse, then the variance of intermediate capital increases expected production of consumption goods (through the exponential transformation of k_i in \hat{K}_i) more than what it decreases expected utility of consumption. With information, agents can adjust labor (working more when there is more intermediate capital available, from Lemma 1). Further, since agents do not wish to decrease volatility of intermediate capital, market-insurance is not desired as it only implies wasting in adjusting costs that reduce expected consumption. These are the reasons agents prefer full information and no trading when $\sigma \leq 1$.

For the case $\sigma > 1$, however, agents dislike variance more that the effect on increasing expected consumption, so a trade-off is operational: information improves labor allocation but induces a larger variance of consumption. In figure 2 we illustrate how technology and supply choices are jointly determined in equilibrium. We illustrate the properties of conditions (17) and (18) with a numerical example and explore three possible specifications of information leakage: no leakage $(f(a_i) = 0)$ in the left panel), intermediate leakage $(f(a_i) = a_i^5)$ in the central panel and full leakage $(f(a_i) = a_i)$ in the right panel). The solid line shows the individual optimal β_i^* given the technology transparency, from (17). The positive slope represents that more transparency (an increase in a_i) induces the agent to sell less raw capital in the market (an increase in β_i^*). The dotted line shows the individual optimal a_i^* given how much agents want to sell, from (18). The weakly positive slope represents that, if agents choose to trade less (an increase in β_i), the agent would like to adopt a more transparent technology (an increase in a_i^*). When the solid line is above the dotted a marginal higher info acquisition is optimal. There is a candidate solution when these two lines intersect, or a corner solution with $a_i^* = 1$ ($a_i^* = 0$) if the solid line is above (below) the dotted line.

Figure 1: Optimal Information and Supply Choices



Plot of β_i^* (in solid) and $\tilde{\beta}_i$ (in dotted) and β^* (in dashed) as a function of a_i for different specification of f(a): f(a) = a on the left, $f(a) = a^5$ on the center, f(a) = 0 on the right. Other parameters are: $\sigma = 4.5, \gamma = 1.9, \alpha = 0.6, \varphi = 1$. The circle denotes the individual best reply, the square the socially preferred local maximum of the individual pay-off function, the diamond the constrained planner solution.

In the left panel there is no leakage and transparent technologies reveal productivity only to the owner. In this case, how much agents want to sell do not affect the level of transparency they want to choose, the curves do not intersect and the only equilibrium is a corner with $a_i^* = 1$. This is intuitive, as information can be used to optimize labor without inducing price volatility on markets. Notice that still the agent does not sell all the raw capital because of the trade-off between reducing consumption risk and accepting a price discount to compensate buyers for the adjustment costs φ , being the optimal supply $\beta_i^*(1)$.

In the central panel, leakages are partial and increase with transparency at an increasing rate. Here there are two intersections. The one at lower levels entail a local individual utility maximum, as agents increase β_i and a_i up to those levels. The highest intersection is instead a local minimum (the solid line cuts the dashed line from below). There is also a corner local maximum at $a_i^* = 1$.

In the right panel there is full leakage, as any information available to the agent is also available to traders. In this case there are two corner local maximum, one at $a_i^* = 0$ with only opaque technologies and one at $a_i^* = 1$ with only transparent technologies. Why these extremes can be local maximum? On the one hand, when

 $a_i^* = 0$ agents choose to sell a lot in the market and then it is optimal to choose an opaque technology to prevent buyers from learning too much. On the other hand, when $a_i^* = 1$ agents choose not to sell anything in the market and it is indeed optimal to choose a transparent technology to make better labor choices.

Notice that the best responses of information and trading from equations (17) and (18) do not contain the choice of other agents, β . Since both choices are made jointly by the individual, and do not depend on others' choices, there is a unique equilibrium in which agents select the global individual utility maximum, which in these three cases is the corner solution of full information. The trading choices of other agents, however, do enter in the utility of individuals, and in the symmetric equilibrium what constitutes an individual global maximum may not be a global maximum when evaluated socially.

In what follows, we characterize utilities, both from individual and social perspectives, and the solution of a social planner that is constrained by the same use of information and trading restrictions in markets that individuals face, but internalizes trading externalities. Given Corollary 2, in what follows we restrict attention to $\sigma > 1$, so agents value a lower variance of non-insurable shocks.

4 Social Evaluations and Social Planning

In this section, we define the problem of a social planner that maximizes the ex-ante expected utility of the representative consumer. First, we solve a *constrained social optimum*, in which the planner is constrained by the same trading restrictions that agents face, i.e. compensation implied by market prices. Even though the planner does not trade in a market, she has to respect the mapping between information and allocations imposed by the market, such that agents with raw capital of known higher productivity receive more intermediate capital in exchange. We show that the planner would like agents to supply more raw capital than in equilibrium, highlighting the nature of an externality in the provision of market-insurance. We then compare welfare of different equilibrium allocations vis-a-vis the welfare achievable with the constrained planner allocations.

Second, we solve an unconstrained social optimum, thus, replicating the allocation with complete markets - a situation in which a transparent technology is always socially desirable. We endow the planner with full-information about productivity shocks and allow her to implement contingent transfers in intermediate capital freely, without being subject to leakages and the allocations implied by market prices. We show that, while markets use information to allocate intermediate capital "regressively" (more intermediate capital to agents with raw capital of higher productivity), a planner would use information to allocate it "progressively" so as to equalize intermediate capital across agents (more intermediate capital to agents with raw capital of lower productivity). This is the deep sense in which efficient capital markets, by fully reflecting all relevant information that is available about the fundamental value of assets, implement allocations that go against valuable insurance.

4.1 Constrained Social Optimum

The constrained planner's problem is identical to the one individuals face in terms of choosing labor, information and the demand of raw capital. The difference comes from the supply of raw capital, as the planner does not take supply of other agents, β as given, and instead solve for the $\beta_i = \beta$ for all agents. Imposing this restriction in the expression of $E[k_i]$ in Lemma 3,

$$E[k_i] = \bar{\theta} - \frac{\varphi}{2}(1 - \beta)^2.$$

This relative small change, in which the constrained planner internalizes the effect of supply on the profits of other agents, makes the problem identical to that in Lemma (20), with the difference that equation (17) becomes

$$\beta^*(a) = \max \left\{ \min \left\{ 1, \frac{A_P}{A_P + 1 - a(1 - \phi) - \phi f(a)} \right\}, 0 \right\}$$
 (21)

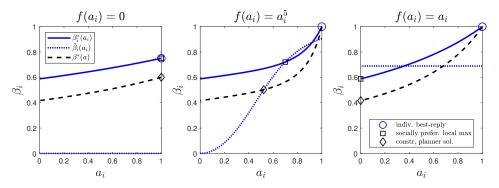
where a is the technology transparency that the planner chooses and

$$A_P = \frac{\varphi}{(1-\alpha)(\sigma-1)} < A \tag{22}$$

This implies that, compared to the decentralized equilibrium, $\beta^* \leq \beta_i^*$, and then the planner weakly prefers to use the market more intensively than individuals do in the decentralized equilibrium. In other words, the planner internalizes that marginally increasing the supply of raw capital of an agent increases the possibilities of diversification and improves insurance for other agents. Individual sellers are not compensated for these "pooling gains" by the market.

Figure 2 shows in dashed black line the optimal raw capital supply of the constrained planner, which is lower than the solid line (the supply choice of individuals) at all levels of transparency. Given that the planner values trading more than individuals, she is biased towards adopting less transparency in the social planning constrained optimal.

Figure 2: Optimal Information and Supply Choices



Plot of β_i^* (in solid) and $\tilde{\beta}_i$ (in dotted) and β^* (in dashed) as a function of a_i for different specification of f(a): f(a) = a on the left, $f(a) = a^5$ on the center, f(a) = 0 on the right. Other parameters are: $\sigma = 4.5, \gamma = 1.9, \alpha = 0.6, \varphi = 1$. The circle denotes the individual best reply, the square the socially preferred local maximum of the individual pay-off function, the diamond the constrained planner solution.

4.2 Welfare Comparisons

How do the individual and social evaluations of welfare differ? Is the equilibrium displaying more information always socially preferred? If not, what are the forces that determine the desirability of restricting the use of information? How does a social planner distorts trading? To make progress on these questions we compare analytically two corner allocations when there is full leakage. One in which agents operate a fully opaque technology (no-information benchmark, $a_i^* = f(a_i^*) = 0$) and one in which they operate a fully transparent technology (full-information benchmark, $a_i^* = f(a_i^*) = 1$). These extremes are useful to isolate the two faces of information: self-insurance vs. market-insurance.

Since the wedge between individual and social evaluations of welfare arise from trading, we show that there is no difference across evaluations in the fully transparent equilibrium, but there is a difference in the fully opaque one. We show that choosing a transparent technology that provides public, perfect and costless information is not always desirable from a social standpoint, and derive a simple condition that characterizes in which situations agents choose information when they are socially better choosing opacity.⁸ The social evaluation of the market equilibrium takes into account how agents' selling choices enter into allowing diversification for the buyers. The planner acts upon this social evaluation and proposes an allocation that displays more trading.

Proposition 2 (The two faces of information). Ex-ante utilities in the two benchmarks, evaluated at individual, social and planning standpoints, $m \in \{I, S, P\}$ respectively, are:

$$E[\mathbb{U}(E_i[K_i])] = \Phi \bar{K} E[e^{(1-\alpha)(1-\sigma)\theta_i}]^{\phi\omega}$$
(23)

where $\bar{K} = \alpha e^{\phi(1-\alpha)(1-\sigma)\bar{\theta}}$

- In the full-information benchmark, for $\sigma > 1$, $\omega = \phi$.
- In the no-information benchmark, for $\sigma > 1$, $\omega = \beta^m$.

⁸Our welfare criterion is based on each identical individual agent from an ex-ante perspective, not the representative agent which is normatively unrepresentative, as explained by Schlee (2001).

with

$$\beta^m = \frac{1}{1 - \frac{1 - \alpha}{\varphi} (1 - \sigma) \chi^m} \tag{24}$$

and the differences between individual, social and planning evaluations given by

$$\chi^{I} = \frac{1}{2};$$
 $\chi^{S} \equiv \frac{3\varphi + (1 - \alpha)(\sigma - 1)}{4\varphi + (1 - \alpha)(\sigma - 1)} \in (3/4, 1);$
 $\chi^{P} \equiv 1, \quad (25)$

Proof. Postponed to Appendix A.3.

This proposition helps contrasting, embodied in ω , the two opposite faces of information. Information induces self-insurance by allowing agents to optimally react to fluctuations of available capital, which is captured by an exponent ϕ from adjusting labor in response to realized productivity in addition to the single ϕ in (23) from adjusting labor in response to expected productivity. Information, however, prevents market-insurance as prices perfectly reflect realized productivity with a discount for adjustment costs. A transparent technology then discourages the use of raw capital markets and the possibility of insurance by diversification.

No-information allows market-insurance by creating scope for raw capital trade which effectively reduces the volatility of labor productivity; captured by β^m , which ameliorates the relative inferiority of an uncontingent labor response (again captured by ϕ). Naturally, the absence of information deters self-insurance, so diversification from the market comes at the cost of preventing contingent labor responses.

Note how β^m in (24) is the mirror image of ϕ in (7), and represents the fraction of own asset that agents do not trade and are exposed to. This analogy is instructive about the similar impact of market-insurance and self-insurance on expected utility. The trade-off that sellers face of lowering variance at a trading cost, which is typically studied in the finance literature, is essentially the same as the trade-off that households face when adjusting labor to reduce variance at a disutility labor cost, which is typically studied in the macro literature.

Proposition 3 (The individual, social and planning value of information). Assume f(a) = a. The allocation attained with full-information is inferior to the one attained

with no-information if and only if $\sigma > 1$ and $\beta^m < \phi$, that is

$$\frac{\alpha}{\gamma} < \frac{1 - \alpha}{\varphi} \chi^m. \tag{26}$$

Proof. With $\sigma > 1$ we have that $\Phi < 0$ (from Lemma 3) and $0 < \phi < 1$ (from equation 7). The proposition is a direct implication of comparing ω in (23).

This proposition can also be explained intuitively from comparing the two channels through which individuals can reduce the variance of consumption.

One channel is self-insurance. When agents know productivity realizations, the raw capital market does not provide insurance, but individuals can self-insure by allocating labor optimally. This reduction of variance is proportional to ϕ , which increases in α/γ . In words, self-insurance is more powerful to reduce variance when labor is more important in the production function (higher α) and when the Frisch elasticity (the elasticity of labor disutility to labor supply) is low such that it is less costly to adjust labor to compensate for lower stochastic productivity (lower γ).

The other channel is market-insurance. When individuals do not know productivity realizations, they cannot self-insure for the own unsold raw capital, but the market can provide insurance for the rest, at an adjustment cost φ . In the absence of information, agents get more market-insurance when $(1 - \alpha)/\varphi$ increases. In words, market-insurance is more powerful to reduce variance when capital is more important in the production function and when adjustment costs are smaller; the former increasing the utility cost of productivity fluctuations and the latter reducing the cost of both trade as well as working with others' capital.

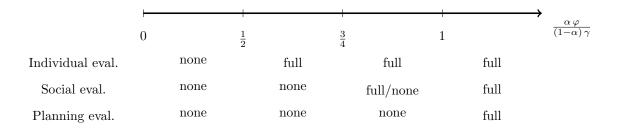
The benefit of market-insurance relative to self-insurance is adjusted by χ^m , which captures the difference in valuation between individuals, society and the social planner. Intuitively, agents do not internalize that, by selling raw capital they effectively increase the amount of raw capital that other agents can use to build safe assets, in the form of perfectly diversified portfolios, when acting as buyers. The planner adjusting allocations accordingly have then a larger valuation of the market insurance.

The implications of this result can be again seen in the last panel of Figure 2.

The unique equilibrium is the one with full information and no trade. The corner with no information is not an equilibrium because it generates a lower individual utility for agents. However, if agents internalize the effect of trading in their utility, they would rather coordinate (from a social perspective) in such situation. This is represented by a square in the figure. This shows that agents may fail to coordinate on the socially preferred equilibrium. As the planner internalizes the role of trading in welfare and would optimally increase trading, it values opacity even further. This is represented by a rhombus in the figure. Can an interior solution, with a combination of transparent and opaque technologies, be socially preferred to full information? The intermediate panel in Figure 2, for instance, displays less leakages than the last panel and shows (again with a square) that the interior equilibrium is socially preferred to the full information equilibrium.

Figure 3 summarizes the Proposition, showing which extreme information regime would be individually, socially and planning preferred.

Figure 3: Individual, Social and Planning preferred information (for given $\chi(\sigma)$



A note on the role of risk aversion. In figure 4 we show the market solution in blue (the equilibrium allocation solid and the socially preferred allocation in dotted) and the constrained planner's solution in black, for different levels of risk aversion σ . The first row corresponds to the technological choices and the second to trading

⁹Notice that in this case the optimal planning allocation also displays an interior level of transparency, but less than the market equilibrium, as the planner would rather exploit trading more.

choices. The figure illustrates the conditions under which these three welfare evaluations may depart. First, when the two blue lines are different, it implies that there is a coordination failure, under which the equilibrium is inferior from a social point of view. When this happens it would be socially preferred to coordinate in the equilibrium with more opacity and trading, for instance by a planner that mandates a higher trading level so agents choose information technologies that indeed maximize social welfare. This divergence tends to happen when agents are more risk averse (higher σ) and when leakages are stronger. Second, when the black and the dotted blue lines are different, the social planner would like to correct the externalities by mandating even more trading, so agents choose even less transparency. This divergence tends to happen for intermediate levels of risk aversion. The reason is that, as σ goes to infinity, the socially preferred market equilibrium also displays lots of trading and little transparency.

A note on implementation with a financial intermediary. A competitive (zero-profit) mutual fund could induce the coordination that sellers cannot achieve in a decentralized market and allow them to reach the constrained socially optimal allocations. Each agent "invests" in the mutual fund $1 - \beta_i$ units of raw capital, the mutual fund pools all the raw capital, and produces intermediate capital subject to identical adjustment costs. Given perfect diversification, the agent receives back $\bar{\theta}(1-\beta_i)-\frac{\varphi}{2}(1-\beta_i)^2$. This implies that the agents' return in terms of expected intermediate capital is lower in expectation, but deterministic. A mutual fund effectively sells insurance at a "fee", $\frac{\varphi}{2}(1-\beta_i)^2$, thereby turning an agent's expected amount of intermediate capital, into $E[k_i] = \bar{\theta} - \frac{\varphi}{2}(1-\beta)^2$, which makes the agent's objective function mathematically identical to that of the constrained social planner; thus, agents optimally contribute to the mutual fund the socially optimal amount. Being that all agents contribute the same amount $1-\beta$, the mutual fund produces in total $\bar{\theta}(1-\beta) - \frac{\varphi}{2}(1-\beta)^2$, which is what it repays to investors, making zero profits. This potential implementation suggests the importance of financial intermediation in increasing the supply of "safe assets" in the economy, for instance, by securitization which indeed follows the logic of an originator pooling assets with idiosyncratic

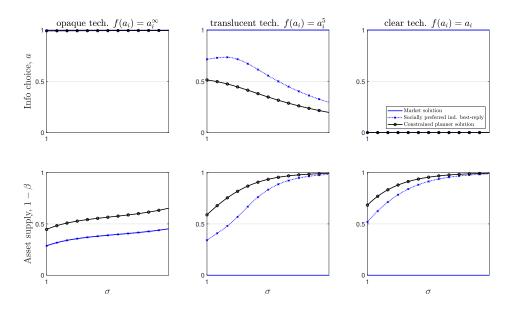


Figure 4: Information Social (Un)desirability

Notes: The picture contrasts the information (first raw) and supply (second raw) choices of individual (simple solid line) and constrained planner (solid line with circle markers) for different values of risk aversion (x-axis) and degree of technological transparency (columns). The last raw shows welfare featured by the preferred planner solution measured from the one obtained at the full-info-full-market corner allocation (solid line with diamond markers) and the individual ex-ante utility at the preferred planner solution measured from the one obtained with an individual full-info-full-market choice. The figure is generated for: $\alpha = 0.6, \gamma = 1.9, \varphi = 1, \bar{\theta} = 10$.

quality and, at a cost, generating a "new asset" of lower variance, as discussed in Gorton and Ordoñez (2020b).

A note on implementation with subsidies. Which tax scheme could a government use to implement the constrained socially optimal supply of raw capital? As we noted, agents' failure to internalize the positive effect of supplying raw capital for other agents' insurance stems from market prices' under-compensation. A government could therefore subsidize the sale of raw capital by an amount $s(\beta_i)$; financing the subsidies with lump-sum taxes T, in terms of intermediate capital. Given this subsidy scheme, the expected amount of intermediate capital becomes

 $E[k_i] = \bar{\theta} - \varphi(1-\beta_i)^2 + \frac{\varphi}{2}(1-\beta)^2 + s(\beta_i) - T$ and the socially optimal supply of raw capital can be implemented by setting $s(\beta_i) = \frac{\varphi}{2}(1-\beta_i)^2$. Note this scheme does not require information on productivity, just on actual supply. In the previously mentioned example of asset backed securities, policymakers should thus not only tax information, to encourage the origination of these safe assets (encourage the trade of certain assets that are used as inputs of private safe assets, such as mortgages for MBS, or bonds for CDOs), but also to subsidize such trading.

A note on the role of increasing returns in the production of capital:

Even though the previous results are mostly based on a set of standard functionalform assumptions in macroeconomics and finance, we have also resorted to specifications that enhanced tractability and expositional clarity. First, the production function of capital is special: exponential on intermediate capital, which implies that when $\sigma < 1$ individuals are risk lovers on intermediate capital (even though being risk averse on consumption goods). Second, the production function of intermediate goods is also special: linear in the productivity of raw capital.

A potentially unattractive implication of combining these two features is that the unconditional distribution of capital is not mean invariant (expected capital production is not the same as the capital production of expected intermediate capital), that is

$$E[\hat{K}_i(\theta_i)] = e^{E[k_i] + \frac{1}{2}V(k_i)} \neq \hat{K}_i(E[\theta_i]) = e^{E[k_i]}.$$

This means that the expected capital available to produce consumption goods increases with the variance of intermediate capital and always exceeds the capital obtained by using the average amount of intermediate capital.

One may wonder to which extent our result about the social undesirability of free and perfect public information could be an artifact of these assumptions. In fact, it is the opposite. The exponential shape of capital production function implies that average production increases with variance, and as information induces more variance by discouraging market-insurance, information is more, not less, desirable. Intuitively, when public information is available prices are volatile. On the one hand, the uncertain amount of capital to produce generates utility losses from consumption

uncertainty: the negative face of information. On the other hand, the uncertain amount of capital to produce generates an increase in expected consumption, and utility gains. This gain from intermediate capital variance is purely mechanical when compared to the more relevant conceptual gain of information that comes from correlating labor to productivity shocks: our positive face of information. Thus, our functional forms overestimate the social benefits of information.

A note on the effects of data processing improvements: In this paper we have focused on how information technologies affects what agents know, both about own and others' raw capital. Data-intensive technologies, however, may also affect the mapping between information and other fundamental parameters. Those additional effects would change the conditions under which information is socially undesirable, but not the fundamental forces. Here we highlight three possibilities.

- i) Data improvements induce transparency (a) to raise productivity $\bar{\theta}$: Data-intensive technologies have been modelled in the literature as increasing productivity though a better allocation of resources. We are already capturing this effect by the better allocation of labor given information about the amount of available intermediate capital. Data-intensive technologies may induce information to raise the amount of intermediate capital directly as well. This extra benefit of information would increase $E(k_i)$ and make information more desirable as data processing improves.
- ii) Data improvements induce transparency (a) to raise leakages f(a): If dataintensive technologies improve the ability of an agent to infer the information available to other agents can be captured by comparative statics on f(a). As we discussed when comparing across columns of Figure 4, more leakages may maintain full-information as an equilibrium, but such equilibrium may become more and more inefficient, as leakages make trading (and then opacity) more desirable.
- ii) Data improvements induce transparency (a) to reduce adjustment costs φ : Data can also help trading and using others assets more effectively, reducing the adjustment costs involved in trading strategies. In this case, data processing technologies would make information more useful to exploit market-insurance, but also makes market-insurance relatively more useful than self-insurance, and then there is

a force that leans towards more opacity. The final result would depend on how much transparency makes trading cheaper but at the same time less useful.

4.3 Unconstrained Social Optimum

Now, we study a planner that seeks to maximize the ex-ante utility of a representative agent, and can freely redistribute intermediate capital (it is not restricted by decentralized markets compensations). Individuals could implement this allocation if they were able to write ex-ante contracts which specify transfers of intermediate capital contingent on productivity realizations.

We have already established that the unconstrained optimal is achieved with a transparent technology that does not leak information to the public, this is f(a) = 0 for all a. In this case it is optimal to only use a transparent technology, obtaining both self- and market-insurance. For this reason we focus on the other extreme, in which the transparent technology always leak information to the public, this is f(a) = a for all a. Even in this extreme, we will show that the unconstrained planner still prefers to use only transparent technologies.

We assume the planner can choose both the proportion of in-house production of intermediate capital β_i and the exchange of intermediate capital after production τ_i . Given that, in this benchmark, the planner's hands are not tied by market compensations, her problem becomes,

$$\max_{\{\beta_i(h),\tau_i\}_{(i,h)\in(0,1)^2}} E[\mathbb{U}(K_i(\theta_i))] = \Phi E[K_i(\theta_i)^{\phi}]$$

subject to

$$k_{i} = (\bar{\theta} + \theta_{i})\beta_{i} + \tau_{i} + \int_{H(i)} \left[(\bar{\theta} + \theta_{h})\beta_{i}(h) - \frac{\varphi}{2}\beta_{i}^{2}(h) \right] dh,$$

$$\int \tau_{i} di = 0,$$

$$1 - \beta_{i} = \int_{H(i)} \beta_{h}(i) dh$$

In other words, the planner maximizes ex-ante utility by controlling the production of intermediate capital, through β_i , and its distribution, through τ_i .

Proposition 4 (Unconstrained planner's solution). The unconstrained planner allocation is characterized by no-trade in raw capital (that is $\beta_i = 1$ for all i) and redistribution of intermediate capital as follows,

$$\tau_i = \begin{cases} 0 & \text{if } \sigma < 1 \\ -\theta_i & \text{if } \sigma \ge 1 \end{cases}$$

Proof. Postponed to Appendix A.4.

Intuitively, an unconstrained planner wants to employ raw capital where it is most productive - with the original owners who don't face adjustment costs - and, having maximized aggregate intermediate capital, go on to achieve perfect insurance, when market-insurance is desired ($\sigma > 1$), by equalizing allocations via redistribution. Further, in stark contrast to the market, which allocates more intermediate capital to the agents with higher productivity (regressive redistribution), the unconstrained planner allocates more intermediate capital to agents with lower raw capital productivity (progressive redistribution).

For the unconstrained planner, it is always optimal to operate transparent technologies, as this allows her to make transfers contingent on productivity (more transfers to less productive agents when $\sigma > 1$) which equalize labor efforts and consumption. In other words, when the planner is not constrained to redistribute, information is unequivocally beneficial as the planner will use it to both increase production and equalize consumption. This is not the case in equilibrium because the market uses information in a way that increases production but prevents risk sharing; indeed, when the planner is constrained by the limitations imposed by the market there are situations in which she would prefer opaque technologies (Proposition ??).

A note on an implementation by a government. With incomplete markets, a government could implement the planner's desired allocation by imposing taxes and

subsidies that achieved zero-trade along with redistribution as per $\tau_i(\theta_i)$. Naturally, the feasibility of such transfers would critically depend on observability, pledgeability, and verifiability of raw capital productivity by the government. This result stresses once more an important assumption of the standard view that information is important for insurance to work properly by facilitating the fulfillment of contingent contracts.¹⁰

5 Final remarks

What is the social value of public, costless, and perfect information about agents' idiosyncratic shocks? An immediate intuition suggests that such information is always socially desirable. We show that in an economy with restrictions for individuals to share risks, the role of information is more nuanced. It has a positive face, by permitting self-insurance, as it improves how agents reallocate their resources (labor, for instance) to face idiosyncratic shocks that affect their consumption. It also has a negative face however, by constraining market-insurance, as it weakens how agents can trade resources (selling volatile assets and buying safe ones) to reduce their exposure to idiosyncratic shocks. We show that this trade off between ex-post optimal labor allocation and ex-ante creation of safe assets makes public information socially desirable only if welfare reacts more to self-insurance than to market-insurance.

When is self-insurance superior? This is the case when consumption depends heavily on resources that can be cheaply adjusted upon idiosyncratic shocks. When is market-insurance superior? This is the case when consumption is heavily exposed to idiosyncratic shocks that can be hedged by buying safe assets that can be cheaply originated and traded. While information is always desirable in the presence of insurance markets, it may be undesirable in their absence, as it improves one insurance alternative at the expense of the other.

¹⁰Notice that the optimal set of taxes and subsidies (*public insurance*) eliminates the need of (*private insurance*), an extreme version of a rich literature that claims that public insurance may crowd out private insurance (such as Golosov and Tsyvinski (2007), Krueger and Perri (2011) and Park (2014)).

This insight shows that a reduction in the cost of originating and trading safe assets should (optimally) be accompanied with steps that discourage the availability of information about idiosyncratic shocks. This is in stark contrast with the information disclosure implication that arises when insurance markets are complete, in which case it would be better to encourage information if it is free, public, and perfect. The application of this insight is relevant, for instance, in the discussion about the design of financial regulations, or the disclosure of information about lending programs.

The trade-off we explore in general equilibrium can also be applied in partial equilibrium to inform recent regulatory reforms. Take the case of banking stress tests, for instance. When regulators reveal to a bank results about stress scenarios, they reveal pieces of information (mostly about sources of systemic risk) that are useful for the bank to rebalance its own portfolio (the positive face of improving self-insurance). Those pieces of information, however, also become available to other banks, who may revise their own beliefs about the bank's individual portfolio and its market valuation, introducing additional volatility and inhibiting the functioning of interbank markets (the negative face of weakening market-insurance). This trade-off is critical in designing information disclosure of stress tests once regulators weight the relevance of portfolio rebalancing vs. interbank market operations.

References

- Amador, Manuel, and Pierre-Olivier Weill. 2010. "Learning from Prices: Public Communication and Welfare." *Journal of Political Economy* 118 (5): 866–907.
- Andolfatto, David, Aleksander Berentsen, and Christopher Waller. 2014. "Optimal disclosure policy and undue diligence." *Journal of Economic Theory* 149:128 152. Financial Economics.
- Angeletos, George-Marios, Luigi Iovino, and Jennifer La'O. 2016. "Real Rigidity, Nominal Rigidity, and the Social Value of Information." *American Economic Review* 106 (1): 200–227 (January).
- Angeletos, George-Marios, and Alessandro Pavan. 2007. "Efficient Use of Information and Social Value of Information." *Econometrica* 75:1103–1142.
- Baeriswyl, Romain, and Camille Cornand. 2010. "The signaling role of policy actions." *Journal of Monetary Economics* 57 (6): 682–695.
- Bai, Jennie, Thomas Philippon, and Alexi Savov. 2016. "Have financial markets become more informative?" *Journal of Financial Economics* 122:625–654.
- Bester, Helmut. 1985. "Screening vs. Rationing in Credit Markets with Imperfect Information." American Economic Review 71:393–410.
- Brynjolfsson, Erik, Wang Jin, and Kristina McElheran. 2021. "The Power of Prediction: Predictive Analytics, Workplace Complements and Business Performance." Working Paper, Stanford University.
- Brynjolfsson, Erik, and Kristina McElheran. 2016. "Data in Action: Data-Driven Decision Making in U.S. Manufacturing." Working Paper, Stanford University.
- Caramp, Nicolas. 2017. "Sowing the Seeds of Financial Crises: Endogenous Asset Creation and Adverse Selection." Working Paper, MIT.
- Chousakos, Kyriakos, Gary Gorton, and Guillermo Ordoñez. 2020. "The Macroprudential Role of Stock Markets." NBER Working Paper 27113.
- Colombo, Luca, Gianluca Femminis, and Alessandro Pavan. 2014. "Information Acquisition and Welfare." Review of Economic Studies 81:1438–1483.
- Dang, Tri Vi, Gary Gorton, Bengt Holmström, and Guillermo Ordoñez. 2017. "Banks as Secret Keepers." American Economic Review 107:1005–1029.
- Davila, Eduardo, and Cecilia Parlatore. 2022. "Identifying Price Informativeness." Working Paper, Yale University.

- Diamond, Douglas. 1984. "Financial Intermediation and Delegated Monitoring." Review of Economic Studies 51:393–414.
- ——. 1991. "Monitoring and Reputation: The Choice between Bank Loans and Directly Placed Debt." *Journal of Political Economy* 99:689–721.
- Eckwert, Bernhard, and Itzhak Zilcha. 2001. "The Value of Information in Production Economies." *Journal of Economic Theory* 100:172–186.
- Farboodi, Maryam, Adrien Matray, Laura Veldkamp, and Venky Venkateswaran. 2022. "Where has all the data gone?" Review of Financial Studies, forthcoming.
- Gaballo, Gaetano. 2016. "Rational Inattention to News: The Perils of Forward Guidance." American Economic Journal: Macroeconomics 8:42–97.
- Golosov, Mihkail, and Aleh Tsyvinski. 2007. "Optimal Taxation with Endogenous Insurance Markets." Quarterly Journal of Economics 122:487–534.
- Golosov, Mikhail, and Luigi Iovino. 2020. "Social Insurance, Information Revelation, and Lack of Commitment." Working Paper, University of Chicago.
- Gorton, Gary, and Guillermo Ordoñez. 2020a. "Fighting Crises with Secrecy." *American Economic Journal: Macroeconomics*, vol. Forthcoming.
- ——. 2020b. "The Supply and Demand for Safe Assets." NBER Working Paper 18732.
- Gorton, Gary, and George Pennacchi. 1990. "Financial Intermediaries and Liquidity Creation." *Journal of Finance* 45 (1): 49–72.
- Gottardi, Piero, and Rohit Rahi. 2014. "Value Of Information In Competitive Economies With Incomplete Markets." *International Economic Review* 55 (February): 57–81.
- Hellwig, Christian. 2005. "Heterogeneous Information and the Welfare Effects of Public Information Disclosures." Working Paper, UCLA.
- Hirshleifer, Jack. 1971. "The Private and Social Value of Information and the Reward to Inventive Activity." *American Economic Review* 61 (4): 561–674.
- Krueger, Dirk, and Fabrizio Perri. 2011. "Public versus Private Risk Sharing." Journal of Economic Theory 146:920–956.
- Kurlat, Pablo, and Laura Veldkamp. 2015. "Should We Regulate Financial Information?" *Journal of Economic Theory* 158:697–720.
- Leland, Hayne, and David Pyle. 1977. "Informational Asymmetries, Financial Structure, and Financial Intermediation." *Journal of Finance* 31:371–387.

- Llosa, Luis Gonzalo, and Venky Venkateswaran. 2017. "Efficiency with Endogenous Information Choice." Working Paper, NYU Stern.
- Lorenzoni, Guido. 2010. "Optimal Monetary Policy with Uncertain Fundamentals and Dispersed Information." Review of Economic Studies 77 (1): 305–338.
- Morris, Stephen, and Hyun Song Shin. 2002. "Social Value of Public Information." American Economic Review 92 (5): 1521–1534.
- Nosal, Jaromir, and Guillermo Ordoñez. 2016. "Uncertainty as Commitment." Journal of Monetary Economics 80:124–140.
- Park, Yena. 2014. "Optimal Taxation in a Limited Commitment Economy." Review of Economic Studies 81:884–918.
- Roca, Mauro F. 2010, April. "Transparency and Monetary Policy with Imperfect Common Knowledge." Imf working papers 10/91, International Monetary Fund.
- Schlee, Edward. 2001. "The Value of Information in Efficient Risk-Sharing Arrangements." *American Economic Review* 91:509–524.
- Vanasco, Victoria. 2017. "The Downside of Asset Screening for Market Liquidit." Journal of Finance 72:1937–1981.
- Walsh, Carl E. 2007. "Optimal Economic Transparency." International Journal of Central Banking 3 (1): 5–36.

A Proofs

A.1 Proof of Lemma 2 and Corollary 1

Proof. Agent h maximizes $\mathbb{U}(E[K_h(\beta_h(i))]) = \Phi E[K_h(\beta_h(i))]^{\phi}$ choosing $\beta_h(i)$ such that, for all i,

$$\Phi\phi\left((1-\alpha)(1-\sigma)\frac{\partial E[\int \Pi_h(i)di]}{\partial \beta_h(i)} + \frac{1}{2}(1-\alpha)^2(1-\sigma)^2\frac{\partial V(\int \Pi_h(i)di)}{\partial \beta_h(i)}\right)e^{\phi E[k_h] + \frac{\phi}{2}V(k_h)} = 0$$

Since portfolio returns enter exponentially in the utility function, constant absolute risk aversion obtains, and the optimal individual asset demand is invariant in the expected value of the rest of the portfolio (total amount of intermediate capital) - as in standard CARA asset pricing models.

In what follows we solve for the profit-maximizing demand of raw capital and then show that it is also the utility-maximizing demand of raw capital satisfying the first order condition above. Suppose instead agent h chooses the quantity $\beta_h(i)$ of raw capital to demand from agent i to maximize her expected profits; then, an interior $\beta_h(i)$ demand (required by $\beta_i \in (0,1)$ and agent homogeneity) must satisfy,

$$\frac{\partial E_h[\Pi_h(i)]}{\partial \beta_h(i)} = \frac{\partial E_h\left[\left(\bar{\theta} + \theta_i\right)\beta_h(i) - \frac{\varphi}{2}\beta_h^2(i) - R_i\beta_h(i)\right]}{\partial \beta_h(i)} = 0$$

$$\implies \beta_h^*(i) = \frac{\bar{\theta} + E_h[\theta_i] - R_i}{\varphi} \tag{27}$$

If the supply of agent i's raw capital is $1 - \beta_i$, market clearing implies,

$$\int_{H(i)} \beta_h^*(i)dh = 1 - \beta_i,$$

and the equilibrium price in the market of agent i's raw capital would be

$$R_i = \bar{\theta} + E_h[\theta_i] - \varphi(1 - \beta_i). \tag{28}$$

Equations (27) and (28) are the same as in Lemma 2, but for agent i. Since all agents have identical information about agent's i raw capital, $E_h[\theta_i]$ is the same for all h. The actual profit of agent h as a buyer of agent i's raw capital can then be rewritten as

$$\Pi_h(i) = (\theta_i - E_h[\theta_i])(1 - \beta_i) + \frac{\varphi}{2}(1 - \beta_i)^2.$$

If agent h is informed, $E_h[\theta_i] = \theta_i$; meanwhile, if not, $E_h[\theta_i] = E[\theta_i] = 0$. By a law of large numbers with a continuum of iid random variables $\int_{(0,1)} E_h[\theta_i] di =$

 $\int_{(0,1)} \theta_i di = 0$ almost surely. As such, as stated in Corollary 1, aggregate portfolio profits,

$$\int_{H(i)} \Pi_h(i)di = \frac{\varphi}{2} \int_{H(i)} (1 - \beta_i)^2 di$$

are deterministic, agents attain perfect diversification, and (since this quantity is strictly positive) agent's total demand for raw capital can be "self-financed".

Now, we prove the conjecture that profit-maximizing demand is the same as utility-maximizing demand. Since portfolio profits are deterministic, $V\left(\int_{H(i)} \Pi_h(i) di\right) = 0$ and,

$$\frac{\partial V(\int_{H(i)} \Pi_h(i)di)}{\partial \beta_h(i)} = 2E \left[\frac{\partial \Pi_h(i)}{\partial \beta_h(i)} \left(\int_{H(i)} \Pi_h(i)di - E \left[\int_{H(i)} \Pi_h(i)di \right] \right) \right] = 0$$

which shows, jointly with (27), that $\beta_h^*(i)$ also satisfies utility-maximizing first-order conditions.

Finally, the expression for the quantity of intermediate capital available to agents at the end of the period, as stated in equation (13) in Corollary 1, comes from substituting the price received from selling raw capital (equation (28)) and the profits from buying raw capital (equation (12)) into equation (8). \Box

A.2 Proof of Lemma 3

Using (1),(2) and (6), we can write the unconditional expected utility as

$$E[\mathbb{U}(E_i[K_i])] \equiv E\left[\frac{K_i E_i[K_i]^{\frac{\phi}{\gamma}\alpha(1-\sigma)}}{\alpha(1-\sigma)} - \frac{1}{\gamma}E_i[K_i]^{\phi}\right].$$

We first note that

$$\frac{\phi}{\gamma}\alpha(1-\sigma) = \phi - 1.$$

¹¹Sun, Yeneng and Yongchao Zhang (2009), "Individual risk and Lebesgue extension without aggregate uncertainty", *Journal of Economic Theory* 144, 432-443.

Let us denote $\tilde{\kappa} = (1 - \alpha)(1 - \sigma)(k_i - E[k_i])$, thus, we can write:

$$E[\mathbb{U}(E_{i}[K_{i}])] = E\left[\frac{e^{\tilde{\kappa}}e^{(\phi-1)\left(E_{i}[\tilde{\kappa}]+\frac{1}{2}V_{i}(\tilde{\kappa})\right)}}{\alpha(1-\sigma)} - \frac{1}{\gamma}e^{\phi\left(E_{i}[\tilde{\kappa}]+\frac{1}{2}V_{i}(\tilde{\kappa})\right)}\right]\bar{K}$$

$$= E\left[\frac{e^{\tilde{\kappa}-E_{i}[\tilde{\kappa}]+\phi E_{i}[\tilde{\kappa}]+\frac{\phi-1}{2}V_{i}(\tilde{\kappa})}}{\alpha(1-\sigma)}\right]\bar{K} - E\left[\frac{1}{\gamma}e^{\phi\left(E_{i}[\tilde{\kappa}]+\frac{1}{2}V_{i}(\tilde{\kappa})\right)}\right]\bar{K}$$

$$= \left(\frac{e^{\frac{1}{2}V_{i}(\tilde{\kappa})+\frac{\phi^{2}}{2}V(E_{i}[\tilde{\kappa}])+\frac{\phi-1}{2}V_{i}(\tilde{\kappa})}}{\alpha(1-\sigma)} - \frac{1}{\gamma}e^{\frac{\phi^{2}}{2}V(E_{i}[\tilde{\kappa}])+\frac{\phi}{2}V_{i}(\tilde{\kappa})}\right)\bar{K}$$

$$= \Phi e^{\frac{\phi^{2}}{2}V(E_{i}[\tilde{\kappa}])+\frac{\phi}{2}V_{i}(\tilde{\kappa})}\bar{K}$$

where

$$\Phi \equiv \frac{\gamma - \alpha(1 - \sigma)}{\gamma \alpha(1 - \sigma)},$$

which is positive for $\sigma < 1$ and negative for $\sigma > 1$,

$$\bar{K} = \alpha e^{\phi(1-\alpha)(1-\sigma)(\bar{\theta}-\varphi(1-\beta_i)^2 + \frac{\varphi}{2}(1-\beta)^2)}$$

gathering all deterministic factors, and $V(E_i[\tilde{\kappa}])$ and $V_i(\tilde{\kappa})$ being the volatility of the conditional mean and the conditional volatility, of $\tilde{\kappa}$ respectively. Using (5), these last two terms obtain as:

$$V(E_{i}[\tilde{\kappa}]) = (1 - \alpha)^{2} (1 - \sigma)^{2} V(E_{i}[k_{i}])$$

$$= (1 - \alpha)^{2} (1 - \sigma)^{2} V\left(\beta_{i}\theta_{i} + (1 - \beta_{i})\sqrt{f(a_{i})}\eta_{c,i}\right)$$

$$= (1 - \alpha)^{2} (1 - \sigma)^{2} \left(\beta_{i}^{2}a_{i} + (1 - \beta_{i})^{2}f(a_{i}) + 2\beta_{i}(1 - \beta_{i})f(a_{i})\right)$$

$$= (1 - \alpha)^{2} (1 - \sigma)^{2} \left(\beta_{i}^{2} (a_{i} - f(a_{i})) + f(a_{i})\right),$$

and

$$V_{i}(\tilde{\kappa}) = (1 - \alpha)^{2} (1 - \sigma)^{2} V_{i}(k_{i})$$

$$= (1 - \alpha)^{2} (1 - \sigma)^{2} V_{i} \left(\beta_{i} \theta_{i} + (1 - \beta_{i}) \sqrt{f(a_{i})} \eta_{c,i}\right)$$

$$= (1 - \alpha)^{2} (1 - \sigma)^{2} \beta_{i}^{2} (1 - a_{i}).$$

A.3 Proof of Proposition 2

To solve the two benchmarks from an individual perspective, let us fix others supply choices to $\{\beta_i\}_{i\neq i}$ and define

$$\hat{\Phi} = \Phi e^{(1-\alpha)(1-\sigma)\phi \frac{1}{2} \int (1-\beta_j)^2 dj}$$

With full-information, there is never trade $\beta_i^*(a_i = 1) = 1$ from equation (18). Further, given that $V_i(k_i) = 1$, according to Lemma (3) we have,

$$\hat{\Phi}E[K_i(\theta_i)^{\phi}] = \hat{\Phi}\alpha e^{(1-\alpha)(1-\sigma)\phi\bar{\theta} + \frac{1}{2}((1-\alpha)(1-\sigma))^2\phi^2} = \\
= \hat{\Phi}\alpha e^{(1-\alpha)(1-\sigma)\phi\bar{\theta}}E[e^{(1-\alpha)(1-\sigma)\theta_i}]^{\phi^2}$$

With no information, trade is possible, as characterized by $\beta_i^*(a_i = 0)$ from equation (18). In this case, $V_i(k_i) = \beta_i^{*,2}$, according to Lemma (3) we have,

$$\hat{\Phi}E[K_{i}(\theta_{i})]^{\phi} = \hat{\Phi}\alpha e^{(1-\alpha)(1-\sigma)\phi(\bar{\theta}-\varphi(1-\beta_{i}^{*})^{2})+\frac{1}{2}((1-\alpha)(1-\sigma))^{2}\phi\beta_{i}^{*,2}} =
= \hat{\Phi}\alpha e^{(1-\alpha)(1-\sigma)\phi\bar{\theta}}e^{\frac{1}{2}(1-\alpha)^{2}(1-\sigma)^{2}\phi(\beta_{i}^{*,2}-\frac{2\varphi}{(1-\alpha)(1-\sigma)}(1-\beta_{i}^{*})^{2})}
= \hat{\Phi}\alpha e^{(1-\alpha)(1-\sigma)\phi\bar{\theta}}E[e^{(1-\alpha)(1-\sigma)\theta_{i}}]^{\phi(\beta_{i}^{*,2}-\frac{2\varphi}{(1-\alpha)(1-\sigma)}(1-\beta_{i}^{*})^{2})}$$

where we define

$$\beta^{I} \equiv \beta_{i}^{*,2} - \frac{2\varphi}{(1-\alpha)(1-\sigma)} (1-\beta_{i}^{*})^{2} = \beta_{i}^{*,2} + A(1-\beta_{i}^{*})^{2}$$

where $A = \frac{2\varphi}{(1-\alpha)(\sigma-1)}$ and $\beta_i^*(a_i = 0) = \frac{A}{1+A}$, from Lemma 4. Then,

$$\beta^{I} = \frac{1}{1 - \frac{1 - \alpha}{\varphi} (1 - \sigma) \chi^{I}}$$

where $\chi^I = \frac{1}{2}$.

From a social point of view, in the symmetric equilibrium, $\int (1-\beta_j)^2 dj = (1-\beta_i^*)^2$. Then

$$\hat{\Phi} = \Phi e^{(1-\alpha)(1-\sigma)\phi \frac{1}{2}(1-\beta_i^*)^2}$$

With full-information, as $\beta_{i,FI}^* = 1$, we have that $\hat{\Phi} = \Phi$ and,

$$\Phi E[K_i(\theta_i)^{\phi}] = \Phi \alpha e^{(1-\alpha)(1-\sigma)\phi\bar{\theta} + \frac{1}{2}((1-\alpha)(1-\sigma))^2\phi^2} = \\
= \Phi \alpha e^{(1-\alpha)(1-\sigma)\phi\bar{\theta}} E[e^{(1-\alpha)(1-\sigma)\theta_i}]^{\phi^2}$$

With no-information, given β_i^* ,

$$\Phi E[K_{i}(\theta_{i})]^{\phi} = \Phi \alpha e^{(1-\alpha)(1-\sigma)\phi(\bar{\theta}-\frac{\varphi}{2}(1-\beta_{i}^{*})^{2})+\frac{1}{2}((1-\alpha)(1-\sigma))^{2}\phi\beta_{i}^{*,2}} =
= \Phi \alpha e^{(1-\alpha)(1-\sigma)\phi\bar{\theta}} e^{\frac{1}{2}(1-\alpha)^{2}(1-\sigma)^{2}\phi(\beta_{i}^{*,2}-\frac{\varphi}{(1-\alpha)(1-\sigma)}(1-\beta_{i}^{*})^{2})}
= \Phi \alpha e^{(1-\alpha)(1-\sigma)\phi\bar{\theta}} E[e^{(1-\alpha)(1-\sigma)\theta_{i}}]^{\phi(\beta_{i}^{*,2}-\frac{\varphi}{(1-\alpha)(1-\sigma)}(1-\beta_{i}^{*})^{2})}$$

where we define

$$\beta^{S} \equiv \beta_{i}^{*,2} - \frac{\varphi}{(1-\alpha)(1-\sigma)} (1-\beta_{i}^{*})^{2} = \beta_{i}^{*,2} + \frac{A}{2} (1-\beta_{i}^{*})^{2}$$

where $A = \frac{2\varphi}{(1-\alpha)(\sigma-1)}$ and $\beta_i^*(a_i = 0) = \frac{A}{1+A}$, from Lemma 4. Then,

$$\beta^S = \frac{1}{1 - \frac{1 - \alpha}{\sigma} (1 - \sigma) \chi^S}$$

where $\chi^S = \frac{3\varphi + (1-\alpha)(\sigma-1)}{4\varphi + (1-\alpha)(\sigma-1)}$. Finally, from the planner's perspective it also takes into account this is a symmetric situation, so the computation is identical to the social perspective analysis, but replacing β_i^* by β^* . Then we define

$$\beta^P \equiv \beta^{*,2} - \frac{\varphi}{(1-\alpha)(1-\sigma)} (1-\beta^*)^2 = \beta^{*,2} + A_P (1-\beta^*)^2$$

where $\beta^*(a=0) = \frac{A_P}{1+A_P}$ from equation (21) and $A_P = \frac{\varphi}{(1-\alpha)(\sigma-1)}$ from equation (22). Then,

$$\beta^P = \frac{1}{1 - \frac{1 - \alpha}{\varphi} (1 - \sigma) \chi^S}$$

where $\chi^S = 1$.

Proof of Proposition 4

Proof. The problem of the unconstrained planner is

$$\max_{\{\hat{\beta}_i(h),\tau_i\}_{(i,h)\in(0,1)^2}} E[\mathbb{U}(K_i(\theta_i))] = \Phi \alpha e^{(1-\alpha)(1-\sigma)\phi E[k_i] + \frac{1}{2}((1-\alpha)(1-\sigma)\phi)^2 V(k_i)}$$
(29)

where

$$k_i = \bar{\theta} + \beta_i \theta_i - \tau_i + \int_{H(i)} \beta_i(h) \theta_h \, dh - \int_{H(i)} \frac{\varphi}{2} \beta_i^2(h) \, dh \,,$$

subject to the resource and balance-budget constraints,

$$1 - \beta_i = \int_{H(i)} \beta_h(i) dh$$
$$0 = \int \tau_i di$$

The first observation is that necessarily in any equilibrium $1 - \beta_i = \beta_h(i) = \beta_j(i)$ for any $h, j \in H(i)$. If this condition were violated, let us say $\beta_h(i) < \beta_j(i)$, the planner could save on quadratic costs without loosing on expected production by moving raw capital type i from agent j to agent h. The result of this observation is that by a law of large numbers result, as in the proof of Proposition ??, and using the constraints,

$$E[k_i] = \bar{\theta} + \int \beta_i \theta_i \, di - \int \tau_i di - \int \int_{H(i)} (1 - \beta_h) \theta_h \, dh \, di - \frac{\varphi}{2} \int \int_{H(i)} (1 - \beta_h)^2 \, dh \, di$$

$$= \bar{\theta} - \frac{\varphi}{2} \int (1 - \beta_i)^2 \, di \,,$$

$$V(k_i) = \int \left(\beta_i \theta_i - \tau_i - \int \beta_i \theta_i \, di\right)^2 \, di.$$

where we used $E[\theta_i] = \int \theta_i di = 0$ and

$$\int_{H(i)} (1 - \beta_h) \theta_h \, dh = \int (1 - \beta_h) \theta_h \, dh \text{ and } \int_{H(i)} (1 - \beta_h)^2 \, dh = \int (1 - \beta_h)^2 \, dh$$

As such,

$$\frac{\partial E[k_i]}{\partial \beta_i} = \varphi(1 - \beta_i)
\frac{\partial V(k_i)}{\partial \beta_i} = 2\theta_i \left(\beta_i \theta_i - \tau_i - \int \beta_i \theta_i di\right)
\frac{\partial V(k_i)}{\partial \tau_i} = 2\left(\beta_i \theta_i - \tau_i - \int \beta_i \theta_i di\right)$$

all of which are equal to zero at $\beta_i = 1, \tau_i = -\theta_i$ and therefore imply that all the necessary first order conditions of problem (29) (factoring in the constraints) for optimality are also satisfied.